ON VISCOELASTIC PROPERTIES OF ASPHALT MIXTURES

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Summary: *This paper comprises the fractional Kelvin-Zener model of viscoelastic body, the Laplace transform and a least squares method, all applied in creep/recovery testing of asphalt mixtures for the purpose of parameter identification. The parameters describing viscoelastic properties of these mixtures are: the order of the fractional derivative, modulus of elasticity, as well as two relaxation constants that obey restrictions that follow from the second law of thermodynamics. Knowing these four parameters one may predict the behavior of an asphalt mixture for different loads. Besides, the pattern of change of these parameters may be related to alterations of the viscoelastic properties of an asphalt mixture due to either aging or different environmental conditions.*

Keywords: *fractional calculus, system identification, viscoelasticity, asphalt mixtures*

1. INTRODUCTION

Predicting a response of real man-made materials for given load is not an easy task. This is usually interrelated with diverse reasons, but the main cause is related to a complex internal structure of these real materials which in combination with complex environmental conditions leads to the complex rheological behavior to be modeled. In this paper, a rather simple procedure based on a constitutive law of fractional order is recommended for both parameter identification and simulation in time domain. In the following the time-dependent viscoelastic behavior of asphalt mixtures is analyzed. In doing so experiments following creep/recovery deformation pattern were used. Namely, the corresponding stress will be given in a form of constant rectangular pulse so the expected strain time evolution will have the viscoelastic or viscoelastoplastic deformation during loading, the instantaneous elastic recovery, i.e. a jump in creep compliance curve, that is followed by the recovery phase, see Fig. 1. This type of studies is widely recognized and important in the analysis of metallic glasses [1,2], asphalt mixtures [3,4] and dental materials [5,6]. However, it must be noted that due to the jump, it is not easy to describe both creep and recovery behavior with the accepted

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constitutive axiom and the same set of parameters. In some cases, when the corresponding stress level is sufficiently small the fractional Kelvin-Zener model (FKZM) of viscoelastic body, see [8], can yield the result, i.e. the creep/recovery deformation pattern can be described by the Mittag-Leffler function and the same set of parameters in both phases. In other situations, with higher stress levels the behavior of asphalt mixture becomes viscoplastic with a significantly greater level of residual strain which can be accompanied with unsatisfactorily results when compared to the experiments, especially in the recovery phase. To overcome this problem one may propose different functions for the recovery phase as in [3, 4] for example. Here we intend to show that the good prediction can be obtained by keeping the same function in both phases but with a change of only two parameters corresponding to recovery phase comparing to the values in the loading phase.

2. PROBLEM FORMULATION

Consider a viscoelastic rod of a given length starting to deform from its virginal state subjected to stress

$$\sigma = \kappa(H(t) - H(t-k)), \tag{1}$$

where κ is constant and where the Heaviside step function is recognized. It is assumed that the rod model is given in the form corresponding to the fractional Kelvin-Zener constitutive axiom, FKZM for short, i.e.

$$\sigma + a\sigma^{(\alpha)} = E(\varepsilon + b\varepsilon^{(\alpha)}) \tag{2}$$

with $0<\alpha<1$, $E>0$, $b>a>0$, respectively being the order of the Riemann-Liouville fractional derivative used, the modulus of elasticity, and the relaxation constants obeying constraints that follow from the Clausius-Duhem inequality.

The straightforward application of the Laplace transform and its inverse yields the corresponding strain in the following form

$$\varepsilon(t) = \begin{cases} \dfrac{\kappa}{E}\left(1-\left(1-\dfrac{a}{b}\right)e_\alpha\left(t,\dfrac{1}{b}\right)\right) & for \quad 0 < t \le k, \\ \dfrac{\kappa}{E}\left(1-\left(1-\dfrac{a}{b}\right)e_\alpha\left(t,\dfrac{1}{b}\right)\right) - \dfrac{\kappa}{E}\left(1-\left(1-\dfrac{a}{b}\right)e_\alpha\left(t-k,\dfrac{1}{b}\right)\right)H(t-k) & for \quad t > k, \end{cases} \tag{3}$$

where $e_\alpha\left(t,\dfrac{1}{b}\right)$ stands for the generalized Mittag-Leffler function, see [7]. From the above expression one may easily get the jump corresponding to instantaneous elastic recovery at *t=k*. It reads

$$\Delta\varepsilon(k) = \frac{\kappa}{E}\frac{a}{b}. \tag{4}$$

Thus, we may speculate that this model may have a chance to capture creep/recovery behavior.
The problem can be cast in dimensionless form and the corresponding creep compliance curve can be easily obtained.

3. RESULTS

In order to relate this model to real data, we are going to find the parameters corresponding to the material-asphalt mixture Hot Rolled Asphalt (HTA). This material is tested at laboratory of University of Nottingham to uniaxial creep compression. The details of the creep/recovery test and the corresponding results were reported in Fig. 8 of [4]. It is important to note that these creep/recovery tests are characterized with high stress intensity of 1MPa and long lasting recovery period. In this work, one of these three tests is going to be considered, namely the test when the unloading phase starts at 25s. The values of material parameters are obtained by use of the fitting procedure known as a method of least squares. As a result, the fitting within the creep phase from the experimental results, the following values of the material FKZM parameters are obtained

$$\alpha = 0.537,\ E = 39.64,\ a = 1.127,\ and\ \ b = 39.64.$$

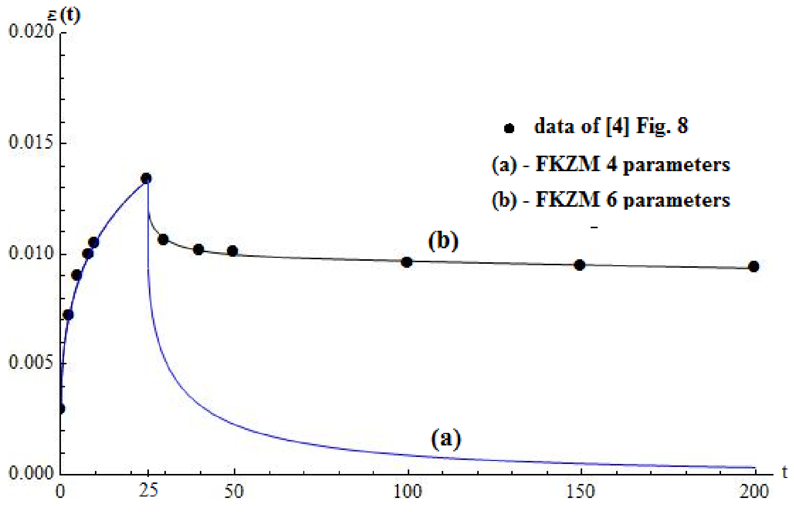

Figure 1. Agreement between the experimental data and this model

The obtained compliant curve (a) for 4 parameters FKZM is shown in Fig. 1. As expected, since the stress intensity was high, there was great mismatch between obtained and experimental results in the recovery phase. To overcome this, we are going to change two parameters in the recovery phase, i.e. the modulus of elasticity E and the relaxation constant b. This new values of the parameters to be used in the recovery phase will be denoted by E_r and b_r. Thus the corresponding strain of the mixture model reads

$$\varepsilon(t)=\begin{cases}\frac{\kappa}{E}\left(1-\left(1-\frac{a}{b}\right)e_{\alpha}\left(t,\frac{1}{b}\right)\right) & for\ \ 0<t\le k,\\ \frac{\kappa}{E}\left(1-\left(1-\frac{a}{b}\right)e_{\alpha}\left(t,\frac{1}{b}\right)\right)-\frac{\kappa}{E_r}\left(1-\left(1-\frac{a}{b_r}\right)e_{\alpha}\left(t-k,\frac{1}{b_r}\right)\right)H(t-k) & for\ \ t>k,\end{cases} \tag{5}$$

having the same functions but now related with 6 parameters. With this type of FKZM the creep compliant curve is much better as shown in Fig. 1. The model (5) and the following material parameters

$$\alpha=0.537,\ E=39.64,\ a=1.127,\ b=39.64,\ E_r=51.98,\ b_r=20.11$$

yield very good agreement between the predicted and experimental results.

4. CONCLUSIONS

The results presented in this paper indicate the potential application of fractional Kelvin-Zener model in rheological characterization of asphalt mixtures following the creep-recovery experiments. It should be noted that the strain history was obtained by use of the Laplace transform and its inverse with two parameters changed in the recovery phase. Then the method of least squares yields acceptable results. An example is given to show the effectiveness of this approach. We believe that this approach has a chance to be effective for some other materials.

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О ВИСКОЕЛАСТИЧНИМ СВОЈСТВИМА АСФАЛТНИХ МЕШАВИНА

Резиме: *У овом раду повезују се методи инверзне Лапласове трансформације и најмањих квадрата за идентификације параметара система, фракциони Келвин-Зенеров модел вискоеластичног тела и тестови течења/релаксације изведени на асфалним мешавинама. Вискоеластична својства мешавине описаће се редом фракционог извода, модулом еластичности, и са две релаксационе константе које испуњавају рестрикције које произилазе из другог закона термодинамике. Познавањем ових параметара може се предвидети понашање асфалтне мешавине за различита оптерећења. Поред тога промене вискоеластичних својстава асфалтне мешавине услед старења или различитих услова експлоатације могу се повезати са обрасцем промене ових параметара.*

Кључне речи: *фракциони рачун, идентификација параметара, вискоеластичност, асфалтне мешавине*