DYNAMIC RESPONSE OF A STRUCTURE TO EARTHQUAKE EXCITATION

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Summary: *Behavior of a column like structure during earthquake excitation is studied. The structure consists of rigid blocks which move translatory during a horizontal ground motion. Fractional derivatives are used within a constitutive model of a viscoelastic damper. Dry friction damping during relative motion of the blocks is also considered. Governing equations of motion of the structure subjected to a single component horizontal ground acceleration are derived. The posed problem is solved by use of mathematical tools for dealing with non-smooth mechanical systems containing fractional derivatives.*

Keywords: *column like structure, earthquake response, fractional Zener model, dry friction*

1. INTRODUCTION

Seismic response of a structure is a very complex process. On the one hand, dynamical models of structures and excitations need to be complex enough to take all necessary effects into account, but on the other hand, dealing with too complicated models can lead to significant difficulties and problems when mathematical analysis and calculations are performed. Various mathematical models of earthquake induced ground motion are developed. Earthquake models which are stationary and non-stationary in both the time and the frequent domain, are presented in [1], [2], [3]. A simplified model based on Ricker's wavelets is given in [4]. Also, there are many different mathematical models and methods for describing the dynamics of structures exposed to seismic excitation. Springs, dashpots and frictional elements are usually used within constructions under consideration. Earthquake response of a multi-storey building, in which the storeys are connected to each other via systems of springs and dashpots, is analyzed in [5].

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Damping system with lumped mass for a longitudinal vibration is studied in [6]. In [7] viscoelastic elements are modelled by the fractional Zener model, while the set-valued Coulomb friction law is used to describe the behavior of a frictional element. In this work we study dynamic response of a column like structure to a horizontal single component ground excitation caused by the real earthquake.
The construction is equipped with a viscoelastic and a friction damper, which both dissipate energy during the relative motion of parts of the structure. A similar problem was treated in [8], where a simplified earthquake excitation was used.

2. SYSTEM UNDER CONSIDERATION

In this paper we analyze seismic response of a structure consisting of two rigid blocks and which is equipped with two types of passive systems for seismic protection. The blocks move translatory in a horizontal direction due to a ground excitation.
The lower block moves together with a foundation, while the upper block of mass *m* slides along the lower one, see Figure 1a. During the relative motion of the blocks, the energy dissipates through both the deformation of the viscoelastic rod and the work done by friction between the blocks.

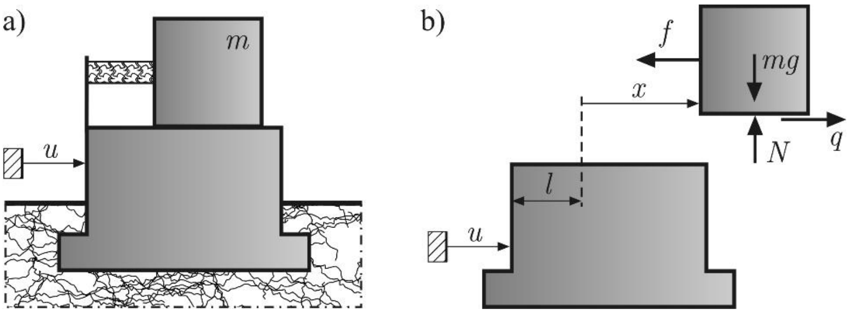

Figure 1. a) A column like structure, b) free body diagram.

The absolute position of the lower block is denoted by *u*, the relative position of the upper block is presented by coordinate *x*, and *l* stands for the length of the rod in undeformed state, see Figure 1b. Normal contact force *N* equals *mg* while the friction force *q* and the force *f* in the viscoelastic rod change during the relative motion of the blocks and need to be calculated.

Governing equations with initial conditions read

$$m \cdot (u^{(2)} + x^{(2)}) = -f + q, \tag{1}$$

$$x(0) = 0, \quad x^{(1)}(0) = 0, \quad f(0) = 0, \tag{2}$$

$$f + \tau_{f\alpha} f^{(\alpha)} = \frac{E_\alpha A}{l}(x + \tau_{x\alpha} x^{(\alpha)}), \tag{3}$$

where equations (1) and (3) represent the fundamental axiom of dynamics and constitutive equation of the viscoelastic rod in the form of the fractional Zener model, see [9]. Designation $(*)^{(\alpha)}$ is used to denote the time derivative of order α of a function (*), i.e. $(*)^{(\alpha)}=d^\alpha(*)/dt^\alpha$, E_α stands for the modulus of elasticity, A represents the cross sectional area of the rod. Constants $\tau_{f\alpha}$ and $\tau_{x\alpha}$ with dimension $[\text{time}]^\alpha$ and E_α must satisfy conditions and $\tau_{x\alpha} > \tau_{f\alpha} > 0$, $E_\alpha > 0$, according to the Clausius-Duhem inequality. In constitutive equation (3) the Riemann-Liouville fractional derivative of the force f and the displacement x is recognized, $0<\alpha<1$, see [10]. The friction force q is modelled by the set-valued Coulomb friction law

$$q \in -\mu N \,\mathrm{Sgn}(x^{(1)}) \tag{4}$$

where Sgn(z) represents the set-valued sign function which is set valued at z=0, meaning that $\mathrm{Sgn}(z) \in [-1,1]$ for z=0, see [11]. Similar problem was studied in [8] where a simplified earthquake model by means of Ricker's waves was used. In this paper, for the ground excitation we use real seismic data, recorded in 1940, during El Centro earthquake. The acceleration over time for North-South component is presented in Figure 2 and used for horizontal ground acceleration $u^{(2)}(t)$ of the column like structure. The data are taken from [12].

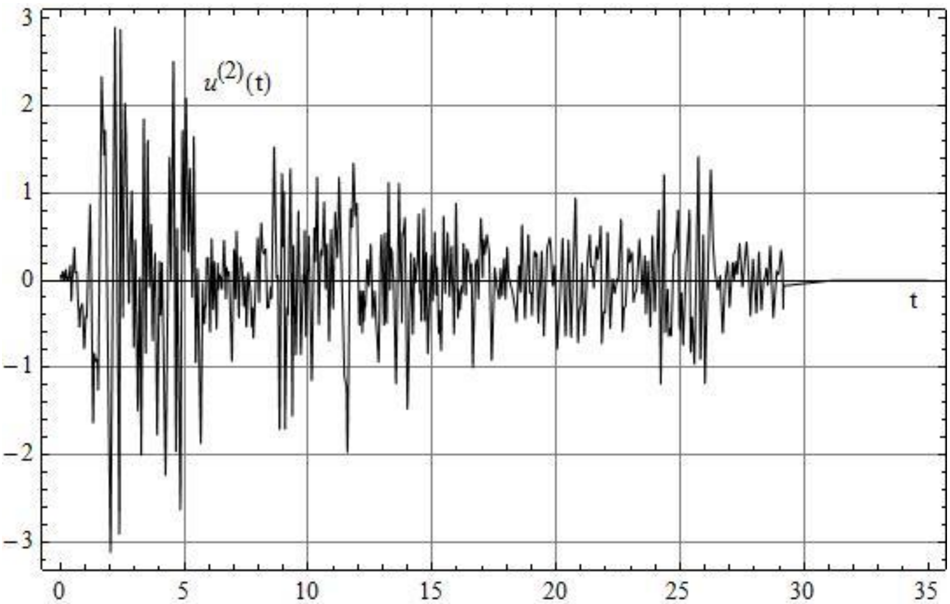

Figure 2. Real earthquake excitation (El Centro North-South component), acceleration in [m/s^2] over time in [s].

Introducing dimensionless quantities as in [9]

$$\bar{t} = t\sqrt{\frac{E_\alpha A}{ml}},\ \bar{x} = \frac{x E_\alpha A}{mgl},\ \bar{u} = \frac{u E_\alpha A}{mgl},\ \bar{p} = \frac{p}{mg},\ \bar{q} = \frac{q}{mg},$$
$$\bar{\tau}_{x\alpha} = \tau_{x\alpha}\left(\frac{E_\alpha A}{ml}\right)^{\frac{\alpha}{2}},\quad \bar{\tau}_{p\alpha} = \tau_{p\alpha}\left(\frac{E_\alpha A}{ml}\right)^{\frac{\alpha}{2}}, \tag{5}$$

and omitting the bar, the problem is presented in the dimensionless form:

$$u^{(2)} + x^{(2)} = -f + q,\qquad f + \tau_{f\alpha} f^{(\alpha)} = x + \tau_{x\alpha} x^{(\alpha)},$$
$$q \in -\mu \operatorname{Sgn}(x^{(1)}) \tag{6}$$

with dimensionless initial conditions which are of the same form as (2) and with dimensionless ground motion *u*. The aim is to determine the functions $x(t)$, $f(t)$ and $q(t)$ for known ground excitation and parameters of the model.

3. THE SOLUTION

System of equations in non-dimensional form (6) together with the initial conditions of the form (2) represent governing equations of a non-smooth fractional order mechanical system. Such problems were treated in [7], [8] and [13], to mention some of them. Different motion phases are characterized by different sets of differential equations. The solution of the posed problem will be obtained by the numerical procedure suggested in [14], which is used in the similar problem, see [8].

After time dicretization with a time step κ, $t_r = r\kappa$, (r=0,1,2,...), using the Grünwald-Letnikov definition of fractional derivatives and following the lines of [8] we obtain the numerical algorithm for calculation of functions $x(t)$ and $f(t)$ in dicretized time instants

$$f_r = \frac{1}{1 + \tau_{f\alpha}\kappa^{-\alpha}}\left\{x_r\left(1 + \tau_{x\alpha}\kappa^{-\alpha}\right) + \kappa^{-\alpha}\sum_{j=0}^{r}\omega_j\left(\tau_{x\alpha}x_{r-j} - \tau_{f\alpha}f_{r-j}\right)\right\}, \tag{7}$$

$$x_{r+1} = \kappa^2(-u_r^{(2)} - f_n - \mu) + 2x_r - x_{r-1},\ \text{ for }\ x_r^{(1)} > 0, \tag{8}$$

$$x_{r+1} = \kappa^2(-u_r^{(2)} - f_n + \mu) + 2x_r - x_{r-1},\ \text{ for }\ x_r^{(1)} < 0, \tag{9}$$

where coefficients ω_j are calculated by the recurrence relationship

$$\omega_0 = 1,\ \omega_j = \left(1 - \frac{\gamma + 1}{j}\right)\omega_{j-1},\ (j = 1,2,3,...). \tag{10}$$

During the stick phase, where $x^{(1)}=0$ and x=const., due to the usage of set-valued sign function, the friction force q_r can be any value from the interval $[-\mu, \mu]$. Applying the fundamental axiom of dynamics $(6)_1$ the friction force within the stick phase is calculated by

$$q_r = p_r + u_r^{(2)},\ r > 0. \tag{11}$$

Different motion phases (sliding to the right and left and stick) alternate during the motion of the system according to its dynamics during the earthquake excitation, implying the usage of different sets of equations during each phase. Also, the time step is changed during the calculation procedure every time the new motion phase takes place, see [9]. Changes of ground acceleration direction are rapid and numerous in real excitation, like in Figure 2, which leads to frequent changes of motion phase. Thus, it is suitable to automatize the procedure for calculation of the position of the block, force in the viscoelastic rod and the friction force. A numerical example is presented in the next section.

4. RESULTS

An example of a dynamic response of the column like structure to the real single component horizontal excitation, by the use of presented numerical algorithm, is shown. The relative position $x(t)$ of the upper block and the force $f(t)$ in the viscoelastic rod are presented in Figure 3, where the system parameters are chosen to be: α=0.23, τ_{xa}=1.183, τ_{pa} =0.004, μ=0.085 and time step κ=0.0005. For the same set of parameters, relative velocity $x^{(1)}(t)$ of the upper block and the friction force are shown in Figure 4. In this figure we see that the system changes all three motion phases during the earthquake excitation. All results are presented in dimensionless form.

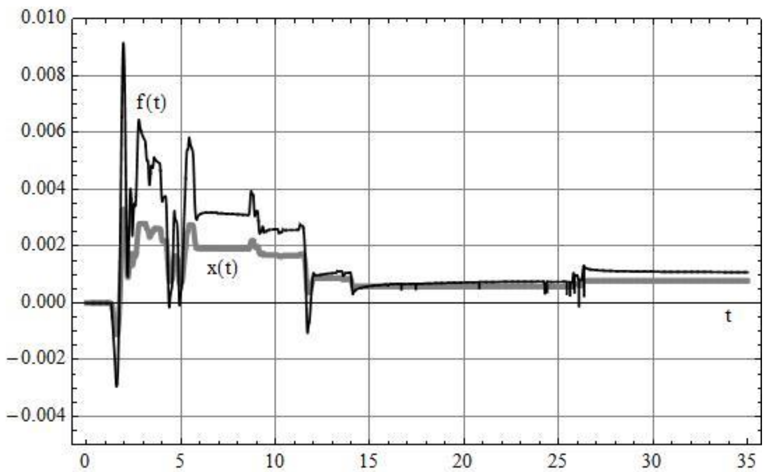

Figure 3. Relative position x(t) and force f(t) in the viscoelastic rod.

The forces *p*(*t*) and *q*(*t*) perform work dissipating the energy during the sliding phases only, i.e. when there exists relative motion between the blocks. In these cases the friction force riches either the upper or the lower limiting value, while $x^{(1)}(t)\neq 0$. Within the stick phase, where $x^{(1)}(t)=0$, the friction force continually changes its magnitude in the state of a relative equilibrium. Figure 5, in which the relative velocity and the friction force are shown during a shorter time interval $t\in[0,6]$, clearly presents these effects.

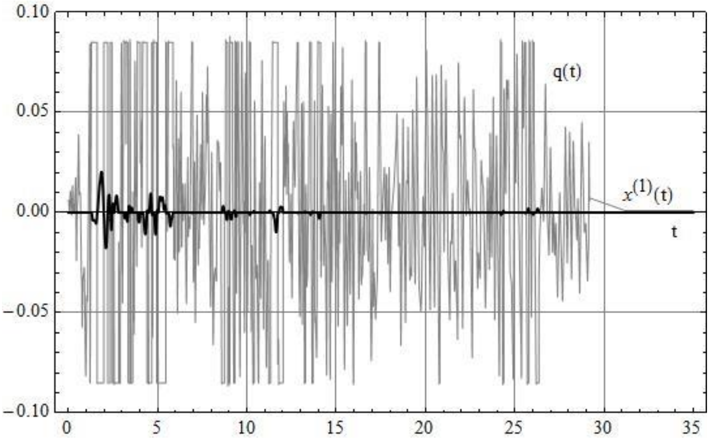

Figure 4. Relative velocity $x^{(1)}(t)$ and friction force q(t).

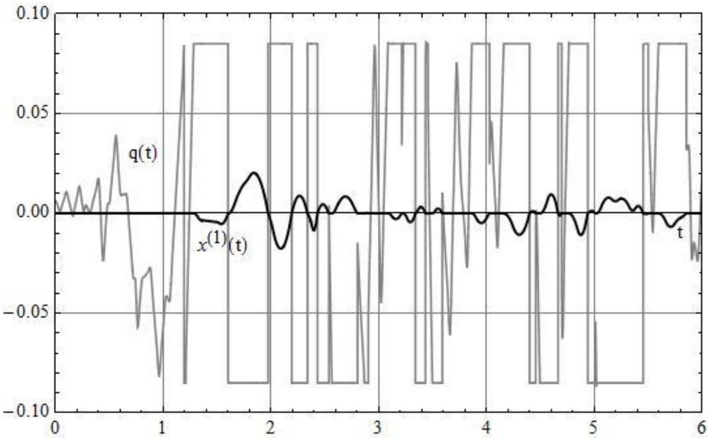

Figure 5. Relative velocity $x^{(1)}(t)$ and friction force q(t), for $t\in[0,6]$.

5. CONCLUSIONS

In this paper the seismic response of a column like structure with a viscoelastic and a friction damper is analyzed. One horizontal component of a real seismic excitation was applied. Governing equations, which include fundamental axiom of dynamics and constitutive laws for the viscoelastic and for the friction damper, are derived in dimensionless form. According to the initial conditions and the restrictions to the parameters of the viscoelastic model, solutions are obtained using the numerical algorithm (7)-(11). For a chosen set of values of system parameters solutions are obtained and shown in Figures (3)-(5). Changes of the ground acceleration of a real excitation data during time are rapid comparing with not so frequent changes within the simplified earthquake model, see [8]. Thus, the integration step was chosen to be very small, in order to take into account fast alterations of motion phases.

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ДИНАМИЧКИ ОДЗИВ КОНСТРУКЦИЈЕ НА ЗЕМЉОТРЕСНУ ПОБУДУ

Резиме: *Проучено је понашање конструкције у облиу стуба при дејству земљотреса. Конструкција се састоји од крутих блокова који се крећу транслаторно током хоризонталног кретања тла. Фракциони изводи су коришћени у конститутивном моделу вискоеластичног пригушивача. Такође је разматрано пригушење услед присуства сувог трења. Изведене су основне диференцијалне једначине кретања конструкције изложене дејству једне компоненте хоризонталног убрзања тла. Постављени проблем је решен помоћу математичких алата намењених за неглатке механичке системе који садрже фракционе изводе.*

Кључне речи: *стубна структура, сеизмички одзив, фракциони Зенеров модел, суво трење*