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RHEOLOGICAL-DYNAMICAL APPROACH IN NONHOMOGENEOUS FINITE STRIP METHOD

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Summary: *This paper presents the finite strip method adapted to the analysis of two-dimensional nonlinear problems of reinforced concrete plate structures. Non-homogenous finite strip, divided into cells along longitudinal direction and layered throughout the thickness, is developed and the interpolation shape functions are evaluated. The concrete is modelled with a biaxial state of stress that acts in the middle plane and includes the following effects: nonlinear stress-strain relationship in compression, compression failure, strength increase in biaxial compression and cracking. The rebar is idealized by smeared biaxial orthotropic steel layer. Nonlinear material behaviour is described by one- and two-dimensional models of concrete and reinforcement using the analytically derived rheological-dynamical viscoelastoplastic constitutive matrix.*

Keywords: *Non-homogeneous finite strip method, rheological-dynamical analogy, reinforced concrete, nonlinear analysis*

1. INTRODUCTION

Problems of the nonlinear analysis of reinforced concrete folded plate structures (RCFPS) have gained importance during the past years due to the application of various C/R materials and their utilization of high working stress. In the past years, the presented problem was treated in the scope of many research projects theoretically and experimentally. These structures generally include both nonlinearities: the geometric nonlinear effects and the nonlinear stress-strain behavior of the composite (concrete and rebar (C/R)) material. The aim of this paper is to present the finite strip method (FSM) adapted for estimation of the ultimate resistance of RCFPS based on the material nonlinear behavior. Here are used working diagrams of concrete and rebar, that represent simultaneous stress-strain pairs, built on the basis of rheological-dynamical analogy

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(RDA) [1]. These working diagrams are presented and compared with recommended diagrams according to Eurocode 2 (EC 2) in Ref. [2]. The C/R material is introduced under the following aspects: C/R are integrated using a biaxial quasi-empirical inelastic, orthotropic model, derived from uniaxial RDA material models; the cracking of concrete is represented by the 'smeared crack' concept rather than by discrete cracks; no relative displacement is allowed between steel layers and the adjacent concrete; viscoplastic (VP) yield stress at the end of the iterations in steel layer is considered as rebar failure. To account for the material nonlinearities a biaxial layered model throughout the thickness of the strip is included [3]. Then, each strip is divided into cells in longitudinal direction and material characteristics which depend of stresses are recorded in these cells [4]. The integrations are performed on each cell domain and their sum provides the four blocks of the strip stiffness matrix in the non-homogeneous finite strip method (NHFSM). Compared with the standard FEM, the main advantages of the NHFSM are in data preparation, program complexity, and execution time. Further advantages of the NHFSM lie in the possibility of modeling by using a small number of harmonics.

2. RHEOLOGICAL-DYNAMICAL APROACH

During the last decade, many papers have been published which have demonstrated that RDA for 1D problems may be easier for application than other nonlinear theories for damage of structures. One simple continuous RDA modulus function is found to approximate the average 1D stress-strain diagram over both the elastic and inelastic range of a specimen

$$E_R = \frac{E_H}{1+\varphi} = \frac{E_H}{1+\sigma_{cr} K_\varphi} \tag{1}$$

The constant K_φ of a specimen must be determined at the elasticity border

$$K_\varphi = \lambda_E \frac{i^3}{I} \frac{1}{E_H} \frac{1}{\gamma} \tag{2}$$

As shown in Fig. 1, all the individual $E_R(\lambda, \sigma_{cr})$ curves coincide, which means that one analytical function $E_R(\sigma_{cr})$ may be used for all concrete cylinder lengths. Fig. 2 shows function $E_R(\sigma_{cr})$ for rebar.

Analytical RDA working diagram of specimen has been identified using previously established analytical relation for $E_R(\sigma_{cr})$, as follows

$$\sigma_{cr} = \frac{1}{2K_\varphi}\left(\sqrt{1+4K_\varphi E_H \varepsilon} - 1\right) \tag{3}$$

The constitutive orthotropic matrix of C/R layers is given by (Ref. [5])

$$\mathbf{D} = \frac{1}{1-\mu^2}\begin{bmatrix} E_{Rx} & \mu\sqrt{E_{Rx}E_{Ry}} & 0 \\ \mu\sqrt{E_{Rx}E_{Ry}} & E_{Ry} & 0 \\ 0 & 0 & \left(1-\mu^2\right)G_H \end{bmatrix} \tag{4}$$

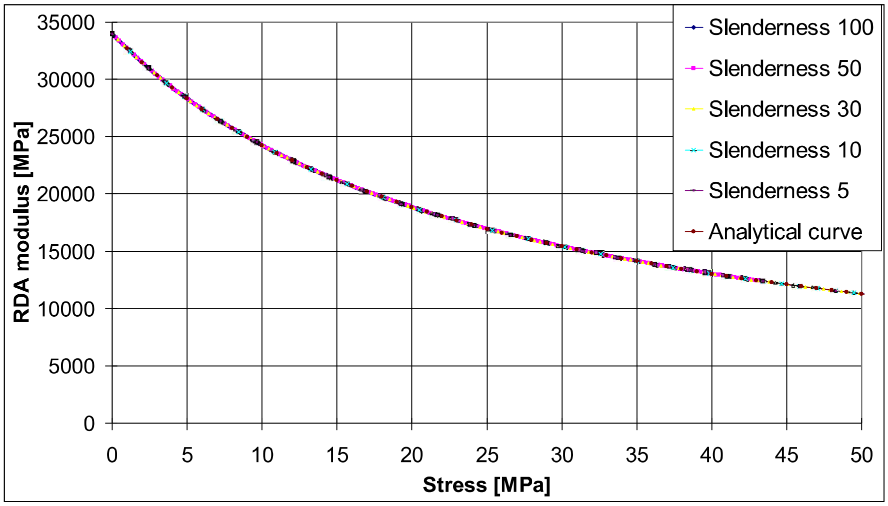

Figure 1. The plots of E_R *v/s* σ_{cr} *for different slenderness ratios* λ *of reference concrete sample (cylinder) made of grade C35/45*

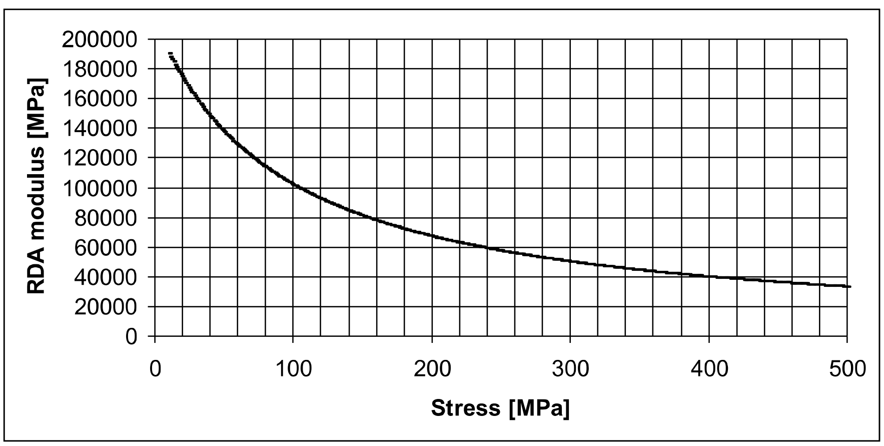

Figure 2. The plots of E_R *v/s* σ_{cr} *of rebar (steel B400)*

where E_{Rx} and E_{Ry} are obtained by RDA working diagrams of C/R, as follows

$$E_{Rx} = \frac{E_H}{1+\sigma_{crx}K_\varphi}\,,\ \sigma_{crx} = \frac{1}{2K_\varphi}\left(\sqrt{1+4K_\varphi E_H \varepsilon_x}-1\right),$$

$$E_{Ry} = \frac{E_H}{1+\sigma_{cry}K_\varphi}\,,\ \sigma_{cry} = \frac{1}{2K_\varphi}\left(\sqrt{1+4K_\varphi E_H \varepsilon_y}-1\right), \qquad (5)$$

$$\mu = \sqrt{\mu_x \mu_y}\,,\ G_H = \frac{1}{4}\left(E_{Rx}+E_{Ry}-2\mu\sqrt{E_{Rx}E_{Ry}}\right)\frac{1}{1-\mu^2}\,,$$

$$\mu_x = 1000\left(1 - \frac{1}{\sqrt[4]{\dfrac{0.0006K_\varphi\left|\sigma_{cr\,x}\right|}{K_\varphi\left|\sigma_{cr\,x}\right|+1}+1}}\right),\ \mu_y = 1000\left(1 - \frac{1}{\sqrt[4]{\dfrac{0.0006K_\varphi\left|\sigma_{cr\,y}\right|}{K_\varphi\left|\sigma_{cr\,y}\right|+1}+1}}\right)$$

The equilibrium equations for a system exhibiting nonlinear behaviour can be written as

$$\mathbf{K}\left(\mathbf{D}\right)\mathbf{q} = \mathbf{Q} \tag{6}$$

The nonlinear term is the stiffness matrix **K** of the system, which depends on the inelastic constitutive matrix **D**. In this paper a new iterative method, which is basically the same as the procedure explained in Ref. [1], is applied. The RDA modulus iteration starts with the form

$$\mathbf{K}\left(\mathbf{D}^{\mathbf{E}}\right)\mathbf{q} = \mathbf{Q} \tag{7}$$

Solving these linear equations, stress $\sigma_{cr}^{(0)}$ and strain $\varepsilon^{(0)}$ in all points of a continuum can be obtained. When the stress component is greater than the elasticity stress σ_E, Eq. (7) may be solved to give a better approximation

$$\sigma_{cr}^{(1)} = \frac{1}{2K_\varphi}\left(\sqrt{1+4K_\varphi E_H \varepsilon^{(0)}} - 1\right) \tag{8}$$

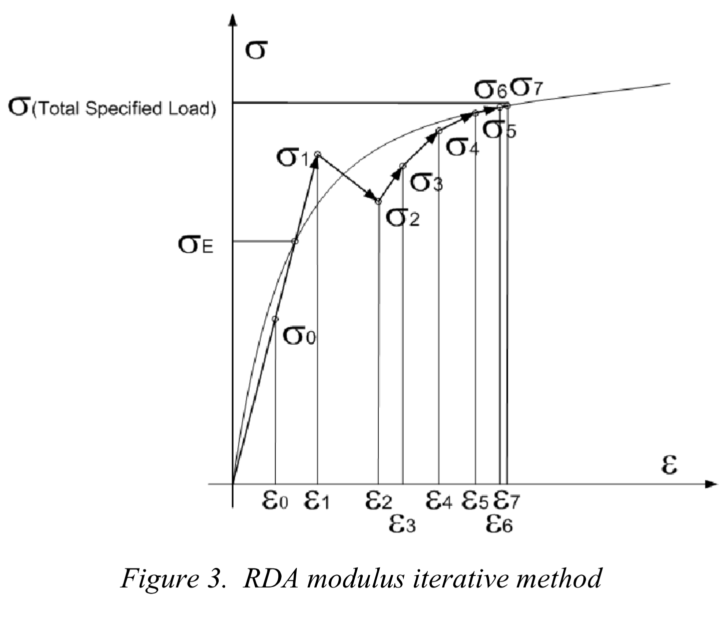

Figure 3. RDA modulus iterative method

The corresponding slope for next iteration is the RDA modulus,

$$E_R^{(1)} = \frac{E_H}{1+\sigma_{cr}^{(1)} K_\varphi} \tag{9}$$

Also,

$$\varepsilon^{(1)} = \frac{\sigma_{cr}^{(1)}}{E_R^{(1)}} \tag{10}$$

The iterative process is deemed to have converged when some measure of change δ in the ε between successive iterations has become tolerably small. Thus,

$$\delta = \sqrt{\left|\varepsilon^{(n)^2} - \varepsilon^{(n-1)^2}\right|} \tag{11}$$

The scheme of the RDA modulus iterative method is illustrated in Fig. 3.

3. NON-HOMOGENEOUS FINITE STRIP MODELLING

In order to apply presented RDA approach on C/R materials, non-homogenous finite strip, divided into cells and layered throughout the thickness, is developed. As shown in Fig. 4 the strip is divided along longitudinal direction into *nc* cells while throughout thickness there are *nl* layers, which constitute non-homogeneous LO2 finite strip.

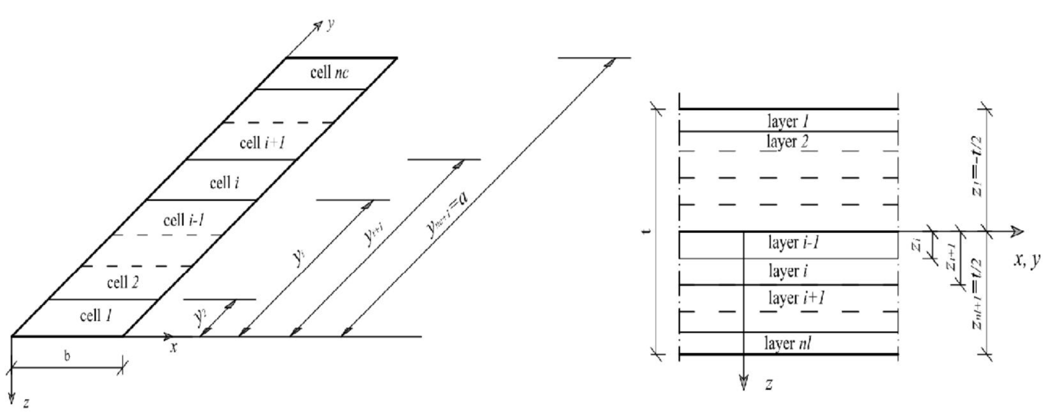

Figure 4. Division of strip into nc *longitudinal cells and* nl *layers throughout thickness*

During the integration required for determination of stiffness matrix, the following two independent integrals should be evaluated for longitudinal non-homogenous strip

$$I_1 = \int_{y_i}^{y_{i+1}} Y_m Y_n dy \qquad I_5 = \int_{y_i}^{y_{i+1}} \frac{dY_m}{dy}\frac{dY_n}{dy} dy \tag{12}$$

For the strip with both ends simply supported Y_m=sin($m\pi y/a$) and Y_m^v=cos($m\pi y/a$), these integrals can be evaluated in closed form, as follows

- for $m=n$

$$\begin{gathered} I_1 = a\left(\frac{h_{i+1}-h_i}{2}+\frac{sin(2h_i m\pi)-sin(2h_{i+1}m\pi)}{4m\pi}\right) \\ I_5 = \frac{1}{a}\left(-\frac{1}{4}m\pi\left[2m\pi(h_i-h_{i+1})+sin(2h_i m\pi)-sin(2h_{i+1}m\pi)\right]\right) \end{gathered} \tag{13}$$

- for $m \neq n$

$$\begin{gathered} I_1 = a\frac{n\left[cos(h_{i+1}n\pi)sin(h_{i+1}m\pi)-cos(h_i n\pi)sin(h_i m\pi)\right]}{m^2\pi-n^2\pi} \\ +a\frac{m\left[cos(h_i m\pi)sin(h_i n\pi)-cos(h_{i+1}m\pi)sin(h_{i+1}n\pi)\right]}{m^2\pi-n^2\pi} \\ I_5 = \frac{1}{a}\frac{mn\pi\left\{n\left[cos(h_i m\pi)sin(h_i n\pi)-cos(h_{i+1}m\pi)sin(h_{i+1}n\pi)\right]\right\}}{m^2-n^2} \\ +\frac{1}{a}\frac{mn\pi\left\{m\left[cos(h_{i+1}n\pi)sin(h_{i+1}m\pi)-cos(h_i n\pi)sin(h_i m\pi)\right]\right\}}{m^2-n^2} \end{gathered} \tag{14}$$

where $y=ha$.

4. CONCLUSIONS

Constitutive compliance matrices to predict the ultimate resistance of RCFPS on the basis of RDA are derived in the frame of non-homogeneous finite strip method.

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РЕОЛОШКО-ДИНАМИЧКО МОДЕЛИРАЊЕ У НЕХОМОГЕНОМ МЕТОДУ КОНАЧНИХ ТРАКА

Резиме: *Рад представља метод коначних трака који је прилагођен анализи дводимензионалних нелинеарних проблема армиранобетонских плочастих конструкција. Развијене су нехомогене коначне траке које су подељене у ћелије у подужном правцу и слојеве по дебљини. За њих су формиране одговарајуће интерполационе функције. Бетон је моделиран у двоаксијалном стању напона у средњој равни коначне траке укључујући следеће ефекте: нелинеарна веза напон-деформација у притиску, лом у притиску, ојачање у притиску и прслине. Арматура је моделирана као размазан ортотропан слој у двоаксијалном стању напона. Нелинеарно понашање материјала је описано једнодимензионалним и дводименsионалним моделима бетона и арматуре коришћењем аналитички изведене реолошко-динамичке конститутивне матрице.*

Кључне речи: *Нехомогени метод коначних трака, реолошко-динамичка аналогија, армирани бетон, нелинеарна анализа*