VERTICAL RESPONSE OF ADJACENT FOUNDATIONS ON LAYERED SOIL BY ITM

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Summary: *In this paper the dynamic interaction of two rigid massless foundations resting on the finite depth soil medium layer. The vertical response of a loaded and unloaded foundation of a layer over a bedrock is calculated by Integral transform method (ITM). This method is based on analytical solution of the Lamé's differential equations of motion. A parametric analysis of vertical vibrations as a function of soil depth is carried out using a computer program developed in MATLAB. The obtained results are presented.*

Keywords: *dynamic soil-structure interaction, group of rigid foundation, layered soil, integral transform method*

1. INTRODUCTION

Soil-Structure Interaction (SSI) is important part of dynamic analysis of structures, especially in a case of structures of special importance and structures founded on a soft soil. Dynamic parameters of foundations resting on a soil are essential for SSI analysis. These parameters are usually given in form of stiffness (impedance) and flexibility (compliance) factors that depend of frequency. Dynamic properties of a single rigid foundation resting on a homogeneous, elastic halfspace had been a major topic over decades and it refers to the majority of published SSI analyses [1]. However, the assumption of homogeneous soil medium could not be satisfied in most cases. Also, the influence of surrounding foundations could not be completely neglected in general. Kausel, Wass and Roesset [2] presented a finite element method (FEM) based approach for determining dynamic properties of the foundation on layered media. Wong and Luco [3], Karabalis and Mohammadi [4] solved the dynamic foundation-soil-foundation interaction (FSFI) using boundary element method (BEM) approach. This paper presents the solution of dynamic FSFI for two adjacent foundations in layered soil medium using Integral Transform Method (ITM) [5]. Parametric study on the effects of layer depth is presented. Only vertical vibrations of the system are taken into account.

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2. FORMULATION OF ITM

2.1. Wave equation in a half space and boundary conditions

ITM is based on Lamé‘s differential equations of motion – partial differential equations with constant coefficients given in terms of spatial coordinates (*x*,*y*,*z*) and time *t*

$$\mu\nabla^2\mathbf{u}+(\lambda+\mu)\nabla\nabla\mathbf{u}=\rho\ddot{\mathbf{u}} \tag{1}$$

where is mass density of the material, **u** is displacement vector and μ and λ are Lamé's material constants given in terms of elasticity modulus *E*, Poisson's coefficient υ and damping ratio ξ

$$\mu=E\frac{1+2i\xi}{2(1+\nu)},\quad \lambda=2E\nu\frac{1+2i\xi}{1-2\nu} \tag{2}$$

With the help of the Helmholtz's principle, Lamé's equations can be written in form of two decoupled wave equations

$$\nabla^2\varphi=\frac{1}{c_p^2}\ddot{\varphi},\quad \nabla^2\boldsymbol{\psi}=\frac{1}{c_s^2}\ddot{\boldsymbol{\psi}} \tag{3}$$

where c_p and c_s are velocities of the dilatational and shear waves

$$c_p^2=\frac{\lambda+2\mu}{\rho},\quad c_s^2=\frac{\mu}{\rho} \tag{4}$$

and φ and $\boldsymbol{\psi}$ are scalar and vector fields that have to satisfy the relation

$$\nabla\mathbf{u}=\nabla\varphi+\nabla\times\boldsymbol{\psi} \tag{5}$$

If we assume that $\psi z = 0$ than Eq (5) could be expand as

$$\begin{aligned}u_x&=\varphi_{,x}-\boldsymbol{\psi}_{y,z}\\u_y&=\varphi_{,y}-\boldsymbol{\psi}_{x,z}\\u_z&=\varphi_{,z}-\boldsymbol{\psi}_{x,y}+\boldsymbol{\psi}_{y,x}\end{aligned} \tag{6}$$

In order to find the solution of the system of equations (3) they are transferred from (*x*,*y*,*z*,*t*) to (k_x,k_y,z,ω) domain by using a threefold Fourier transform

$$\hat{f}(k_x,k_y,z,\omega)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y,z,t)e^{-i(k_x x+k_y y+\omega t)}dxdydt \tag{7}$$

In (k_x,k_y,z,ω) domain they represent a system of three ordinary differential equations

$$\begin{aligned}&-\lambda_1^2\hat{\varphi}+\frac{\partial^2\hat{\varphi}}{\partial z^2},\\ &-\lambda_2^2\hat{\boldsymbol{\psi}}_i+\frac{\partial^2\hat{\boldsymbol{\psi}}_i}{\partial z^2},\quad i=x,y\end{aligned} \tag{8}$$

where

$$\begin{aligned}\lambda_1^{\ 2}&=k_x^2+k_y^2-k_p^2,\quad k_p=\frac{\omega}{c_p}\\ \lambda_2^{\ 2}&=k_x^2+k_y^2-k_s^2,\quad k_s=\frac{\omega}{c_s}\end{aligned} \tag{9}$$

The solution of the system of equations (8) is given as

$$\begin{aligned}\hat{\varphi}&=A_1e^{\lambda_1 z}+A_2e^{-\lambda_1 z},\\ \hat{\boldsymbol{\psi}}_i&=B_{1i}e^{\lambda_2 z}+B_{2i}e^{-\lambda_2 z},\quad i=x,y\end{aligned} \tag{10}$$

where A_1, A_2, B_{1x}, B_{1y}, B_{2x}, B_{2y} are unknown coefficients of integration.
The displacement field in the transferred domain can be obtained by substituting Eq (10) into Eq (6) previously transferred into (k_x,k_y,z,ω) domain by using Eq (7). Therefore

$$\hat{\mathbf{u}}=\mathbf{A}_u\mathbf{C} \tag{11}$$

where $\hat{\mathbf{u}}$ is a displacement field in the transferred domain, Eq (12), **C** is a vector of unknown coefficients of integration, Eq (13), and $\mathbf{A}^u$ is a correlation matrix, Eq (14).

$$\hat{\mathbf{u}}^{\mathsf{T}}=\left\{\hat{u}_x\quad \hat{u}_y\quad \hat{u}_z\right\} \tag{12}$$

$$\mathbf{C}^{\mathsf{T}}=\left\{A_1e^{z\lambda_1}\quad A_2e^{-z\lambda_1}\quad B_{1x}e^{z\lambda_2}\quad B_{2x}e^{-z\lambda_2}\quad B_{1y}e^{z\lambda_2}\quad B_{2y}e^{-z\lambda_2}\right\} \tag{13}$$

$$\mathbf{A}_u=\begin{bmatrix} ik_x & ik_x & 0 & 0 & -\lambda_2 & \lambda_2\\ ik_y & ik_y & \lambda_2 & -\lambda_2 & 0 & 0\\ \lambda_1 & -\lambda_1 & -ik_y & -ik_y & ik_x & ik_x\end{bmatrix} \tag{14}$$

Using well known relations between stress and displacement fields, one can obtain

$$\hat{\boldsymbol{\sigma}} = \mathbf{A}_{\sigma}\mathbf{C} \tag{15}$$

where $\hat{\boldsymbol{\sigma}}$ is a stress field in the transferred domain, Eq (16), $\mathbf{C}$ is a vector of unknown coefficients of integration, Eq (13), and $\mathbf{A}^{\sigma}$ is a correlation matrix, Eq (17).

$$\hat{\boldsymbol{\sigma}} = \left\{ \hat{\sigma}_x \quad \hat{\sigma}_y \quad \hat{\sigma}_z \quad \hat{\tau}_{xy} \quad \hat{\tau}_{yz} \quad \hat{\tau}_{zx} \right\} \tag{16}$$

$$\mathbf{A}_{\sigma} = \mu \begin{bmatrix} -(2k_x^2 + \frac{\lambda_1}{\mu}k_p^2) & -(2k_x^2 + \frac{\lambda_1}{\mu}k_p^2) & 0 & 0 & -2ik_x\lambda_2 & -2ik_x\lambda_2 \\ -(2k_y^2 + \frac{\lambda_1}{\mu}k_p^2) & -(2k_y^2 + \frac{\lambda_1}{\mu}k_p^2) & 2ik_y\lambda_2 & -2ik_y\lambda_2 & 0 & 0 \\ 2(k_x^2 + k_y^2) - k_s^2 & 2(k_x^2 + k_y^2) - k_s^2 & -2ik_y\lambda_2 & 2ik_y\lambda_2 & 2ik_x\lambda_2 & -2ik_x\lambda_2 \\ -2k_xk_y & -2k_xk_y & ik_x\lambda_2 & -ik_x\lambda_2 & -ik_y\lambda_2 & ik_y\lambda_2 \\ 2ik_y\lambda_1 & -2ik_y\lambda_1 & \lambda_2^2 + k_y^2 & \lambda_2^2 + k_y^2 & -k_xk_y & -k_xk_y \\ 2ik_x\lambda_1 & 2ik_x\lambda_1 & k_xk_y & k_xk_y & -(\lambda_2^2 + k_x^2) & -(\lambda_2^2 + k_x^2) \end{bmatrix} \tag{17}$$

The unknown coefficients are obtained by taking into account boundary conditions (BC). There are three types of BC: (1) natural, on the top surface, (2) mixed, at the contact surface between adjacent layers and (3) essential, at the bottom of the soil medium in the case of a bedrock, or Sommerfeld's radiation BC in the case of an infinite halfspace [6]. Natural boundary conditions at the top surface of the soil medium, *z=0*, can be written by taking into account applied surface load $\mathbf{p}(x, y, \omega)$

$$\begin{Bmatrix} \hat{\sigma}_{zx}(k_x, k_y, 0, \omega) \\ \hat{\sigma}_{zy}(k_x, k_y, 0, \omega) \\ \hat{\sigma}_{zz}(k_x, k_y, 0, \omega) \end{Bmatrix} = \begin{Bmatrix} \hat{p}_x(k_x, k_y, \omega) \\ \hat{p}_y(k_x, k_y, \omega) \\ \hat{p}_z(k_x, k_y, \omega) \end{Bmatrix} = \hat{\mathbf{p}}(k_x, k_y, \omega) \tag{18}$$

where $\hat{\mathbf{p}}(k_x, k_y, \omega)$ is a Fourier transform of the applied surface load. These BC give a complete system of equations in the case of halfspace, since coefficients A_2, B_{2x} and B_{2y} vanish due to the Sommerfeld's radiation condition. In the case of a soil layer of finite depth *h* resting on a rigid bedrock, in addition to BC at the surface, Eq (18), BC at the bottom of the soil layer, *z=h*, should be defined in order to obtain vector **C**:

$$\begin{Bmatrix} \hat{u}_x(k_x, k_y, h, \omega) \\ \hat{u}_y(k_x, k_y, h, \omega) \\ \hat{u}_z(k_x, k_y, h, \omega) \end{Bmatrix} = \{0\} \tag{19}$$

In the case of a multi-layered soil medium, additionally defined BC that preserve the continuity of stress and displacement fields for every contact surface between layers

must be taken into account. Once vector **C** is calculated considering all BC the problem is solved.
Originally, ITM solution is obtained in (k_x,k_y,z,ω) domain, but it could be transferred into (x,y,z,t) domain by using a threefold inverse Fourier transform

$$f(x,y,z,t)=\frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\hat{f}(k_x,k_y,z,\omega)e^{i(k_x x+k_y y+\omega t)}dxdydt \qquad (20)$$

The schematic presentation of ITM procedure is given in Figure 1.

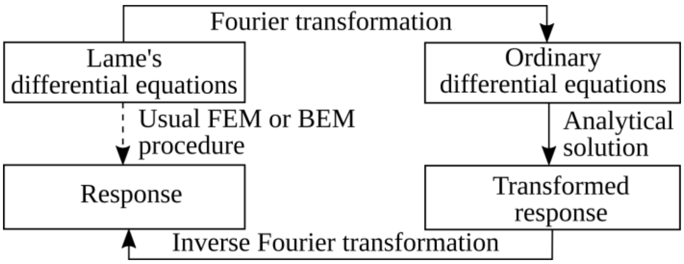

Figure 1. ITM scheme

3. DYNAMIC RESPONSE OF SURFACE FOUNDATIONS

In this paper, term *foundation* refers to a part of the soil surface that represents the contact area between soil and foundation.
Dynamic response of the surface of the halfspace subjected to the unit harmonic force is obtained with the help of ITM. The analysis considers three load cases, as shown in Figure 2. The results are three displacement fields (u_x,u_y,u_z) used for derivation of the flexibility matrix of a flexible foundation, $\mathbf{F_F}$.

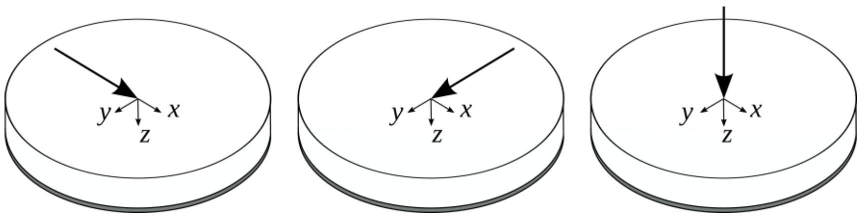

Figure 2. Load cases

3.1. Rigid foundation

The flexibility matrix of a rigid foundation, FR, is obtained by using the kinematic consideration derived from the principle that equates the deformation energies of both rigid and flexible foundation systems.

FR = tt FF t (21)

In Eq (21) term **t** refers to the kinematic matrix that represents the relationship between DOF of rigid and DOF of flexible foundation. Rigid foundation has six DOF located at the centroid – three translations and three rotations. Hence, the kinematic matrix **t** is defined as

$$\mathbf{t}^{\mathrm{T}} = \{\mathbf{t}_1 \cdots \mathbf{t}_i \cdots \mathbf{t}_n\} \tag{22}$$

where $\mathbf{t}_i$ is equal

$$\mathbf{t}_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -y_i \\ 0 & 1 & 0 & 0 & 0 & x_i \\ 0 & 0 & 1 & y_i & -x_i & 0 \end{bmatrix} \tag{23}$$

while x_i and y_i are coordinates of the node i.
The flexibility matrix of the rigid foundation is a square, 6x6, quasi diagonal matrix. Nondiagonal elements exist because horizontal translations are coupled with rocking and vice versa.

$$\mathbf{F_R} = \begin{bmatrix} F_{xx} & 0 & 0 & 0 & F_{x,my} & 0 \\ 0 & F_{yy} & 0 & F_{y,mx} & 0 & 0 \\ 0 & 0 & F_{zz} & 0 & 0 & 0 \\ 0 & F_{mx,y} & 0 & F_{mx} & 0 & 0 \\ F_{my,x} & 0 & 0 & 0 & F_{my} & 0 \\ 0 & 0 & 0 & 0 & 0 & F_{mz} \end{bmatrix} \tag{24}$$

Stiffness matrix of the rigid foundation $\mathbf{K_R}$ can be obtained directly from $\mathbf{F_R}$ since

$$\mathbf{K_R} = \mathbf{F_R}^{-1} \tag{25}$$

Although the calculation of dynamic properties of a rigid foundation using ITM has advantages, one should be aware of aliasing and collocation problems that might arise during the numerical calculation [7].

3.2. Group of foundations

Assessment of dynamic parameters of a group of *n* foundations follows the previously mentioned process with minor differences. The number of DOF of *n* foundations is *n* times higher than the number of DOF of a single foundation. Therefore, the order of flexibility matrix $\mathbf{F_R}$ is (6*n*)x(6*n*). Also, the kinematic equation, Eq (3), must be rewritten as

$$\mathbf{F_R} = \mathbf{T}^T \mathbf{F_F} \mathbf{T} \tag{26}$$

where **T** is a diagonal block matrix consisting of the kinematic matrices $\mathbf{t}_i$ for each foundation in a group of *n* foundations [7].

4. NUMERICAL RESULTS

The dynamic compliance of the system of two adjacent foundations is calculated using presented approach. The two considered foundations are shown in Figure 3. They are rigid, square and placed on the surface of homogeneous soil limited by a substratum. The distance between centroids of foundations is X. The soil layer of height H is viscoelastic, linear and characterized by its mass density ρ, shear modulus G, damping coefficient ξ and Poisson's ratio ν. The goal is to obtain the vertical compliance functions of the two footings and to calculate the influence of the adjacent foundation as well as the influence of the layer depth on the vertical response. The numerical calculation is performed by own program developed using software package Mathworks MATLAB [8].

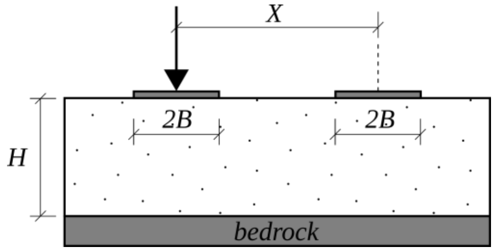

Figure 3. Model disposition

In order to analyse the influence of the layer depth the compliances are calculated for relative depth H/B = 2, 4, 8 and 10^5, at the relative distance X/B = 4 between two footings, versus dimensionless frequencies a_0. The results of the analysis are shown in Figure 4. The symbols F_{ij}^k indicate the vertical compliance of the foundation *i* in a group of *k* foundations when the foundation *j* is loaded with a vertical force. The dimensionless frequency a_0 is defined as

$$a_0 = \frac{\omega B}{c_s} \tag{27}$$

where ω is the angular frequency, B is the half-width of the foundation and c_s is share waves velocity.

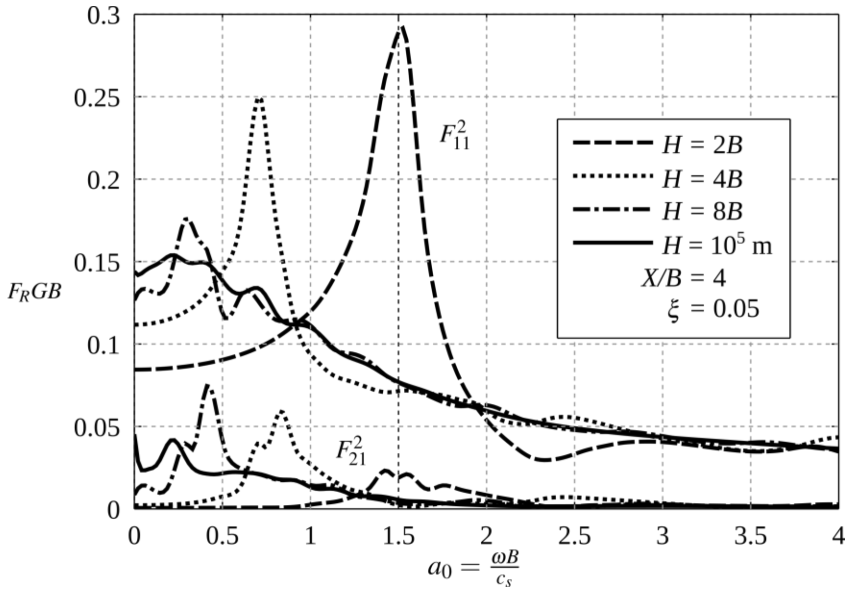

Figure 4. Vertical compliance of adjacent foundations for varying depths of the layer

For the foundation loaded with the vertical unit force the static stiffness increases (a_0 = 0) but the magnitude of the resonant peak decreases when the soil layer depth increases. Also, a remarkable shift in the resonant frequency occurs. That behaviour is expected as shallow layers are stiffer than deep ones. In case of a very deep layer (H = 10^5 m) the solution is similar to the solution of a foundation resting on an infinite halfspace. The amplitudes of the compliances of the adjacent foundation F_{21}^2 are significantly lower than the amplitudes of the foundation loaded with unit force F_{11}^2. However, when the soil layer depth increases, the amplitudes F_{21}^2 increases. This behaviour could be explained by observing waves reflection of the bedrock. Regarding F_{11}^2, P-waves are directly reflected of the bedrock, having zero deflection angles and significantly amplifying the response of the system. In the case of shallow layer, the shorter deflection path results in a smaller dumping effect and higher amplification. Regarding F_{21}^2, S-waves are predominant waves as the reflection angle of P-waves is higher. Therefore, the increase of layer depth results in decrease of the stiffness of the system giving the higher amplitudes. The analysis also showed that the influence of the unloaded foundation on the dynamic response of the foundation loaded with the unit force is negligible. However, the vice versa effect is significant and it should not be neglected in general.

5. CONCLUSIONS

In this paper, the dynamic behaviour of a two square foundations resting on a layered viscoelastic soil medium and subjected to vertical harmonic force of unit amplitude is presented. The analysis is carried out in frequency domain using Integral Transform

Method. It shows that ITM is an efficient method for obtaining the dynamic parameters of foundations. The advantages lay in the fact that the displacements in the soil can be obtained straightforward by solving the system of linear equation in a frequency –wave number domain. However, some difficulties can arise in the numerical perform of the Fourier transformation. The parametric study shows the influence of layer depth on the vertical response of the system, reflected in frequency shifting and variation of peak amplitudes. The conclusion, based on the obtained results, is that a detailed soil structure interaction should be taken in the account for the analysis of any structure sensitive to the supports displacements.

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ВЕРТИКАЛНИ ОДГОВОР ГРУПЕ ТЕМЕЉА НА СЛОЈЕВИТОМ ТЛУ ПРИМЕНОМ ITM

Резиме*: У овом раду анализирана је динамичка интеракција два крута темеља, без масе, фундирана на површини слоја тла коначне дубине. Динамички одговор у вертикалном правцу је одређен применом Методе интегралне трансформације (Integral Transform Method, ITM) која се заснива на аналитичком решењу Ламéових* *диференцијалних једначина кретања. Параметарска анализа вертикалних вибрација система у функцији дебљине слоја је спроведена применом програма написаног у Матлаб-у. Резултати те анализе су приказани у раду.*

Кључне речи: *динамичка интеракција тла и објекта, група крутих темеља, слојевито тло, метода интегралне трансформације*