DYNAMIC ANALYSIS AND RESPONSE OF SYSTEMS UNDER IMPACT LOADS

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Summary: *Many structures may be exposed to impact loads. Their behavior under such extreme loading conditions is crucial for the safety of people and the environment. In modern design of structures, these loads are paid great attention. The paper presents a short theoretical and mathematical formulation of several methods by which the problem of impact can be analyzed, analysis of the time response of a simply supported beam was performed and also the comparison of the obtained results. In addition, linear and nonlinear dynamic analysis was performed using multifunctional Abaqus and Diana software.*

Keywords: *Dynamic analysis, impact loads, FEM*

1. INTRODUCTION

In modern design of structures, impact loads are paid great attention. Although they are mainly extreme load cases, with a low likelihood of occurrence during the lifetime of a structure, so-called incident loads are gaining in importance both as a consequence of human activity and because of natural disasters. Dangers that occur during impacts, explosions, landslides, etc. for people and the environment must not be ignored.

The impact load in the wider sense represents every sudden change in the already existing load, that is, the sudden start of a new load. The most important task in the impact analysis is to assess the behaviour of the structure during and after the impact. Since there is a significant change in the load intensity in a very short period of time, the exact calculation of the stresses arising as a result of impact is an extremely complex problem that is difficult to solve by using the usual methods. With the purpose of safe and rational design, it is necessary to know the extreme values and the time flow of the impact change in the structure caused by impact loads. Proper design of the structure must provide satisfactory load capacity and, what is very important, the appropriate ductility. From an economic point of view, it would not make sense to produce an oversized structure and expect an elastic response to the load, whose probability of occurrence in the lifetime of the structure is small.

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2. PHYSICAL CONCEPT OF IMPACTS

The simplest example of an impact is the impact of the ball with mass m, which freely falls from the height h to the surface (Figure 1.a). An ideal elastic impact implies preservation of momentum and conservation of energy. Momentum is defined as the product of mass and speed of the body and it remains constant before and after the impact. At the moment when the ball is in the initial position, its potential energy has a maximum value, while the kinetic energy is zero, and at the moment of impact, the kinetic energy reaches its maximum, while the potential equals zero. Energy only transfers from one form to another, and its value remains unchanged over time.

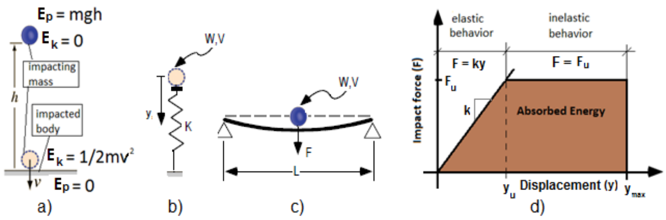


Figure 1. Physical parameters of impact

Now let mass with its weight W act upon an ideal elastic spring with a spring rate k (Figure 1.b). The incoming kinetic energy will be completely converted to the strain energy of deformation within the impacted structure. The impact force F carried by the spring and its equal and opposite reaction, act to slow down the mass and compress the spring to a maximum distance y_{max}. From the equation for the kinetic energy of mass and deformation energy of a spring, it follows:

$$\frac{1}{2}mv^2 = \int_0^y F du \rightarrow \frac{1}{2}\frac{W}{g}v^2 = \int_0^{y_{max}} kudu \qquad (1)$$

After sorting, we get the maximum compression of the spring:

$$y_{max} = v\sqrt{\frac{W}{kg}} = \sqrt{\frac{2E_k}{k}} \qquad (2)$$

The previous approach can be further expanded. The impact of dropped mass on the gird (Figure 1.c) will be considered. Due to the rapid emergence of load on the beam, inertial forces that can not be ignored are occurring. The absorbed energy is equal to the sum of the incoming kinetic energy and the work performed by the weight W. From the equation of absorbed energy and elastic deformation work, we get the maximal displacement of the beam:

$$E_k + W \cdot y_{max} = \frac{1}{2}k \cdot y_{max} \qquad (3)$$

$$y_{\max} = \frac{W}{k}\left(1+\sqrt{1+\frac{2kE_k}{W^2}}\right) = y_{st}\left(1+\sqrt{1+\frac{2E_k}{Wy_{st}}}\right) = y_{st}\cdot\lambda \tag{4}$$

Where y_{st} is static displacement of the beam due to the weight W.
The term in brackets represents magnification factor λ and increases the static displacement of the beam. This takes into account the dynamic effect due to the sudden emergence of the load. In case the mass velocity is zero (the kinetic energy is zero), the dynamic factor has a maximum value $\lambda = 2$. Which means that the displacement due to the sudden emergence of a load is twice that of the displacement that would occur if the load was applied gradually.
In practice, it is almost impossible to avoid plastic deformation, especially local one at the point of impact. In the analysis of inelastic impact, the law on energy conservation does not apply, while the law on the conservation of momentum still applies. The kinetic energy of the impacting body will be partially converted to strain energy in impacted structure and partially dissipated through friction and local plastic deformation. Figure 1.d shows the ideal elastoplastic behaviour of the structural element. The shaded surface represents the absorbed energy. From the equation for kinetic and strain energy, we obtain the maximum displacement of the structure:

$$y_{\max} = \frac{E_k + \frac{F_u^2}{2k}}{F_u} \tag{5}$$

The ratio of total deformation and deformation at the elasticity yield point is the factor of ductility and it is the best indicator of the ability of the structure to withstand plastic deformation. In practice, the maximum displacement limits, depending on the ability of the structure to withstand plastic deformations without loss of stability, and this is most often done through the mentioned ductility factor:

$$\mu = \frac{y_{\max}}{y_u} \tag{6}$$

From the equation of the work of the impact force and the absorbed energy of the system, we get:

$$F\cdot y_{\max} = \frac{1}{2}F_u\cdot y_u + F_u(y_{\max} - y_u) \rightarrow \frac{F_u}{F} = \frac{2\mu}{2\mu - 1} \tag{7}$$

Limiting the value of the ductility factor to 5 will result in the occurrence of medium and minor damage after impact, so it may be possible to use it during repair. If this value is adopted within the limits 5-10, one can expect the appearance of very large damage, and in some cases collapse of the structure or its part due to loss of stability.

3. DYNAMIC ANALYSIS AND RESPONSE OF SYSTEM

By applying the expression from the previous chapter, a maximum response of the system in the event of an impact can be determined, but no information can be obtained at which moment in time it will occur. It is often necessary to know the complete time response of the system, and for this, it is necessary to perform a more detailed dynamic analysis. In this chapter, we briefly outline some methods by which the analysis of the complete time response of the system can be performed.

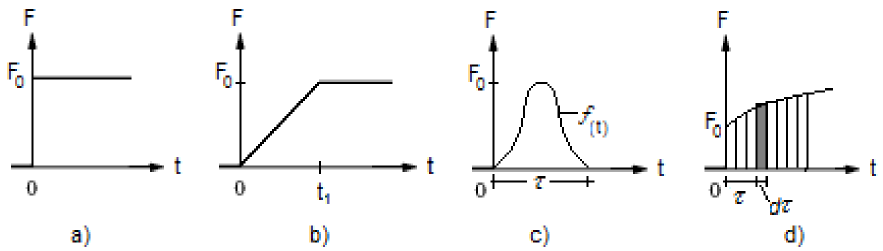


Figure 2. Some forms of load that can simulate impact load

Step Force (Figure 2.a):

At some point in time, the load suddenly starts to act in its full intensity. It can also be displayed using the Heaviside function (step function).
By solving the differential equation of the dynamic equilibrium we obtain:

$$y_{(t)} = y_{st}(1 - \cos\omega t) = y_{st} \cdot \lambda_{(t)} \tag{8}$$

By introducing damping, the previous expression takes the following form:

$$y_{(t)} = y_{st}\left[1 - \frac{e^{-\zeta\omega t}}{\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}\,\omega t + \alpha)\right] = y_{st} \cdot \lambda_{(t)} \tag{9}$$

Where:
ω is natural circular frequency
$\alpha = arctg\dfrac{\sqrt{1-\zeta^2}}{\zeta}$ is phase angle
The envelope curves are given by:

$$y_{(t)} = y_{st}\left(1 \mp \frac{e^{-\zeta\omega t}}{\sqrt{1-\zeta^2}}\right) \tag{10}$$

Step Force with finite rise time (Figure 2.b):

A load that occurs suddenly is rarely seen in practice. Generally, it increases linearly over a shorter or longer time interval and reaches a maximum value at some point in time t_1. Boundary cases of linear force increase are a sudden load ($t_1 \rightarrow 0$) and static load ($t_1 \rightarrow \infty$). For undamped system the displacement (motion) is:

$$y_{(t)} = y_{st}\left(\frac{t}{t_1} - \frac{\sin\omega t}{\omega t_1}\right) = y_{st}\cdot\lambda \rightarrow for(t \le t_1)$$
$$y_{(t)} = y_{st}\left[1 + \frac{1}{\omega t_1}(\sin\omega(t-t_1) - \sin\omega t)\right] = y_{st}\cdot\lambda \rightarrow for(t > t_1) \tag{11}$$

Pulse Force (Figure 2.c):

Impulse load is a short-term high-intensity load that does not change the direction of action and whose time-integral value is a finite value. The force impulse, or the time-integral of the force, is defined by the expression:

$$I = \int_0^{t_1} F_{(t)}dt = \int_0^{t_1} F_0 f_{(t)}dt \tag{12}$$

The function that defines the unit impulse is called the Dirac (delta) function.
For a system without damping, the motion is given by the equation:

$$y_{(t)} = \frac{I}{m\omega}\sin\omega t \tag{13}$$

Arbitrary variable force (Fig. 2.d):

When considering the linear behaviour of the system, the action of some load can be shown as a superposition of the action of infinitely elementary impulses, so the equation of the motion of the system is:

$$y_{(t)} = y_0\cos\omega t + \frac{v_0}{\omega}\sin\omega t + \frac{1}{m\omega}\int_0^t F_{(\tau)}\sin\omega(t-\tau)d\tau \tag{14}$$

$$y_{(t)} = e^{-\zeta\omega t}\left(y_0\cos\omega_d t + \frac{\dot{y}_0 + \zeta\omega\cdot y_0}{\omega_d}\sin\omega_d t\right) +$$
$$\frac{1}{m\omega_d}\int_0^t F_{(\tau)}e^{-\zeta\omega(t-\tau)}\sin\omega_d(t-\tau)d\tau \tag{15}$$

These terms represent well-known Duhamel's integral (integral of convolution), which describes the response of the system due to arbitrary dynamic load.
By using the expressions (12-15), all previously presented expressions can be derrived.
Direct application of the Duhamel integral is possible if the dynamic load is given in the analytic form and if the sub-integral function is integrable. In practice, these conditions are often not fulfilled and numerical integration is applied - step by step integration.

Central Difference Method (CDM):

This method is based on a finite difference approximation of time derivatives of displacement, velocity and acceleration. For known system characteristics (m, c, k) and known initial conditions, a time interval is adopted (Δt), and then the motion are determined:

$$y_1 = y_0 + \dot{y}_0 \Delta t + \frac{\Delta t^2}{2m}\left(F_0 - c\dot{y}_0 - ky_0\right) \tag{16}$$

$$y_{i+1} = \frac{1}{\dfrac{m}{\Delta t^2} + \dfrac{c}{2\Delta t}}\left[\left(\frac{2m}{\Delta t^2} - k\right)y_i + \left(\frac{c}{2\Delta t} - \frac{m}{\Delta t^2}\right)y_{i-1} + F_i\right] \tag{17}$$

Newmark's - Average Acceleration Method (AAM):

Newmark developed a family of time-stepping methods. Below is a method based on the assumption that the acceleration during a single interval has a certain value. For known system characteristics and initial conditions, the initial acceleration is determined, and then the motion, velocity and acceleration for other moments of time are determined.

$$y_{i+1} = \frac{1}{\dfrac{4m}{\Delta t^2} + \dfrac{2c}{\Delta t} + k}\left[\left(\frac{4m}{\Delta t^2} + \frac{2c}{\Delta t}\right)y_i + \left(\frac{4m}{\Delta t} + c\right)\dot{y}_i + m\ddot{y}_i + F_{i+1}\right] \tag{18}$$

$$\dot{y}_{i+1} = \frac{2}{\Delta t}\left(y_{i+1} - y_i\right) - \dot{y}_i \tag{19}$$

$$\ddot{y}_{i+1} = \frac{1}{m}\left(F_{i+1} - c\dot{y}_{i+1} - ky_{i+1}\right) \tag{20}$$

Nonlinear response:

There are several methods used for non-linear dynamic system analysis [1], [3], [4]. All these methods are quite complex and can be applied only for simpler problems, while for the analysis of more complex problems the application of appropriate software is necessary. The incremental form of equation of equilibrium is:

$$m\Delta\ddot{y}_{(t)} + c_{(t)}\Delta\dot{y}_{(t)} + k_{(t)}\Delta y_{(t)} = \Delta f_{(t)} \quad (21)$$

In general, various methods for numerical integration of this equation are used. They are all based on certain approximations, and errors that occur due to the same are small if a sufficiently small time interval, step, is adopted. However, the accumulation of errors in some cases can lead to the instability of the solution, and various procedures for correcting the accuracy are therefore applied.

4. NUMERICAL EXAMPLE

In this section, a dynamic analysis was performed and the complete response of a dynamic system was presented using the previous expressions, and then a comparison of the obtained results was performed. In addition, linear and non-linear analysis was performed using multifunctional Abaqus and Diana TNO software based on the finite element method. The steel beam HEB200 with a span of 2m was selected and the steel is quality S235. By adopting this profile and span, a great stiffness of the system has been achieved, and the stability problem that is particularly present in steel structures will not be significant. In this way, it is possible to better understand the response of this dynamic system over time. In order to examine the effect of damping, an analysis of un-damped and damped (damping ratio 7%) system was performed. The impact load is simulated with a trapezoidal impulse of a total duration of 0.05s. The load increases to 0.001s then remains constant up to 0.049s and then drops to zero at a time of 0.05s. The force was selected which, in static effect, causes maximum sress (yield) of only the ends of the cross-sectional area, while the weight of the beam is neglected. The static displacement due to the load in the center of the beam is about 4.0 mm. Figure 3 shows responses of system during load. It seems that after a sudden excitement, the system starts to oscillate around a statically equilibrium position. The amplitude of the oscillations in the non-damped oscillations remains constant, while with damped it progressively decreases within the limits (10).

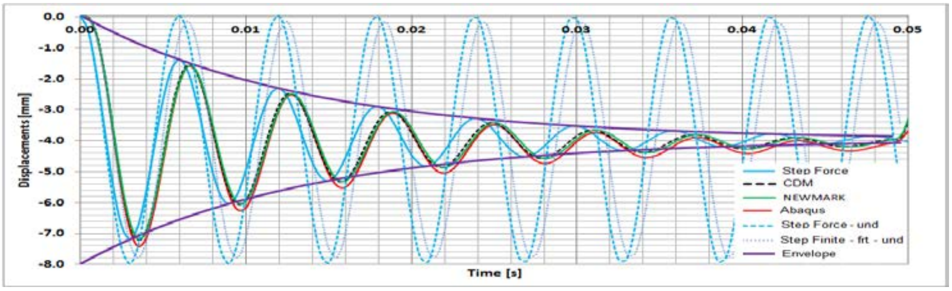


Figure 3. Response of system during load action

Upon termination of the load, the response of the system can be obtained through use of expressions used to analyze free vibrations, numerical integration, or by adding a new impulse load of the opposite character, which will eliminate the effect of the initial impulse. Figure 4 shows the complete response of system during and after the load action. As can be seen after a sudden cessation of load action, the system can not immediately return to the initial position, but it starts to oscillate around the initial position.

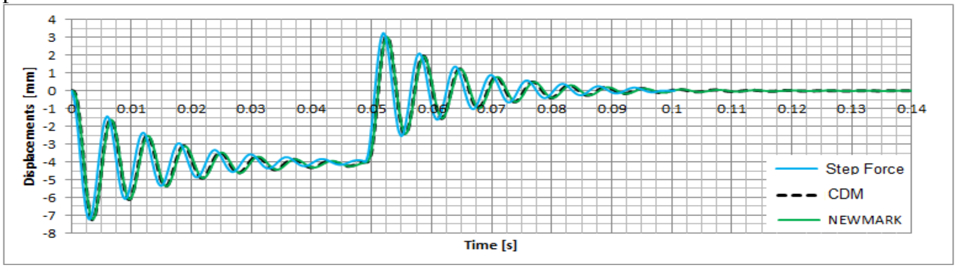


Figure 4. Complete temporal response of system

From here follows the definition that the impact load in the wider sense represents any sudden change in the existing or sudden start of a new load.
Figure 5 shows a change in the dynamic factor over time for a undamped and damped system.

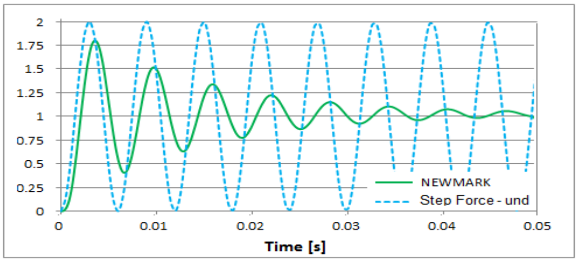


Figure 5. Change of dynamic (magnification) factor over time

The linear theory describes well the dynamic response of system to a certain load intensity. Although its simplicity has found wide application in practice, in some cases the application of a more precise (nonlinear) theory is required.
Figure 6 shows the nonlinear response of the system obtained by numerical analysis. The red line shows the motion of the point on the joint of the upper flange and the web in the cross-section at the centre of the span, while the orange line shows the motion of the point at the edge of the upper flange in the same cross-section. The upper flange behaves like a double cantilever fixed at the point of the joint with the web.

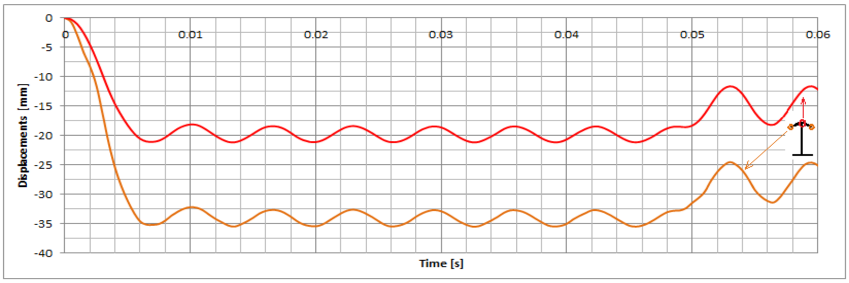
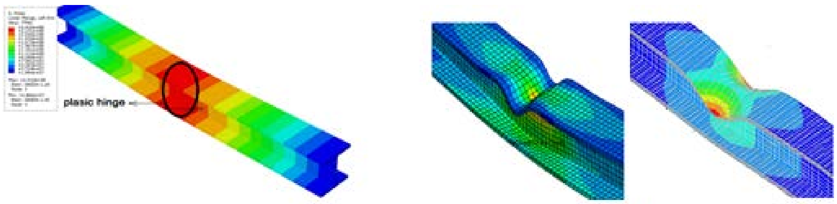


Figure 6. Nonlinear response of system - Diana TNO

Given the very nature and the intensity of the impact load, the appearance of a plastic joint is certain (Figure 7.a). In this case, the beam does not return to its initial position after unloading but remains permanently deformed.
As said, it is impossible to avoid local damage at the point of impact. These damages are shown in Figure 7.b and can be classified as minor or medium damage.



a) b)

Figure 7. Display of stress value at the moment of formation of a plastic hinge (a) and local damage - Abaqus and Diana TNO (b)

5. CONCLUSION

The dynamic effects of impact loads are very complex, and they are characterized by an even more complicated response of system. A large energy wave is converted into high stresses in a very short time interval, followed by a large deformation that a structure or a structural component needs to withstand without losing bearing capacity or stability. Dynamic response of system (local or global) implies a complex interaction of physical-mechanical characteristics of materials, along with the geometric characteristics of the elements.
Linear analysis has shown that the response of the dynamic system to the effect of impact load can be successfully analyzed in several relatively simple ways. All the methods presented provide mostly solutions of satisfactory accuracy. The concurrence of the results obtained using the expressions presented in this paper with the results of the numerical analysis in the Abaqus and Diana software is very good. The slight deviations that occur are mainly the result of the adopted form of the impulse load (trapezoid-rectangle). The effect of damping on the linear behaviour of materials is of great importance and should be taken into account since the energy dissipated by the elastic deformation is not too large.
When it comes to a non-linear calculation that includes the post-elastic behaviour of materials, the analysis becomes much more complex and practically impossible even for simpler systems. Using modern software and non-linear dynamic analysis is carried out without major problems. Also, for more complex linear systems, the application of the appropriate software is the only solution. It can be seen that the effect of damping, in this case, is considerably lower. The largest part of the energy dissipated is dissipated through deformation energy of the beam, as well as through friction and local plastic deformations, so the effect of damping can be ignored. Due to a large amount of kinetic energy dissipated at the very beginning, the vibrations in this area are considerably smaller.

When it comes to stresses, especially at the point of impact, the correct answer can be given only by numerical analysis. It can be said that the application of the software in analyzing the impact load problem allows for a better understanding of the stress and gives a broader picture of the behaviour of the structure during and after the load action.

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DINAMIČKA ANALIZA I ODGOVOR SISTEMA PRI UDARNOM OPTEREĆENJU

Rezime: *Mnoge konstrukcije mogu biti izložene udarnim opterećenjima. Njihovo ponašanje pod takvim ekstremnim uslovima opterećenja je od presudnog značaja za bezbednost ljudi i životnu sredinu. U savremenom projektovanju konstrukcija ovim opterećenjima se posvećuje velika pažnja. U radu je prikazana kratka teorijska i matematička formulacija nekoliko metoda pomoću kojih se može analizirati problem udara, izvršene su analize vremenskog odgovora jednog grednog nosača i upoređivanje dobijenih rezultata. Pored toga, izvršena je linearna i nelinearna dinamička analiza korišćenjem višenamenskih programa Abaqus i Diana.*

Ključne reči: *Dinamička analiza, udarno opterećenje, MKE*