ANALYSIS OF THE EXCENTRICALLY PRESSED MEMBER CALCULATION ACCORDING TO EC3

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Summary: *The stability of eccentrically pressed elements can, according to EC3, be analyzed using methods based on the application of second-order theory as well as first-order theory methods on models with global imperfections. The stability control of eccentrically pressed elements is performed by means of intercaction formulas on insulated elements, with the buckling length which is determined depending on the boundary conditions and the lateral displacement of the system. The paper presents a detailed analysis of the application of Annex A and Annex B according to EC 3.*

Keywords: *excentric stress, EC 3, steel structures.*

1. CALCULATION OF STABILITY OF ECCENTRICALLY PRESSED ELEMENTS ACCORDING TO EUROCODE

The stability of eccentrically pressed elements can be analyzed in several ways, depending on how the second order influences are taken due to global (P-Δ influences) and local imperfections (P-δ influences). Eurocode 3 provides the following three options for controlling the stability of eccentrically pressed elements:

1. Calculation of the influence according to the theory of the second order on the model with global and local imperfections and control of the bearing capacity of the relevant cross sections;
2. Calculation of the influence according to the theory of the second order on the model with global imperfections and control of the stability of eccentrically pressed elements using interaction formulas for isolated elements, with a buckling length equal to the system length of the elements;

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3. Calculation of influences according to the theory of the first order on the model with global imperfections and stability control of eccentrically pressed elements using interaction formulas for isolated elements, with buckling length determined depending on boundary conditions of support and lateral displacement of the system;

The first approach is not practical for engineering applications in systems with a larger number of elements for the simple reason that a very large number of combinations of local imperfections of the elements need to be examined, depending on their orientation.

In the second variant, the construction is modeled with global imperfections which are mainly replaced by an equilibrium system of equivalent horizontal forces, and the influence of local imperfections of the elements is taken through interactive formulas for controlling the stability of the eccentrically pressed element. The influence in the construction is determined by applying the theory of the second order, and the stability of the elements is determined on the basis of the system length, ie considering that the system is with immovable nodes.

The third variant has the greatest application in everyday engineering practice and best suits our previous habits. It is based on the calculation of the impact according to the theory of the first order, taking into account the global imperfections of the system. The stability control of the eccentrically pressed element is performed using interaction formulas, but with a buckling length that indirectly takes into account the influences of the second order. Thus, with the exception of the first approach, which has already been said to have more theoretical-research than practical significance, in both remaining variants the calculation is based on interaction formulas applied to individual (isolated) elements considered as separated from the system. In the general case, according to Eurocode 3, the elements exposed to the combined action of bending and axial pressure should satisfy both of the following conditions: The stability control of the eccentrically pressed element is performed using interaction formulas, but with a buckling length that indirectly takes into account the influences of the second order. Thus, with the exception of the first approach, which has already been said to have more theoretical-research than practical significance, in both remaining variants the calculation is based on interaction formulas applied to individual (isolated) elements considered as separated from the system. . In the general case, according to Eurocode 3, the elements exposed to the combined action of bending and axial pressure should satisfy both of the following conditions: The stability control of the eccentrically pressed element is performed using interaction formulas, but with a buckling length that indirectly takes into account the influences of the second order. Thus, with the exception of the first approach, which has already been said to have more theoretical-research than practical significance, in both remaining variants the calculation is based on interaction formulas applied to individual (isolated) elements considered as separated from the system. In the general case, according to Eurocode 3, the elements exposed to the combined action of bending and axial pressure should satisfy both of the following conditions: in both remaining variants, the calculation is based on interaction formulas applied to individual (isolated) elements that are considered as being separated from the system.

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In the general case, according to Eurocode 3, the elements exposed to the combined action of bending and axial pressure should satisfy both of the following conditions:

$$\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}} + k_{yy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}} + k_{yz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \leq 1$$
$$\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}} + k_{zy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}} + k_{zz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \leq 1 \qquad (1)$$

where:

- N_{Ed}, $M_{y,Ed}$ and $M_{z,Ed}$ are the calculated values of the compressive force and maximum moments around the yy and zz axes along the elements,
- N_{Rk}, M_y, R_k and M_z, R_k are the characteristic values of the corresponding loads (Table 1),
- ΔM_y, E_d and $\Delta M_{z,Ed}$ moments of eccentricity due to center of gravity displacement at cross sections of class 4,
- χ_y and χ_z reduction coefficients due to flexion buckling around yy and zz axes,
- χ_{LT} reduction coefficients due to lateral torsional buckling,
- k_y, k_{yz}, k_{zy} and k_{zz} interaction coefficients.

The interaction interaction coefficients k_y, k_{yz}, k_{zy} and k_{zz} depend on the calculation method chosen. Namely, Eurocode 3 provides two alternative methods for calculating the interaction coefficients, according to:

- Annex A, which is of a general nature, comprehensive, but also more complicated to implement (this annex was developed by French and Belgian researchers);
- Annex B, which is simpler to apply, but is limited to certain types of cross-sections such as symmetrical I and X sections and rectangular hollow profiles (this annex was developed by German and Austrian researchers).

2. NUMERICAL EXAMPLE: LOAD CAPACITY OF ECCENTRICALLY PRESSED ELEMENTS

Check the load capacity of the element shown in the figure, from a rolled profile HE 140 A made of steel S 235 loaded with eccentric pressure, according to SRPS EN 1993.

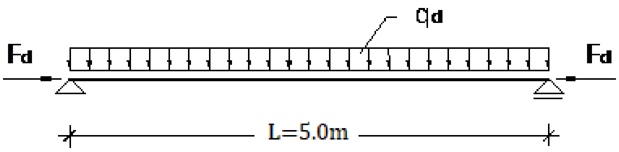
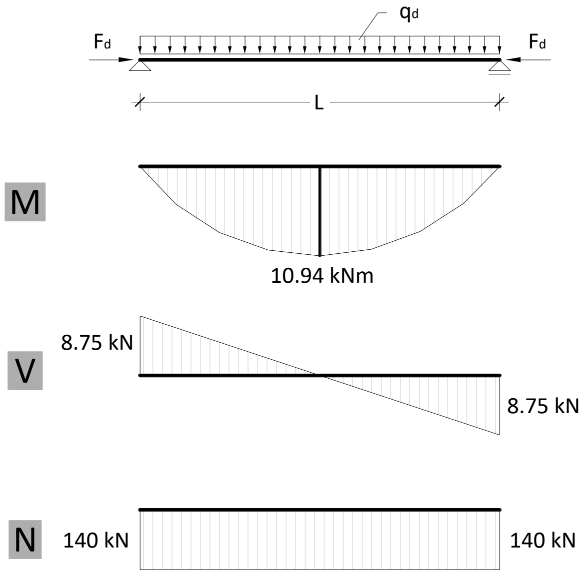


Figure 1. Static system

$F_d = 140\ kN$ (pressure force)



$q_d = 3,5\ kN/m$

Figure 2. Diagrams D, V, N

M - N interaction (bending and axial pressure)

$$\frac{N_{Ed}}{\chi_y \cdot N_{Rk}/\gamma_{M1}} + k_{yy} \cdot \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \cdot \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz} \cdot \frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \leq 1,0$$
$$\frac{N_{Ed}}{\chi_z \cdot N_{Rk}/\gamma_{M1}} + k_{zy} \cdot \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \cdot M_{y,Rk}/\gamma_{M1}} + k_{zz} \cdot \frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \leq 1,0 \quad (2)$$

2.1. Method 1 according to Annex A

$$k_{yy} = C_{my} \cdot C_{mLT} \cdot \frac{\mu_y}{1 - \frac{N_{Ed}}{N_{Cr,y}}} \cdot \frac{1}{C_{yy}} \quad (3)$$

$$k_{zy} = C_{my} \cdot C_{mLT} \cdot \frac{\mu_z}{1 - \frac{N_{Ed}}{N_{Cr,y}}} \cdot \frac{1}{C_{zy}} \cdot 0.6\sqrt{\frac{w_y}{w_z}} \quad (4)$$

If it is:

$$\bar{\lambda}_0 \geq 0.2 \cdot \sqrt{C_1} \cdot \sqrt[4]{\left(1 - \frac{N_{Ed}}{N_{Cr,TF}}\right) \cdot \left(1 - \frac{N_{Ed}}{N_{Cr,z}}\right)} \quad (5)$$

$$C_{my} = C_{my,0} + (1 - C_{my,D}) \frac{\sqrt{\varepsilon_y} a_{LT}}{1 + \sqrt{\varepsilon_y} a_{LT}}$$
$$C_{mz} = C_{mz,0} \quad (6)$$
$$C_{m,LT} = C_{my}^2 \frac{a_{LT}}{\sqrt{\left(1 - \frac{N_{Ed}}{N_{cr,z}}\right)\left(1 - \frac{N_{Ed}}{N_{cr,T}}\right)}} \geq 1$$

$k = 1.0 => C_1 = 1.132 \,;\, C_2 = 0.459$
Critical moment for lateral torsional buckling:

$$M_{Cr,E} = C_1 \cdot \frac{\pi^2 \cdot E \cdot I_z}{(kL)^2} \cdot \sqrt{\left(\frac{k}{k_w}\right)\frac{I_w}{I_z} + \frac{(kL)^2 G \cdot I_t}{\pi^2 \cdot E \cdot I_z})} \quad (7)$$

$$M_{Cr,E} = 1.113 \cdot \frac{\pi^2 \cdot 21000 \cdot 389.3}{(1.0 \cdot 500)^2} \cdot \sqrt{\left(\frac{1.0}{1.0}\right)\frac{15060}{389.3} + \frac{(1.0 \cdot 500)^2 \cdot 8077 \cdot 8.13}{\pi^2 \cdot 21000 \cdot 389.3}} = 5692\ kNcm \quad (8)$$

$$\bar{\lambda}_0 = \sqrt{\frac{4077}{5692}} = 0.85$$

Elastic critical force for lateral torsional buckling

$$N_{Cr,TF} = N_{Cr,T} = \frac{1}{i_R^2} \cdot (G \cdot I_t + \pi^2 \cdot \frac{E \cdot I_w}{l_T^2}) \quad (9)$$

$$N_{Cr,TF} = N_{Cr,T} = \frac{1}{45.27} \cdot \left(8077 \cdot 8.13 + \pi^2 \cdot \frac{21000 \cdot 15060}{500^2}\right) = 1726.34\ kN \quad (10)$$

$$0.2 \cdot \sqrt{1.132} \cdot \sqrt[4]{\left(1 - \frac{140}{1726.34}\right) \cdot \left(1 - \frac{140}{322.7}\right)} = 0.181 \leq \bar{\lambda}_0 = 0.85 \quad (11)$$

Maximum relative slenderness

$$\bar{\lambda}_{max} = \max\left(\bar{\lambda}_y, \bar{\lambda}_z\right) = 1.513 \quad (12)$$

$$C_{yy} = 1 + (1.12 - 1)\left[\left(2 - \frac{1.6}{1.12} \cdot 1.0^2 \cdot 1.513 - \frac{1.6}{1.12} \cdot 1.0^2 \cdot 1.513^2\right) 0.190 - 0\right] = 0.92 \quad (13)$$

$$C_{zy} = 1 + (1.12 - 1)\left[\left(2 - 14\,\frac{1.0^2 \cdot 1.513^2}{1.12^5}\right)0.19 - 0\right] = 0.631 \quad (14)$$

$$\mu_y = \frac{1 - \frac{N_{Ed}}{N_{cr,y}}}{1 - \chi_y \cdot \frac{N_{Ed}}{N_{cr,y}}} = \frac{1 - \frac{140}{856.4}}{1 - 0.643 \cdot \frac{140}{856.4}} = 0.935 \quad (15)$$

$$\mu_z = \frac{1 - \frac{N_{Ed}}{N_{cr,z}}}{1 - \chi_z \cdot \frac{N_{Ed}}{N_{cr,z}}} = \frac{1 - \frac{140}{322.7}}{1 - 0.310 \cdot \frac{140}{322.7}} = 0.654 \quad (16)$$

Interaction factors k_{yy} and k_{zy}

$$k_{yy} = 1.0 \cdot 1.36 \cdot \frac{0.935}{1 - \frac{140}{856.4}} \cdot \frac{1}{0.92} = 1.652 \quad (17)$$

$$k_{zy} = 1.0 \cdot 1.36 \cdot \frac{0.654}{1 - \frac{140}{856.4}} \cdot \frac{1}{0.631} \cdot 0.6 \cdot \sqrt{\frac{1.12}{1.50}} = 0.874 \quad (18)$$

Control of element load capacity on the combined effect of pressure force and bending torque

Interaction formulas
I condition

$$\frac{N_{Ed}}{\chi_y \cdot \frac{N_{Rk}}{\gamma_{M1}}} + k_{yy} \cdot \frac{M_{y,Ed}}{\chi_{LT} \cdot M_{y,Rk}/\gamma_{M1}} \le 1,0 \quad (19)$$

$$\frac{140\,kN}{0,643 \cdot 738,4\frac{kN}{1,0}} + 1,652 \cdot \frac{10,94\,kNm}{1,0 \cdot 40,77\,kNm/1,0} = 0,74 \quad (20)$$

$$0,74 < 1,0 \quad (21)$$

II condition

$$\frac{N_{Ed}}{\chi_z \cdot \frac{N_{Rk}}{\gamma_{M1}}} + k_{zy} \cdot \frac{M_{y,Ed}}{\chi_{LT} \cdot \frac{M_{y,Rk}}{\gamma_{M1}}} \le 1,0 \quad (22)$$

$$\frac{140\,kN}{0,310 \cdot 738,4\frac{kN}{1,0}} + 0874 \cdot \frac{10,94\,kNm}{1,0 \cdot 40,77\frac{kNm}{1,0}} = 0,85 \quad (23)$$

$$0,24 < 1,0 \quad (24)$$

2.2. Method 2 according to Annex B

$$k_{yy} = C_{my} \cdot \left[1 + \left(\bar{\lambda}_y - 0,2\right) \cdot \frac{N_{Ed}}{\chi_y \cdot N_{Rk}/\gamma_{M1}}\right] \leq C_{my} \cdot \left[1 + 0,8 \cdot \frac{N_{Ed}}{\chi_y \cdot N_{Rk}/\gamma_{M1}}\right] \quad (25)$$

$$k_{zy} = 0,6 \cdot k_{yy} \quad (26)$$

$$k_{yy} = 0,95 \cdot \left[1 + (0,929 - 0,2) \cdot \frac{140\ kN}{0,643 \cdot 738,4\ kN/1,0}\right] = 1,15 \quad (27)$$

$$k_{yy} = 1,15 \leq 0,95 \cdot \left[1 + 0,8 \cdot \frac{140\ kN}{0,643 \cdot 738,4\ kN/1,0}\right] = 1,17 \quad (28)$$

$$1,15 < 1,17 \quad \Rightarrow \quad k_{yy} = 1,15 \quad (29)$$

$$k_{zy} = 0,6 \cdot 1,15 = 0,69 \quad (30)$$

I condition

$$\frac{140\ kN}{0,643 \cdot 738,4 \frac{kN}{1,0}} + 1,15 \cdot \frac{10,94\ kNm}{1,0 \cdot 40,77\ kNm/1,0} = 0,29 + 0,31 = 0,60 \quad (31)$$

$$0,60 < 1,0 \quad (32)$$

II condition

$$\frac{140\ kN}{0,310 \cdot 738,4 \frac{kN}{1,0}} + 0,69 \cdot \frac{10,94\ kNm}{1,0 \cdot 40,77 \frac{kNm}{1,0}} = 0,61 + 0,19 = 0,80 \quad (33)$$

$$0,80 < 1,0 \quad (34)$$

Control of element load capacity to the combined effect of pressure force and bending torque according to Annex B

Interaction formulas
I condition

$$\frac{140\ kN}{0,643 \cdot 738,4 \frac{kN}{1,0}} + 1,15 \cdot \frac{10,94\ kNm}{1,0 \cdot 40,77\ kNm/1,0} = 0,29 + 0,31 = 0,60 \quad (35)$$

$$0,60 < 1,0 \quad (36)$$

II condition

$$\frac{140\ kN}{0,310 \cdot 738,4 \frac{kN}{1,0}} + 0,69 \cdot \frac{10,94\ kNm}{1,0 \cdot 40,77 \frac{kNm}{1,0}} = 0,61 + 0,19 = 0,80 \quad (37)$$

$$0,80 < 1,0 \quad (39)$$

3. CONCLUSION

Utilizations of section load capacity on combined comparison effect of torque and normal pressure force according to Annex A and according to Annex B
The following table will show the values of voltage utilization at eccentric pressure, as well as the ratio of the obtained values of interaction factors.

Table 1. Comparative presentation of results according to Annex A and Annex B, EC 3

	kyy	kzy	I condition	II condition
According to the attachment A	1,65	0,87	0,74	0,85
According to the attachment B	1,15	0,69	0,60	0,80
%	43	26	23	6

We can conclude that according to Annex A, which is comprehensive and can be used for all types of cross-sections, a stricter result was obtained compared to Annex B, which is essentially much simpler to apply but is of limited application and can only be used for symmetrical I, X and rectangular hollow sections.

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АНАЛИЗА ПРОРАЧУНА ЕКСЦЕНТРИЧНО ПРИТИСНУТИХ ЕЛЕМЕНАТА ПРЕМА ЕС3

Резиме: *Стабилност ексцентрично притиснутих елемената може се према ЕС3, вршити помоћи метода kоје се базирају па примени теорије другог реда kао и метода базираних на теорији првог реда на математичким моделита са глобалним/локалним имперфекцијама система. Контрола стабилности ексцентрично притиснутих елемената врши се помоћу интеракционих формула па изолованит елементита са одговарајућим дужинама извијања, које се одређују у зависности од граничних услова. У раду је приказана детаљна анализа и дата компарација резултата прорачуна применом правила датих у Анексу А и Анексу Б, ЕС 3.*

Кључне речи: *ексцентрично напрезање, ЕС 3, челичне конструкције.*