VIBRATION ABSORBER OF SEISMIC ACTIONS IN BUILDINGS

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Summary: *Various mthods for elimination vibrations cased by seismic perturbation are developed. Beside the passive vibration absorbers significant number of active vibration absorbers are suggested. Unfortunately, these are extremely expensive due to the fact that they work randomly but their maintance has to be continual. It gives the priority to development of passive absorbers. In this paper we suggest a new type of nonlinear absorber based on the piano-wire vibration concept. The absorber is connected to the floor element. It contains a straightforward translatory moving mass settled on a steel wire installed in the direction perpendicular to slider motion. Due to slider motion in the wire a restoring force occurs. The force is a nonlinear deflection function. Using the principle of elimination of building motion, parameter of nonlinearity is computed. The model of the system are two coupled second order nonlinear differential equations. The nonlinearity is of cubic order. Based on the exact steady state solution that corresponds to nonlinear resonant vibration, we developed an approximate solving method for the perturbed equations. Finally, the conditions for vibration elimination are computed.*

Keywords: *Passive absorber, piano-wire concept, nonlinear resonant vibration, Jacobi elliptic function*

1. INTRODUCTION

Mechanical vibration is a common disturbance that occurs in buildings due to dynamic loads like earthquake, wind, etc. The interval of vibration intensity is very wide: from quite small oscillations up to very large and destructive one. Earthquakes are well known for their destruction. It is the reason that the buildings in the seismic regions are constructed according to some specific rules and in addition special devices are installed to reduce vibrations. A popular means to mitigate excessive structural vibrations is the attachment of a lightweight spring-mass element known as a vibration absorber or tuned mass damper. Designing new types of vibration absorbers that outperform the classical

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linear tuned mass damper is a challenging ongoing research field [1]. To try and enhance the performance of a tuned mass damper, nonlinear stiffness can be introduced to good use [2]. It is found that the nonlinearity has to be of pure type to obtain the system that is a nonlinear energy sink [3]. Using these results in this paper we suggest a vibration absorber with strong nonlinear spring characteristics. The nonlinearity is of geometrical type. The absorber is suggested to be installed on the building that is forced to vibrate. Model of the system are two coupled strong nonlinear differential equations. Condition for steady state solution for nonlinear resonant case is determined. Using this result we developed an approximate solving procedure for the perturbed equations. The necessary parameters of the mechanism for vibration absorption are computed.

2. MODEL OF THE NONLINEAR ABSORBER

The nonlinear vibration absorber is designed as a mass-spring system shown in Fig.1.

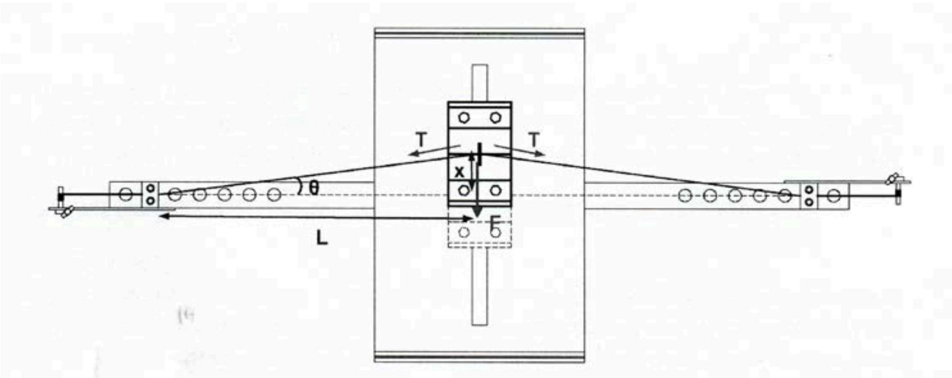

Figure 1. Model of the absorber [4]

The mass is realized with a slider which has a straightline motion. Across the slider in the perpendicular direction a steel wire is installed. The ends of the wire are fixed. The slider is in the middle of the wire. The displacement of slider causes deformation of the wire

$$\Delta \mathrm{L} = \sqrt{L^2 + x^2} - L \tag{1}$$

where L is the length of the wire and x is the displacement. Due to elastic elongation the tension in both parts of the wire is

$$\mathrm{T} = \frac{EA\Delta L}{L} \tag{2}$$

where E is the Young's modulus and A is the cross-section of the wire. For the angle θ, formed between the initial and deformed position of the wire, and $\sin\theta = x/\sqrt{x^2 + L^2}$ the restition force in x direction is obtained as

$$F = 2T\sin\theta = 2\frac{EA}{L}x(1-\frac{L}{\sqrt{L^2+x^2}}). \tag{3}$$

Using the first term of the series expansion of (3) the relation simplifies into

$$F = k_{na}x^3, \quad k_{na} = \frac{EA}{L^3}, \tag{4}$$

where k_{na} is the coefficient of nonlinearity. Thus, the designed nonlinear absorber is of nonlinear energy sink type and modeled with a pure nonlinear spring characteristic.

3. MODEL OF THE BUILDING-ABSORBER SYSTEM

The building-absorber is modeled as a nonlinear oscillator with attached nonlinear tuned mass damper two degree of freedom system (Fig.2)

$$\begin{aligned} &m\ddot{x} + kx^3 + k_{na}(x-x_{na})^3 + c\dot{x} + c_{na}(\dot{x}-\dot{x}_{na}) = F(t), \\ &m_{na}\ddot{x}_{na} + k_{na}(x_{na}-x)^3 + c_{na}(\dot{x}_{na}-\dot{x}) = 0, \end{aligned} \tag{5}$$

where *m*, *c* and *k* denote the mass, damping and stiffnes of the building, '*na*' is used for absorber and *F*(*t*) is the excitation force.

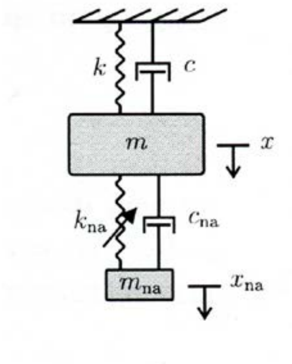

Figure 2. Model of the building-absorber system [1]

Introducing the relative displacement $z=x-x_{na}$ the equations (5) are transformed into

$$\begin{aligned} &\ddot{z} + \frac{k}{m}x^3 + \frac{k_{na}}{\mu}z^3 = \frac{F(t)}{m} - \frac{c}{m}\dot{x}\frac{c_{na}}{\mu}\dot{z}, \\ &m_{na}(\ddot{x}-\ddot{z}) - k_{na}z^3 = c_{na}\dot{z}, \end{aligned} \tag{6}$$

where $\mu=m_{na}m/(m_{na}+m)$. Let us assume that the damping is neglected and the excitation is a polyharmonic function. Using the condition of the nonlinear resonant case [6] the exact steady state vibration exist if the system is forced with the function

$$F(t) = F_0 cn^3(\Omega t, 1/2), \tag{7}$$

where cn is the cosine Jacobi elliptic function, Ω is the frequency of the function and F_0 and F_1 are amplitudes of excitation forces. Model of the system simplifies into

$$\ddot{z} + \frac{k}{m}x^3 + \frac{k_{na}}{\mu}z^3 = \frac{F_0}{m}cn^3, \quad m_{na}(\ddot{x}-\ddot{z}) - k_{na}z^3 = 0, \tag{8}$$

where $cn=cn(\Omega t,1/2)$. Introducing the solutions

$$x = Acn(\Omega t, 1/2), \quad z = Bcn(\Omega t, 1/2) \tag{9}$$

into (8) and equating the terms with the same order of the cn function, two algebraic equations follow

$$-B\Omega^2 + \frac{k}{m}A^3 + \frac{k_{na}}{\mu}B^3 = \frac{F_0}{m}, \quad m_{na}(A-B)\Omega^2 + k_{na}B^3 = 0. \tag{10}$$

After some transformation it is

$$A = B - \frac{k_{na}B^3}{\Omega^2 m_{na}}, \quad B^3\frac{k}{m}\left(1-\frac{k_{na}B^2}{\Omega^2 m_{na}}\right)^3 + \frac{k_{na}}{\mu}B^3 - B\Omega^2 - \frac{F_0}{m} = 0. \tag{11}$$

Solving the sixth order algebraic equation for B, the exact solution for A is obtained.

4. PARAMETERS OF ABSORBER FOR BUILDING MOTION ELIMINATION

The intention of our investigation is to determine parameters of the absorber that eliminate the motion of the building. Namely, if the amplitude of vibration of the building is zero and A=0, according to (11) we have

$$B = \Omega / \sqrt{\frac{k_{na}}{m_{na}}}, \quad \frac{\Omega^3}{F_0} = \sqrt{\frac{k_{na}}{m^3{}_{na}}}. \tag{12}$$

After some transformation we express

$$B = \frac{F_0}{m_{na}\Omega^2}, \quad k_{na} = m_{na}^3\frac{\Omega^6}{F_0^2}. \tag{13}$$

Substituting the relation $(4)_2$ into $(13)_2$ the necessary conditions for vibration elimination are obtained

$$\frac{EA}{m_{na}^3 L^3} = \frac{\Omega^6}{F_0^2}. \tag{14}$$

The relation (14) gives the parameters of the absorber for the case when amplitudes of vibration are zero. To achieve this requirement automatic control of the length of the wire has to be introduced. It is possible to be realized by moving the fixed ends of the wire according to the measured parameters of the excitation force: intensity and frequency.

5. TRANSIENT MOTION FOR THE SYSTEM WITH NONLINEAR ABSORBER

Damping in the building-absorber acts as the small perturbation for motion. The mathematical model of the system (6) transforms into

$$\begin{aligned} &\ddot{z} + \frac{k}{m}x^3 + \frac{k_{na}}{\mu}z^3 = \frac{F_0}{m}cn - \frac{\varepsilon c}{m}\dot{x} - \frac{\varepsilon c_{na}}{\mu}\dot{z}, \\ &m_{na}(\ddot{x}-\ddot{z}) - k_{na}z^3 = \varepsilon c_{na}\dot{z}, \end{aligned} \tag{15}$$

where ε<<1 is a small parameter. Let us introduce the analytic solution of (15) and the first derivatives as

$$x = A(t)cn\psi(t), \quad \dot{x} = -A(t)\Omega sn\psi(t)dn\psi(t),$$
$$z = B(t)cn(\psi(t)), \quad \dot{z} = -B(t)\Omega sn\psi(t)dn\psi(t), \tag{16}$$

with

$$\dot{\psi}(t) = \Omega + \dot{\theta}(t), \tag{17}$$

where A(t)=A and ψ(t)=ψ are time variable amplitude and phase and sn and dn are Jacobi elliptic functions. Comparing the assumed and computed first time derivatives of the solution (9) the constraint follows

$$\dot{A}cn - A\dot{\theta}_1 sndn = 0, \quad \dot{B}cn - B\dot{\theta}_2 sndn. \tag{18}$$

Substituting (16) into (15) we have

$$-(\dot{B}\Omega + B\dot{\Omega})sndn - B\Omega\dot{\theta}_2 cn^3 + \frac{k}{m}A^3cn^3 = \frac{\varepsilon c}{m}A\Omega sndn + \frac{\varepsilon c_{na}}{\mu}B\Omega sndn,$$
$$-(\dot{A}\Omega + A\dot{\Omega})sndn - A\Omega\dot{\theta}_1 cn^3 + (\dot{B}\Omega + B\dot{\Omega})sndn + B\Omega\dot{\theta}_2 cn^3 = -\frac{\varepsilon c_{na}}{m_{na}}B\Omega sndn. \tag{19}$$

Eliminating $\dot{\theta}_1$ and $\dot{\theta}_2$ from (18) and (19) it is

$$-(\dot{B}\Omega + B\dot{\Omega})sn^2dn^2 - \dot{B}\Omega cn^4 + \frac{k}{m}A^3cn^3sndn = (\frac{\varepsilon c}{m}A + \frac{\varepsilon c_{na}}{\mu}B)\Omega sn^2dn^2,$$
$$-(\dot{A}\Omega + A\dot{\Omega})sn^2dn^2 - \dot{A}\Omega cn^4 + (\dot{B}\Omega + B\dot{\Omega})sn^2dn^2 + \dot{B}\Omega cn^4 = -\frac{\varepsilon c_{na}}{m_{na}}B\Omega sn^2dn^2. \tag{20}$$

Using the derivative for Ω and averaging over the period of the elliptic functions

$$T = \frac{4K(1/2)}{\Omega}, \tag{21}$$

the equations transform into

$$\dot{B} + \beta\frac{\varepsilon c_{na}}{\mu}B = -\beta\frac{\varepsilon c}{m}A, \quad \dot{A} + \beta\frac{\varepsilon c}{m}A = \beta\frac{\varepsilon c_{na}}{m}B, \tag{22}$$

where K(1/2) is the complete elliptic integral of the first kind and β= $\frac{1}{T}\int_0^T sn^2dn^2dt$. The averaged equation of amplitude variation of the building is

$$\ddot{A} + 2\delta\dot{A} + \omega^2 A = 0, \tag{23}$$

where

$$2\delta = \beta(\frac{\varepsilon c}{m} + \frac{\varepsilon c_{na}}{\mu}), \quad \omega^2 = \beta^2\frac{(\varepsilon c)(\varepsilon c_{na})}{m}(\frac{1}{m} + \frac{1}{\mu}). \tag{24}$$

Solution of the equation is in general

$$A = \mathrm{R}e^{-\delta t}\sin(\kappa t + \alpha), \tag{25}$$

where $\kappa = \sqrt{\omega^2 - \delta^2}$, R and α are constants of integration. Using the condition of the unperturbed motion when A=0 it is concluded that the initial phase angle is zero, i.e., α=0. The relation (25) represents the transient motion to the steady state.

6. CONCLUSION

In the paper a new type of nonlinear absorber based on the piano-wire vibration concept is suggested. It is of nonlinear enrgy sink type. It contains a straightforward translatory moving mass settled on a steel wire installed in the direction perpendicular to slider motion. Due to slider motion in the wire a restoring force occurs. The force is a nonlinear deflection function. Using the principle of elimination of building motion, parameter of nonlinearity is computed. The model of the system are two coupled second order nonlinear differential equations. The nonlinearity is of cubic order. Based on the exact steady state solution that corresponds to nonlinear resonant vibration, we developed an approximate solving method for the perturbed equations. Finally, the conditions for vibration elimination are computed. We suggest the length of the wire as the control parameter in this absorber.

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ВИБРАЦИОНИ АБСОРБЕР ЗА ЗГРАДУ ПРИ СЕИЗМИЧКОМ ПОРЕМЕЋАЈУ

Резиме: *Постоји велики број различитих метода за елиминацију вибрација проузрокованих сеизмичким поремећајима. Поред пасивних абсорбера вибрација постоји и велики број активних абсорбера. На жалост, ови други су врло скупи, јер их је неопходно стално одржавати иако раде само повремено у случајним приликама. То даје предност пасивним абсорберима. У овом раду је предложен* *нови тип нелинеарног абсорбера чији је рад заснован на концепту осциловања клавирске жице. Абсорбер је везан за зграду. Састоји се од клизача, који се креће транслаторно праволинијски, и од челичне жице која је провучена кроз клизач и има правац управан на кретање клизача. Услед кретања клизача жица се деформише и у њој се генеришереституциона сила. Сила је нелинеарна функција деформације. Користећи принцип елиминације кретања зграде срачуна се потребан параметар нелинеарности. Модел система су две спрегнуте нелинеарне диференцијалне једначине другог реда. Нелинеарност је трећег реда. Одреди се устаљено осцилаторно кретање за случај нелинеарног резонантног режима. Користећи овај резултат у раду је развијен аналитички метод за решавање поремећене једначине кретања. Решење је претпостављено у форми Јакоби елиптичке функција. Најзад, одређени су услови за елиминацију вибрација.*

Кључне речи: *Пасивни абсорбер, концепт клавирске жице, нелинеарни резонантни режим, Јакоби елиптичка функција*