# ANALYSIS AND DESIGN OF RC DEEP BEAM USING THE "ST METHOD" PROGRAM

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***Summary:*** *This paper presents a design of reinforced concrete deep beam by means of Strut-and-Tie method. Numerical analyzes were conducted with the "ST method" program, which automatically defines Strut-and-Tie models and dimensioning of plane RC members. The reinforced concrete deep beam with opening was analyzed. Nodal zones of Strut-and-Tie model were analyzed, as hydrostatic, by "hand" calculation, and as nonhydrostatic, using the "ST method" program. The obtained results of the both design approaches were compared.*

***Keywords:*** *rc deep beam, "ST method", Strut-and-Tie model, nodal zone*

## 1. INTRODUCTION

The Strut-and-Tie method can be used for design of reinforced concrete members, with static or geometric discontinuities [1]. The method replaces the real member with equivalent, truss member, which consists of elements, with determined dimensions, compressed or tensioned, and their places of intersection, the so-called nodal zones also with defined dimensions. The basic principle of the method is that the compressed elements replace concrete parts of reinforced concrete member, while reinforcements are modeled with tensioned elements. Crushing of reinforced concrete member may appear due to capacity loss of compression elements, nodal zones or due to exceeding the yield strength of reinforcement, i.e. tensioned elements. For analysis purposes, "ST method" program was made that automatically generates models, calculates forces in the elements of replaced equivalent member, defines and controls stress state in the elements of Strut-and-Tie model. Algorithmic approach of the reinforced concrete plane member design, applied in the "ST method" is presented with more details in the paper [2]. In this paper, the rc wall with an opening has been analyzed in two ways. "Hand" calculation is done according to [3], where the nodes are analyzed as hydrostatic. Thus obtained results were

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compared with the results from the "ST method" program, where the nodes are analyzed as nonhydrostatic. Nodal zone description of Strut-and-Tie model is presented in the following section.

## 2. NODAL ZONE IN STRUT-AND-TIE MODEL

### 2.1 Introduction

Nodes of Strut-and-Tie model are points where the axial forces, of the replaced truss, intersect, while the nodal zones are an area around the associated node surface where the struts and ties are connected. For vertical and horizontal equilibrium of plane member, such as deep beam, in the node at least three forces must act.
The nodes are divided according to the type of forces that intersect in the node. CCC node contains three struts, CCT node contains two struts and one tie, and CTT node contains one strut and two ties, Fig. 1 [4]. In nodal zone that is under pressure, it is assumed that the widths of the nodal zone edges are equal to the widths of the strut ends that are crossed.

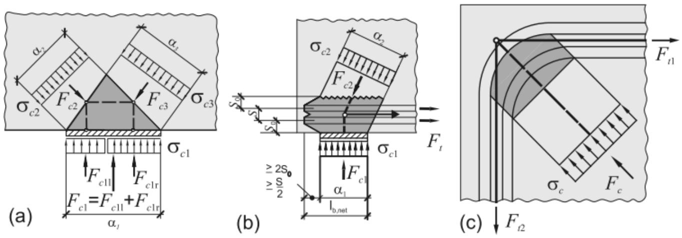



*Fig. 1 - Nodes: a) CCC, b) CCT and c) CTT, [4]*

Nodal zones can be hydrostatic and nonhydrostatic, Fig. 2. Hydrostatic nodal zones are defined so that all the stresses acting on their sides are equal, and in nonhydrostatic that condition does not need to be fulfilled [5].

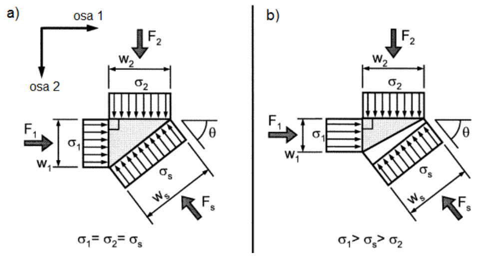



*Fig. 2 – Nodal zones: a) hydrostatic and b) nonhydrostatic [5]*

One disposition of the nodal zone design is shown in the paper [6]. In the nodal zones of Strut-and-Tie model, where plane stress state exists, discontinuous stress field occurs, Fig. 3 [6]. The line of discontinuity separates two continuous stress fields.
Besides the basic equations of equilibrium, there are additional equations in place of discontinuity, in order to satisfy stress continuity at the border of each continuity, Fig. 3 [6].

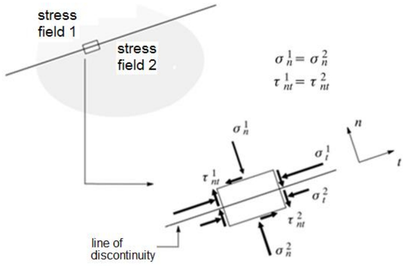



*Fig. 3. Stress discontinuity, according to [6]*

Next chapter of this paper presents a solution of the nodal zone by means of finite element method within the „ST method“ program.

## 2.2. Nodal zone in "ST method" program

In "ST method" program, the nodal zones are analyzed as nonhydrostatic. The nodal zone is defined as polygon, whose geometry is defined by the width of rods that are crossing at the node. Polygon vertices are obtained by interconnection of points defined with edge line sections of struts and ties, Fig. 4 (left). Stress state in so defined nodal zone is controlled by Kupfer criterion (1969), Fig. 4 (middle), for biaxial stress state in concrete [4].

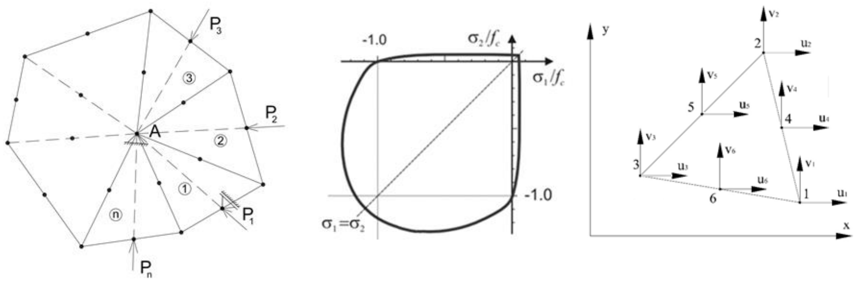



*Fig. 4 - Nodal zone (left), allowed biaxial stress state in concrete (middle [4]) and triangular finite element (right [8])*

Polygon, Fig. 4 (left), that represents the nodal zone, is modeled with multiple triangular finite elements that have a common point, node A, where all main axis of corresponding Strut-and-Tie model elements intersect. In Fig. 4 (right) triangular element is shown, with 12 freedom degrees of displacement, two in each node, and the nodes are arranged in the vertices and in the middle of the triangular element edge. Characteristics of this finite element are given in [7] and [8]. The axial forces of compression or tension represent load, which originate from the elements that are crossing in the node A, and act in the middle of the polygon edges. According to that, selected finite element is used, Fig. 4 (right). In order to solve the stress state of the node, one support is set in the point A, and the second support is set in the middle of one polygon edge, Fig. 4 (left). In this way displacements of the polygon, as a rigid body, are eliminated, and this behavior approximately corresponds to the behavior of the isolated node from concrete mass of the member.

In the nodal zones, displacements of the system nodes, then deformations, and according to the stress-strain relationship the stress values $\sigma_x$, $\sigma_y$ and $\tau_{xy}$, ($\tau_{xy}$ appears if nodal zones are not hydrostatic type) are determined. Finally, calculation of the principal stress values, for compression or tension, is followed by equations (1). Obtained principal stresses are used for fracture control according to Kupfer criterion, Fig. 4 (middle).

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (1)$$

## 3. NUMERICAL ANALYSIS

In this paper a reinforced concrete wall with an overhang, with an opening and thickness of 30 cm, loaded with two concentrated forces, is analyzed, Fig. 5 (left). Strut-and-Tie model, obtained with "ST method" program, is shown in Fig. 5 (right) [3].

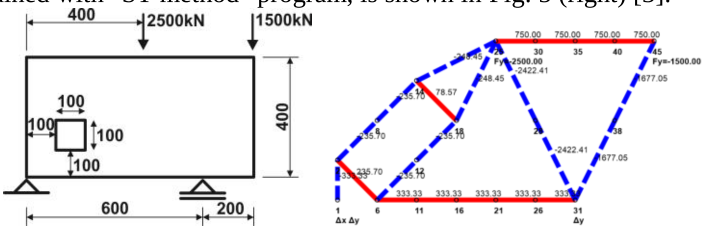
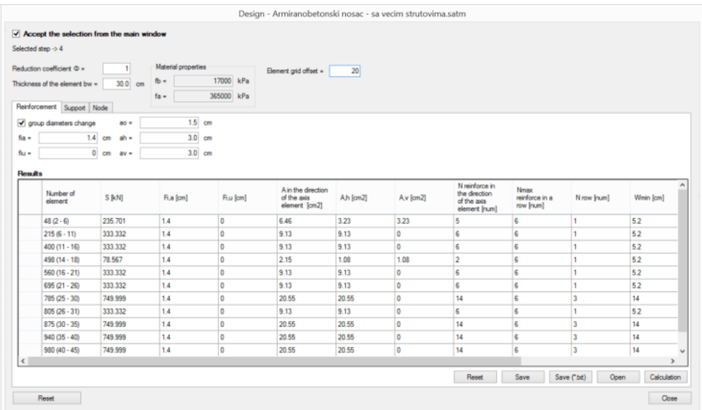



*Fig. 5 - RC member (left) and Strut-and-Tie model (right), [3]*

The mechanical material properties for concrete C30/37 are characteristic pressure strength of the cylinder $f_{ck}$ = 30 MPa and design pressure strength $f_{cd}$ = 17 MPa, while for reinforcement characteristic yielding strength is $f_{yk}$ = 420 MPa and design yielding strength is $f_{yd}$ = 365MPa, according to [3]. Required amounts of reinforcement, determined on the basis of obtained forces in the struts, with required number of reinforcement bars and the minimum required dimensions of the struts, are shown in Fig. 6. Dimensions can be further corrected if stress control of nodal zones and member geometry require so. In Fig. 7 and Fig. 8, local concrete stress controls in locations of bearing plates are shown (nodes 1, 25, 31 and 45, Fig. 5 right).




| Number of element | S [kN] | Fi,a [cm] | Fi,u [cm] | A in the direction of the axis element [cm2] | A,h [cm2] | A,v [cm2] | N reinforce in the direction of the axis element [num] | Nmax reinforce in a row [num] | N row [num] | Wmin [cm] |
|---|---|---|---|---|---|---|---|---|---|---|
| 48 (2 - 6) | 235.701 | 1.4 | 0 | 6.46 | 3.23 | 3.23 | 5 | 6 | 1 | 5.2 |
| 215 (6 - 11) | 333.332 | 1.4 | 0 | 9.13 | 9.13 | 0 | 6 | 6 | 1 | 5.2 |
| 400 (11 - 16) | 333.332 | 1.4 | 0 | 9.13 | 9.13 | 0 | 6 | 6 | 1 | 5.2 |
| 498 (14 - 18) | 78.567 | 1.4 | 0 | 2.15 | 1.08 | 1.08 | 2 | 6 | 1 | 5.2 |
| 560 (16 - 21) | 333.332 | 1.4 | 0 | 9.13 | 9.13 | 0 | 6 | 6 | 1 | 5.2 |
| 695 (21 - 26) | 333.332 | 1.4 | 0 | 9.13 | 9.13 | 0 | 6 | 6 | 1 | 5.2 |
| 785 (25 - 30) | 749.999 | 1.4 | 0 | 20.55 | 20.55 | 0 | 14 | 6 | 3 | 14 |
| 805 (26 - 31) | 333.332 | 1.4 | 0 | 9.13 | 9.13 | 0 | 6 | 6 | 1 | 5.2 |
| 875 (30 - 35) | 749.999 | 1.4 | 0 | 20.55 | 20.55 | 0 | 14 | 6 | 3 | 14 |
| 940 (35 - 40) | 749.999 | 1.4 | 0 | 20.55 | 20.55 | 0 | 14 | 6 | 3 | 14 |
| 980 (40 - 45) | 749.999 | 1.4 | 0 | 20.55 | 20.55 | 0 | 14 | 6 | 3 | 14 |

*Fig. 6 – Required amounts of reinforcement and min dimensions of struts*

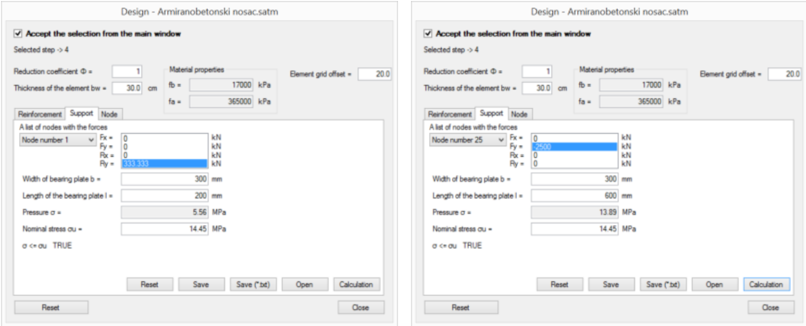


*Fig. 7 – Local concrete stress control in places of bearing plates: node 1 (left) and node 25 (right), Fig. 5 (right)*

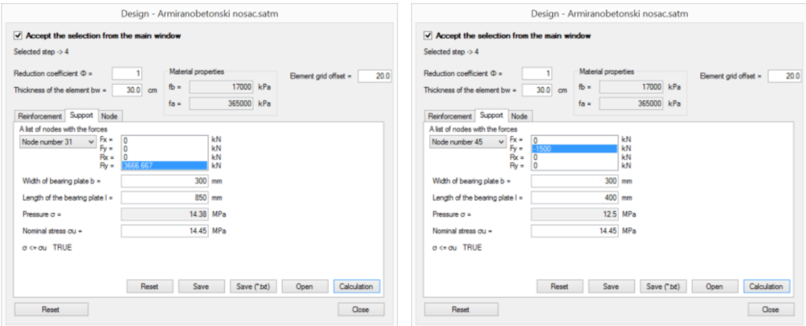


*Fig. 8 - Local concrete stress control in places of bearing plates: node 31 (left) and node 45 (right), Fig. 5 (right)*

In Fig. 9 and Fig. 10, widths of struts determined from satisfying stress conditions that are defined according to forces in elements and node type, are shown.

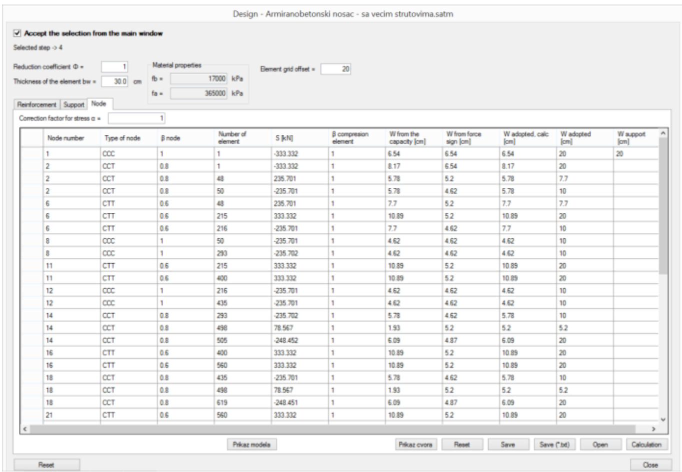
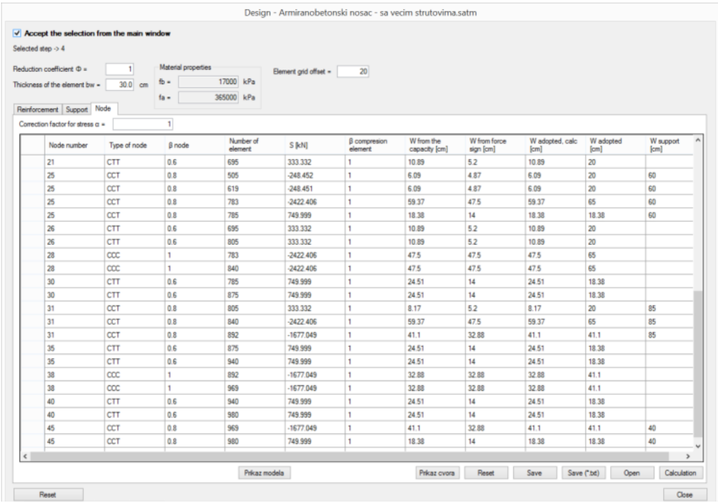


*Fig. 9 – Element widths of Strut-and-Tie model*




| Node number | Type of node | β node | Number of element | S [kN] | β compresion element | W from the capacity [cm] | W from force sign [cm] | W adopted, calc [cm] | W adopted [cm] | W support [cm] |
|---|---|---|---|---|---|---|---|---|---|---|
| 21 | CTT | 0.6 | 695 | 333.332 | 1 | 10.89 | 5.2 | 10.89 | 20 | |
| 25 | CCT | 0.8 | 505 | -248.452 | 1 | 6.09 | 4.87 | 6.09 | 20 | 60 |
| 25 | CCT | 0.8 | 619 | -248.451 | 1 | 6.09 | 4.87 | 6.09 | 20 | 60 |
| 25 | CCT | 0.8 | 783 | -2422.406 | 1 | 59.37 | 47.5 | 59.37 | 65 | 60 |
| 25 | CCT | 0.8 | 785 | 749.999 | 1 | 18.38 | 14 | 18.38 | 18.38 | 60 |
| 26 | CTT | 0.6 | 695 | 333.332 | 1 | 10.89 | 5.2 | 10.89 | 20 | |
| 26 | CTT | 0.6 | 805 | 333.332 | 1 | 10.89 | 5.2 | 10.89 | 20 | |
| 28 | CCC | 1 | 783 | -2422.406 | 1 | 47.5 | 47.5 | 47.5 | 65 | |
| 28 | CCC | 1 | 840 | -2422.406 | 1 | 47.5 | 47.5 | 47.5 | 65 | |
| 30 | CTT | 0.6 | 785 | 749.999 | 1 | 24.51 | 14 | 24.51 | 18.38 | |
| 30 | CTT | 0.6 | 875 | 749.999 | 1 | 24.51 | 14 | 24.51 | 18.38 | |
| 31 | CCT | 0.8 | 805 | 333.332 | 1 | 8.17 | 5.2 | 8.17 | 20 | 85 |
| 31 | CCT | 0.8 | 840 | -2422.406 | 1 | 59.37 | 47.5 | 59.37 | 65 | 85 |
| 31 | CCT | 0.8 | 892 | -1677.049 | 1 | 41.1 | 32.88 | 41.1 | 41.1 | 85 |
| 35 | CTT | 0.6 | 875 | 749.999 | 1 | 24.51 | 14 | 24.51 | 18.38 | |
| 35 | CTT | 0.6 | 940 | 749.999 | 1 | 24.51 | 14 | 24.51 | 18.38 | |
| 38 | CCC | 1 | 892 | -1677.049 | 1 | 32.88 | 32.88 | 32.88 | 41.1 | |
| 38 | CCC | 1 | 969 | -1677.049 | 1 | 32.88 | 32.88 | 32.88 | 41.1 | |
| 40 | CTT | 0.6 | 940 | 749.999 | 1 | 24.51 | 14 | 24.51 | 18.38 | |
| 40 | CTT | 0.6 | 980 | 749.999 | 1 | 24.51 | 14 | 24.51 | 18.38 | |
| 45 | CCT | 0.8 | 969 | -1677.049 | 1 | 41.1 | 32.88 | 41.1 | 41.1 | 40 |
| 45 | CCT | 0.8 | 980 | 749.999 | 1 | 18.38 | 14 | 18.38 | 18.38 | 40 |

*Fig. 10 – Element widths of Strut-and-Tie model - continue*

Nodal zones, polygons, of Strut-and-Tie model are shown in Fig. 11 to Fig. 13 and Fig. 14 (left). Stress utilization in each nodal zone is shown in Fig. 14 (right).

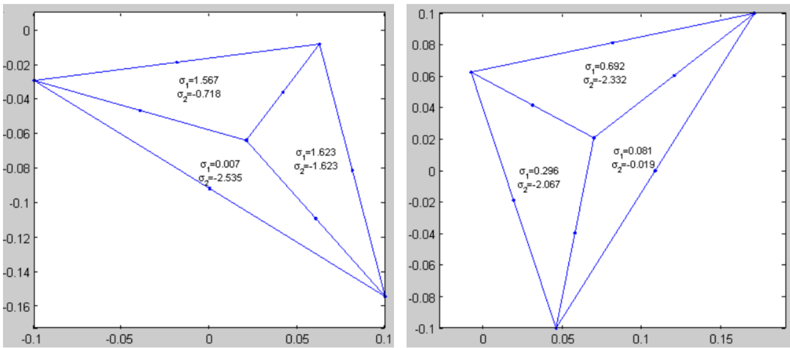



*Fig. 11 – Polygon of node 2 (left) and node 6 (right)*

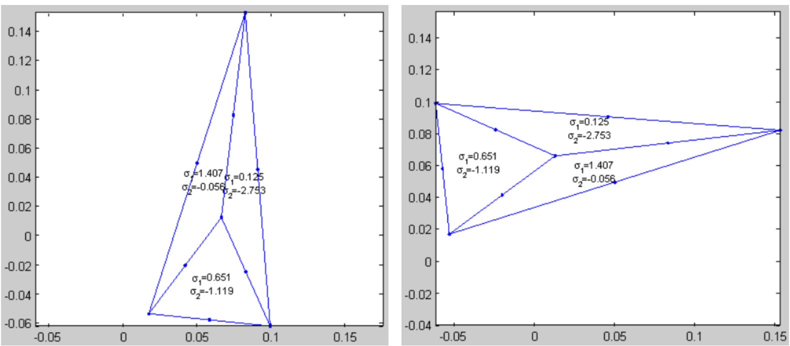



*Fig. 12 - Polygon of node 14 (left) and node 18 (right)*

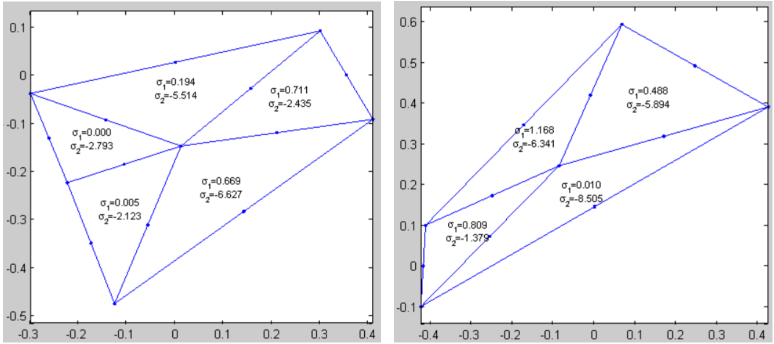



*Fig. 13 - Polygon of node 25 (left) and node 31 (right)*

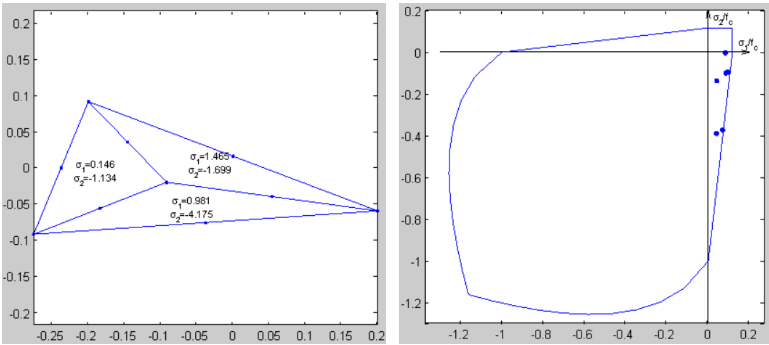



*Fig. 14 – Polygon of node 45 (left), stress utilization in concrete for all nodal zones of Strut-and-Tie model (right)*

Based on the results obtained by "ST method" program, it can be concluded that the dimensions of all Strut-and-Tie model elements satisfy required stress criterions. Finite dimensions of elements in Strut-and-Tie models are graphically presented in Fig. 15.

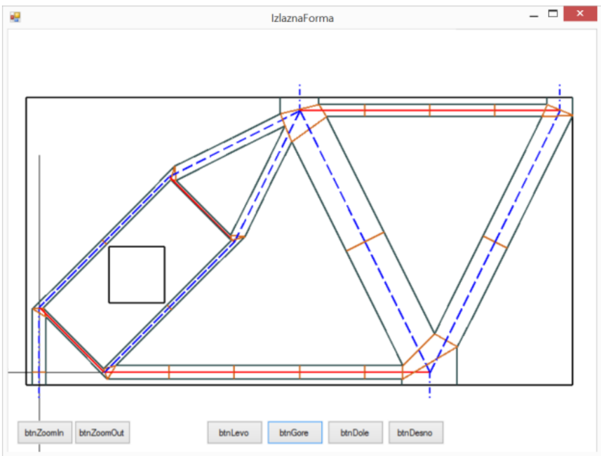



*Fig. 15 – Finite dimensions of Strut-and-Tie model elements*

Based on the comparative analysis of the results of "hand" calculation (hydrostatic nodes) [3], and the results obtained in "ST method" program (nonhydrostatic nodes), it can be concluded that the dimensions of elements and nodal zones differ. Percentage of dimension differences of elements are given in Tab. 1, and dimensions of bearing plates obtained by "ST method" program and "hand" calculation, are shown in Tab. 2. The differences range from -188% to approx. 46%, for dimensions of elements, and approximately -25% to -75% for dimensions of bearing plates. Shapes of the nodal zones for node 25 that are determined by "hand" calculation (hydrostatic node) and "ST method" program (nonhydrostatic node) are shown in Fig. 16. The biggest difference in the dimension of the bearing plate occurs in this node and is equal -75%.
In general, dimensions of elements and bearing plates in Strut-and-Tie model, determined by "ST method" program, are smaller when compared to the results of "hand" calculation.

*Tab. 1 – Finite dimensions of rods*

| element (initial node - end node) | w [cm] "hand" calculation (hydrostatic nodes) | w [cm] "ST method" (nonhydrostatic nodes) | percentage difference (relative to "ST method") [%] |
|---|---|---|---|
| 2-6 | 8.0 | 7.7 | -3.7 |
| 14-18 | 4.4 | 5.2 | 15.4 |
| 6-31 | 10.8 | 20.0 | 46.0 |
| 25-45 | 22.7 | 18.4 | -23.47 |
| 1-2 | 25.0 | 20.0 | -25.0 |
| 2-14 | 28.8 | 10.0 | -188.0 |
| 6-18 | 28.8 | 10.0 | -188.0 |
| 14-25 | 27.0 | 20.0 | -35.0 |
| 18-25 | 28.6 | 20.0 | -43.0 |
| 25-33 | 76.6 | 65.0 | -17.8 |
| 33-45 | 55.0 | 41.1 | -33.8 |

*Tab. 2 – Finite dimensions of bearing plates*

| Node | l [cm] "hand" calculation (hydrostatic nodes) | l [cm] "ST method" (nonhydrostatic nodes) | percentage difference (relative to "ST method") [%] |
|---|---|---|---|
| 1 | 25 | 20 | -25.0 |
| 25 | 105 | 60 | -75.0 |
| 31 | 115 | 85 | -35.3 |
| 45 | 50 | 40 | -25.0 |

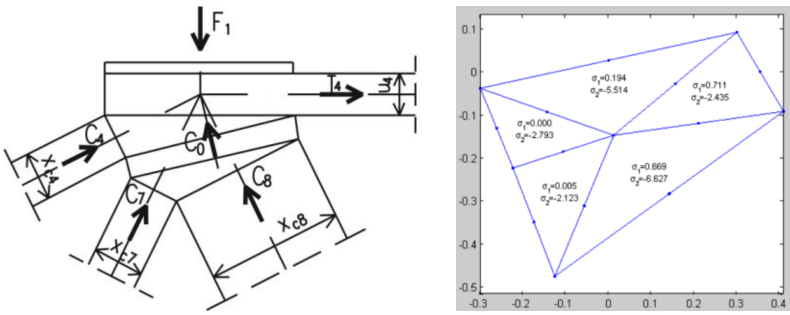



*Fig. 16 – Nodal zone shape for node 25: "hand" calculation (left [3]) and "ST method" (right)*

## 4. CONCLUSION

Strut-and-Tie method can be applied for design of reinforced concrete members, with static or geometric discontinuities [1].
For analysis of RC plane member, "ST method" program is made, that automatically generates and designs Strut-and-Tie models [2].
In this paper, rc wall with an opening is analyzed and designed in two ways, by "hand" calculation according to [3], where the nodes are analyzed as hydrostatic and by "ST method" program, where the nodes are analyzed as nonhydrostatic.
In both design approaches dimensions of Strut-and-tie model elements fulfill criteria required by Strut-and-Tie method. Based on the comparative analysis of the dimensions of Strut-and-Tie model elements obtained by "hand" calculation in relation to the dimensions obtained by "ST method" program, it can be concluded that the differences range from approximately -188% to 46%, and that the differences in the dimensions of bearing plates are approximately -25% to -75%. The differences are determined in relation to the results obtained with the "ST method" program. Comparing the results of analysis, in general, it can be concluded, that dimensions of the Strut-and-Tie model elements obtained by "ST method" program (nonhydrostatic node) in most cases, are smaller than dimensions of the elements obtained by "hand" calculation (hydrostatic nodes). As dimensions of the Strut-and-Tie model elements with hydrostatic nodes are larger, there is a question "Can such dimensions always fit in geometric limitations of member dimensions?". In those cases, design with nonhydrostatic nodes proves to be a better solution, and that corroborates the advantage of "ST method" program application.

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# ANALIZA I DIMENZIONISANJE AB ZIDNOG NOSAČA PRIMENOM PROGRAMA "ST METHOD"

***Rezime:*** *U ovom radu je prikazano dimenzionisanje armiranobetonskog zidnog nosača primenom Strut-and-Tie metode. Numeričke analize su urađene u programu "ST method" koji ima mogućnost automatskog određivanja Strut-and-Tie modela i dimenzionisanja ravanskih AB nosača. Analiziran je armiranobetonski zidni nosač sa otvorom. Čvorne zone Strut-and-Tie modela analizirane su, kao hidrostatičke, u ručnom proračunu, i nehidrostatičke, primenom programa "ST method". Upoređeni su dobijeni rezultati oba načina proračuna.*

***Ključne reči:*** *ab zidni nosač, ST method, Strut-and-Tie model, čvorna zona*