A 2-D FLOW MODEL FOR ALLUVIAL WATERCOURSES

Zoltan Horvat[1]

UDK: 532.5:627.1

DOI: 10.14415/konferencijaGFS2014.089

Summary: *This work presents the development and calibration of a two dimensional flow model for alluvial watercourses. The governing depth-averaged flow equations are given in orthogonal curvilinear coordinates and solved using the fractional step method, which resulted in three successive steps (advection, diffusion and propagation). The numerically most challenging advection step, that can produce numerical oscillations and damping, was solved utilizing the method of characteristics, while for the other two steps finite difference schemes were employed combined with the ADI method. The developed model is assessed using field measurements conducted on the reach of the Danube River located in the border area between Hungary and Serbia. Analysis of the measured and computed velocities confirmed the developed model's reliability.*

Keywords: *Numerical model, flow equations, field measurements*

1. INTRODUCTION

The development of a stable and accurate numerical model for water flow in alluvial watercourses is essential when computing transport processes in rivers, thus making it one of the most significant fields in hydraulic engineering. Two dimensional depth-averaged hydraulic models are widely used [1,2,3], since they can produce more detailed results than 1-D models, and much faster than 3-D models.

The split-operator approach, when applied to river modeling, typically results in a three-step solution, with separate advection, diffusion and propagation step [4]. Using this method in water flow simulations provides the possibility to treat each of the obtained steps with the most appropriate numerical method. The objective of this paper is to present a developed two dimensional depth averaged model and its implementation and assessment using field measurements.

2. MODEL FORMULATION

After deriving the full set of the two dimensional Reynolds-averaged Navier-Stokes equations in orthogonal curvilinear coordinate system, the obtained equations were depth

[1] Zoltan Horvat, dipl.inž. građ., University of Novi Sad, Faculty of Civil Engineering Subotica, Kozaračka 2a, Subotica, Serbia, tel: +381 24 554 300, e-mail: horvatz@gf.uns.ac.rs

averaged [1,5]. Applying the split-operator approach [6], the water flow equations were divided in three subsequent steps, the advection step presented with Eq. (1),

$$\begin{aligned}
\frac{u^a - u^n}{\Delta t} &= -\frac{u}{h_\xi}\frac{\partial u}{\partial \xi} - \frac{v}{h_\eta}\frac{\partial u}{\partial \eta} - \frac{1}{h_\xi h_\eta}\frac{\partial h_\xi}{\partial \eta} u\, v + \frac{1}{h_\xi h_\eta}\frac{\partial h_\eta}{\partial \xi} v^2, \\
\frac{v^a - v^n}{\Delta t} &= -\frac{u}{h_\xi}\frac{\partial v}{\partial \xi} - \frac{v}{h_\eta}\frac{\partial v}{\partial \eta} - \frac{1}{h_\eta h_\xi}\frac{\partial h_\eta}{\partial \xi} u\, v + \frac{1}{h_\eta h_\xi}\frac{\partial h_\xi}{\partial \eta} u^2,
\end{aligned} \tag{1}$$

diffusion step given with Eq. (2)

$$\begin{aligned}
\frac{u^d - u^a}{\Delta t} &= \frac{1}{h_\xi h_\eta}\left[\frac{1}{d}\frac{\partial\left(2 h_\eta \nu_t T_\xi d\right)}{\partial \xi} + \frac{1}{d}\frac{\partial\left(h_\xi \nu_t T_{\xi\eta} d\right)}{\partial \eta} - \frac{\partial h_\eta}{\partial \xi} 2 \nu_t T_\eta + \frac{\partial h_\xi}{\partial \eta}\nu_t T_{\xi\eta}\right], \\
\frac{v^d - v^a}{\Delta t} &= \frac{1}{h_\eta h_\xi}\left[\frac{1}{d}\frac{\partial\left(2 h_\xi \nu_t T_\eta d\right)}{\partial \eta} + \frac{1}{d}\frac{\partial\left(h_\eta \nu_t T_{\xi\eta} d\right)}{\partial \xi} - \frac{\partial h_\xi}{\partial \eta} 2 \nu_t T_\xi + \frac{\partial h_\eta}{\partial \xi}\nu_t T_{\xi\eta}\right],
\end{aligned} \tag{2}$$

and the propagation step that also consists the continuity equation, given by Eq. (3)

$$\begin{aligned}
\frac{u^{n+1} - u^d}{\Delta t} &= -\frac{g}{h_\xi}\frac{\partial}{\partial \xi}\left(z_b + d\right) - \frac{C_f\, u\sqrt{u^2 + v^2}}{d\cos\varphi_\xi}, \\
\frac{v^{n+1} - v^d}{\Delta t} &= -\frac{g}{h_\eta}\frac{\partial}{\partial \eta}\left(z_b + d\right) - \frac{C_f\, v\sqrt{u^2 + v^2}}{d\cos\varphi_\eta}, \\
h_\xi h_\eta \frac{\partial d}{\partial t} &+ \frac{\partial}{\partial \xi}\left(h_\eta\, u\, d\right) + \frac{\partial}{\partial \eta}\left(h_\xi\, v\, d\right) = 0.
\end{aligned} \tag{3}$$

These three steps are respectively marked with upper indexes *a*, *d* and *p*. In Eqs. (1), (2) and (3) u and v denote the depth-averaged velocity components in ξ and η coordinate direction, t represents the time, h_ξ and h_η are the geometric coefficients, d is the water depth, z_b denotes the bed elevation, C_f is the friction factor, φ_ξ and φ_η present the angle of the bed slope in both coordinate directions, g is the gravitational acceleration, ν_t is the turbulent viscosity, while Δt denotes the computational time step.

The advection step equations can be solved with the characteristics method using the principles given by Ref. [7]. This method transforms Eqs. (1) into

$$\frac{Du}{Dt} = -\frac{1}{h_\xi h_\eta}\frac{\partial h_\xi}{\partial \eta} v\, u + \frac{1}{h_\xi h_\eta}\frac{\partial h_\eta}{\partial \xi} v^2, \qquad \frac{Dv}{Dt} = -\frac{1}{h_\eta h_\xi}\frac{\partial h_\eta}{\partial \xi} u\, v + \frac{1}{h_\eta h_\xi}\frac{\partial h_\xi}{\partial \eta} u^2 \tag{4}$$

along the characteristic curves (5) that also represent the trajectory of the water particle

$$\frac{d\xi}{dt}=\frac{u}{h_\xi},\quad \frac{d\eta}{dt}=\frac{v}{h_\eta}. \tag{5}$$

Integration of Eqs. (4) requires known trajectory coordinates that can be retrieved by integrating Eqs. (5) from the departure D to the arrival point A. In order to shorten the computation time, Eqs. (5) are solved in a way that allows the characteristic curve to stretch through multiple computational cells. This is achieved by developing an algorithm that divides the considered trajectory into an arbitrary number of straight segments, l=1, 2,..., L, each bounded on both ends by neighboring computational cells. Finally, integration of Eqs. (4) along the trajectory yields the velocity equations, which are solved using the Newton-Raphson iterative algorithm [8].

The diffusion step is not numerically challenging, therefore it is not given in much detail. Equations (2) are discretized with the Crank-Nicolson scheme. Implementing the alternating direction implicit (ADI) method [5] the equations are divided, each in two orthogonal directions, and solved using the double-sweep algorithm [9].

Using the time weighting coefficient θ in propagation step Eqs. (3) one can extract explicit expressions for velocity components from the discretized continuity equation, and obtain a single propagation step equation in terms of unknown depth increment Δd

$$\begin{aligned} & h_\xi h_\eta \frac{\Delta d}{\Delta t}+\theta\frac{\partial}{\partial\xi}\left[h_\eta\left(\alpha_1\frac{\partial\Delta d}{\partial\xi}+\beta_1\Delta d+\gamma_1 d^n\right)\right]+ \\ & +\theta\frac{\partial}{\partial\eta}\left[h_\xi\left(\alpha_2\frac{\partial\Delta d}{\partial\eta}+\beta_2\Delta d+\gamma_2 d^n\right)\right]+\left(1-\theta\right)\left[\frac{\partial\left(h_\eta u^n d^n\right)}{\partial\xi}+\frac{\partial\left(h_\xi v^n d^n\right)}{\partial\eta}\right]=0, \end{aligned} \tag{6}$$

where α, β, γ are known coefficients. To avoid numerical oscillations, Eq. (6) is discretized on a staggered grid and then divided in two orthogonal directions using the ADI method, and solved using the double-sweep algorithm.

3. FIELD MEASUREMENTS

Field measurements were conducted on the reach of the Danube River located in the border area between Hungary and Serbia. The selected sight was bounded by Bezdan (rkm 1425.5) in Serbia and Mohacs (rkm 1446.9) in Hungary. Within this reach seven data ranges were selected for detailed flow field and sediment data measurements (Figure 1). These ranges were placed between rkm 1438 and rkm 1432, at 1 km apart, while bathymetry data was collected at 100m intervals. The data collection campaign took place during five days (23-27 May 2011). Each range had seven verticals, where velocity distribution profiles were collected with the use of ADCP. The velocity distribution measurements were carried out in accordance with Ref.[10].

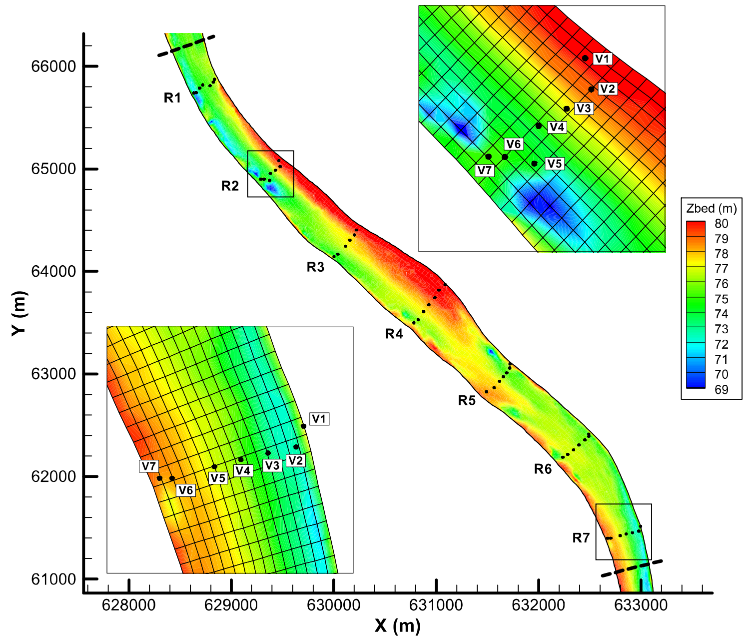

Figure 1. Selected data ranges

4. RESULTS

The simulation time was five days, with two additional days that were allocated as the stabilization period. The initial conditions were horizontal free-surface elevation and zero-velocity field. The upstream and downstream boundary conditions are respectively known discharge and free-surface elevation. The implemented algorithm in the water advection computation enabled using larger time steps, thus finally selecting 30sec.

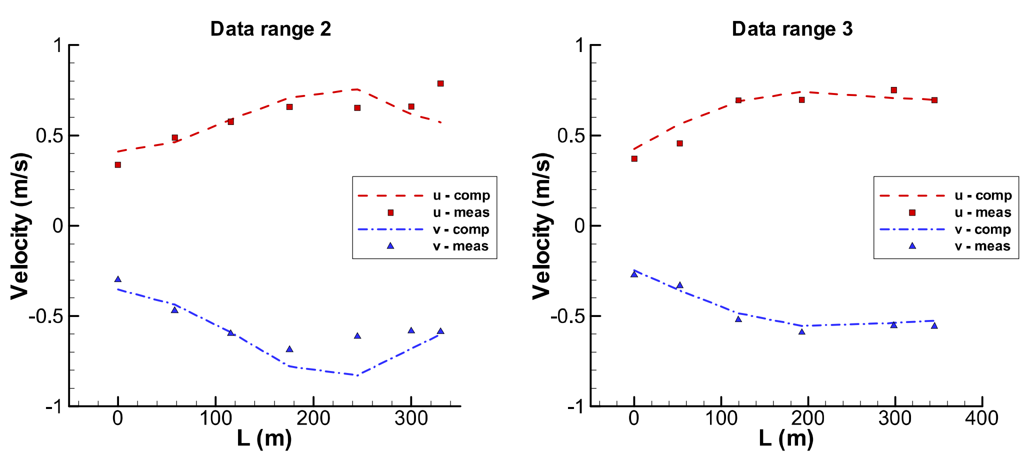

Figure 2. Velocity components at data ranges 2 and 3

The calibration of the hydraulic model was done by changing the value of the Manning's coefficient, until attaining reasonable accordance between measured and computed free-surface elevations. Following this process, the disagreement of measured and computed values decreased to +0.7cm, -1.7cm. Since free-surface elevation was the calibration criteria, comparison of computed and measured velocity components is a suitable indicator of the calibration quality. As an internal consistency indicator, the continuity equation error was monitored. The largest relative error for a single computational point throughout the simulation stayed under 0.03%, while the cumulative value of the relative error for the whole computational domain did not surpass $4 \cdot 10^{-8}$ %. The analysis of the measured and computed velocities (Figs. 2 and 3) leads to the conclusion that the developed numerical model is trustworthy.

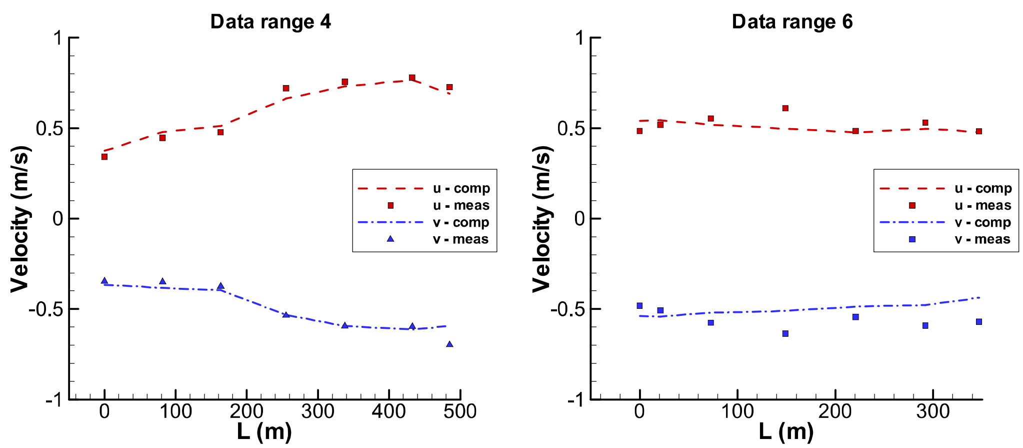

Figure 3. Velocity components at data ranges 4 and 6

5. CONCLUSION

A 2-D (depth-averaged) numerical model simulating water flow was developed. The model implements the split-operator approach, allowing separate treatment of the troublesome advection terms. The advection step was solved using the improved algorithm for the characteristic method allowing trajectories to extend through multiple computational cells. Since the simulation results show good agreement with the measurements, and the continuity equation error was negligible, the presented model is suitable for application in natural watercourses.

ACKNOWLEDGEMENTS

This work was funded by the Ministry of Education and Science of the Republic of Serbia, project number TR 37009.

REFERENCES

[1] Wang, G., Xia, J., Wu, B.: Numerical Simulation of Longitudinal and Lateral Channel Deformations in Braided Reach of the Lower Yellow River. *Journal of Hydraulic Engineering*, **2008.**, vol. 134, № 8, p.p. 1064-1078.
[2] Hung, M.C., Hsieh, T.Y., Wu, C.H., Yang, J.C.: Two-Dimensional Nonequlibrium Noncohesive and Cohesive Sediment Transport Model. *Journal of Hydraulic Engineering*, **2009.**, vol. 135, № 5, p.p. 339-382.
[3] Zhou, G., Wang, H., Shao, X., Jia, D.: Numerical Model for Sediment Transport and Bed Degradation in the Yangtze Channel Downstream of Three Gorges Reservoir. *Journal of Hydraulic Engineering*, **2009.**, vol. 135, № 9, p.p. 729-740.
[4] Isic, M., Horvat, Z., Spasojevic, M.: Advection step in the split-operator approach applied to river modeling. *Applied Numerical Mathematics*, **2013.**, vol. 72, p.p.1-18.
[5] Hsieh, T.Y., Yang, J.C.: Implicit Two-Step Split-Operator Approach for Modelling Two-Dimensional Open Channel Flow. *Journal of Hydroscience and Hydraulic Engineering*, **2004.**, vol. 22, № 2, p.p. 113-139.
[6] Yanenko, N.N.: *The Method of Fractional Steps*, Translated by Holt, M., Springer-Verlag, New York, **1971**.
[7] Benqué, J.P., Cunge, J.A., Feuillet, J., Hauguel, A., Holly, F.M.Jr.: New Method for Tidal Current Computation. *Journal of the Waterway*, *Port, Coastal and Ocean Division*, **1982.**, vol. 108, № WW3, p.p. 396-417.
[8] Biringen, S., Chow, C.-Y.: *An Introduction to Computational Fluid Mechanics by Example*, John Wiley & Sons, Inc., Hoboken, **2011**.
[9] Hirsch, C.: *Numerical Computation of Internal and External Flows – Volume 1 – Second edition,* Elsevier, Oxford, **2007**.
[10] Muste, M., Yu, K., Pratt, T., Abraham, D.: Practical aspects of ADCP data use for quantification of mean river flow characteristics; Part II: fixed-vessel measurements. *Flow Measurement and Instrumentation*, **2004.**, vol. 15, p.p. 17-28.

РАВАНСКИ МОДЕЛ ТЕЧЕЊА У АЛУВИЈАЛНИМ ВОДОТОЦИМА

Резиме: *Овај рад представља развој и калибрацију раванског модела течења воде у алувијалним водотоцима. Једначиње течења осредњене по дубини тока, су дате у ортогоналном криволинијском координатном систему, након чега су решаване применом методе разломљених корака, која даје три рачунска корака (адвективни, дифузиони и пропагациони). Са становишта нумеричког решавања најзахтевнији је адвективни корак, који је подложан нумеричким осцилацијама и дифузији, због чега је за његово решавање примењена метода карактеристика. У случају друга два рачунска корака коришћена је метода коначних разлика у комбинацији са ADI методом. Тачност развијеног нумеричког модела је процењена користећи мерења спроведена на деоници реке Дунав у пограничној области између Мађарске и Србије. Поузданост развијеног нумеричког модела је потврђена упоређивањем мерених и срачунатих вредности брзина.*

Кључне речи: *Нумерички модел, једначине течења, теренска мерења*