FREE VIBRATIONS OF RECTANGULAR PLATES WITH CUTOUTS USING FINITE STRIP METHOD

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УДК: 534.1:624.073
DOI:10.14415/konferencijaGFS 2015.042

Summary: *Free vibrations of rectangular plates with uniform thickness and with rectangular cutouts are investigated. Plates are modeled using finite strip method where the displacement field is approximated with product of trigonometric and polynomial functions. In order to analyze the influence of polynomial order, two types of strips are utilized: LO2 and HO3. Strips are divided into cells in longitudinal direction, and the zero stiffness is given for cells which represent cutouts. Presented approach is coded using Wolfram Mathematica. Numerical tests contain comparison of results and convergence properties for LO2 and HO3 strips. Obtained results are in good agreement with data available in literature.*

Keywords: *finite strip method, free vibrations, plates with cutouts*

1. INTRODUCTION

Bending of thin plates is commonly used during initial testing and developing of new analytical and numerical procedures. Finite strip method (FSM) is one of the most suitable procedures for analysis of thin plates with regular geometries, [HYPERLINK \l "DDM97" [1]]. One of the main drawbacks of this method is its inability to describe discontinuities such as abrupt changes of thickness, cutouts and similar. Some authors already dealt with this problem, using the negative stiffness approach, [2]], [HYPERLINK \l "Bui09" [3]],[4]]. Only the free vibration and buckling analyses are performed since, for them, the effects of localization and stress concentration can be ignored without prejudicing the accuracy of the solution.
In this paper, two types of finite strips are utilized and their performance is examined through detailed numerical test of free vibrations of rectangular plate with two cutouts. Presented approach uses standard trigonometric functions which allow modeling of all

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types of boundary conditions, contrary to classical FSM approach where the simply supported strip is mostly used.
Brief theory overview is given, followed with numerical example and conclusions.

2. THEORY OVERVIEW

Rectangular low order finite strip with two nodal lines (LO2), Figure 1, has four degrees of freedom in bending, [HYPERLINK \l "DDM97" 1]. Degrees of freedom in the FSM are displacement parameters of nodal lines which represent weighting coefficients of each series term Y_m in total displacement field, (1). This series is chosen as free vibration eigenfunctions of Bernoulli-Euler beam, which enables modeling of all types of boundary conditions.

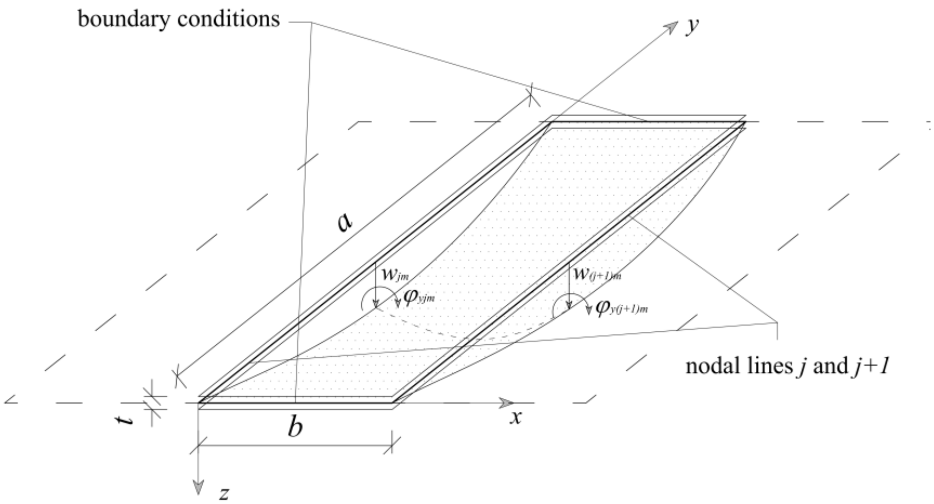


Figure 1. Flat rectangular LO2 finite strip for m=1 - part of the discretised structure

$$w(x,y)=\sum_{m=1}^{nst} w_m(x)Y_m(y). \quad (1)$$

Besides LO2 strip, high order strip with three nodal lines (HO3) is also utilized here. This strip has additional nodal line in the middle of the strip. Accordingly, interpolating polynomial, w_m, for LO2 strip is of the third, and for HO3 strip of the fifth order.
In order to model cutouts, strip is divided into cells, Figure 2. During the evaluation of stiffness and mass matrices, integration is performed across each cell and then summed, (2). In this way, strip with different properties in longitudinal direction is introduced. It should be noted that, with this approach, displacement function is continuous over cutouts. This implies that large number of series terms should be used in analysis if local effects are considered.

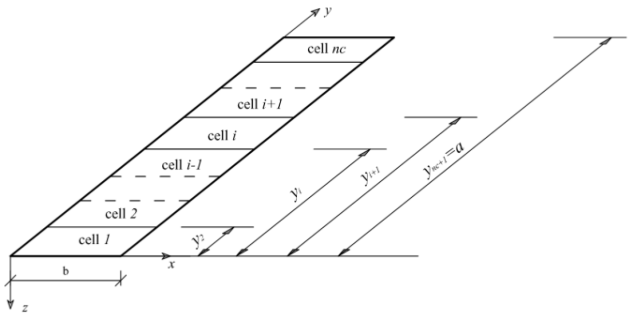


Figure 2. Division of strip into cells in longitudinal direction

$$\int_0^a f(\mathrm{y})dy = \sum_{i=1}^{ns} \int_{y_i}^{y_{i+1}} f(y)dy. \qquad (2)$$

3. NUMERICAL EXAMPLE

Presented approach is programmed using software package Wolfram Mathematica. The code is built upon software LEDA presented in [HYPERLINK \l "Bor131" [2]].
The numerical research includes simply supported squared plates with two rectangular cutouts, [3]], with disposition of cutouts as in Figure 3. The plates were modeled with 20 strips and three cells in longitudinal direction. In order to examine the convergence between two types of strips, numerical calculation was carried out by varying number of series terms. Natural frequency coefficient $(\rho h/D)^{1/2}\omega_i a^2$ is given as a result.

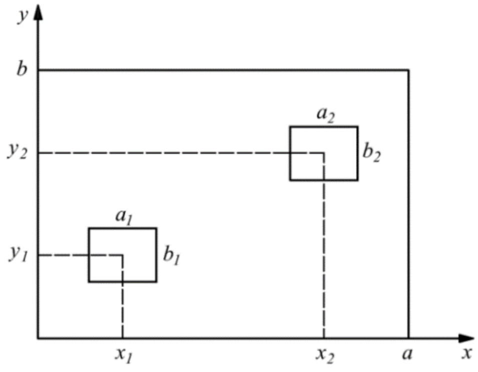
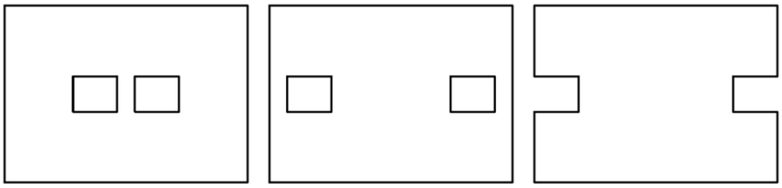


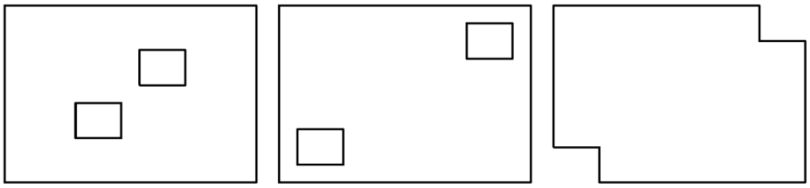
Figure 3. Disposition of cutouts and notations

First numerical test is performed for cutouts located along the middle horizontal axis of the plate (C1). Three plates having different space between cutouts were monitored, see Figure 2. Results are presented in Table 1. Then the other case of plates, using cutouts diagonally, is observed (C2). This case and disposition of cutouts are displayed on Figure 3 while results are listed in Table 2. Convergence of relative difference between FSM and Ref. [HYPERLINK \l "Ava03" [3]] results for case C1 is given in Figure 6.



a) b) c)

Figure 4. Case of cutouts placed along the middle horizontal axis - C1



a) b) c)

Figure 5. Case of cutouts located along the plate diagonal - C2

Table 1. The first four frequency coefficients for case C1

Size of the cutouts	Cutouts position	FSM		Reference [3]
		LO2	HO3	
$a_1/a=0.1=b_1/b$ $a_2/a=0.1=b_2/b$	Figure 4 a)	19.542	19.542	19.324
		48.939	48.937	48.769
	$x_1/a=0.4$ $y_1/b=0.50$	49.297	49.297	49.136
	$x_2/a=0.6$ $y_2/b=0.50$	78.184	78.184	78.035
	Figure 4 b)	19.639	19.639	19.550
		48.608	48.604	48.265
	$x_1/a=0.2$ $y_1/b=0.50$	49.027	49.027	48.946
	$x_2/a=0.80$ $y_2/b=0.50$	78.822	78.821	78.621
	Figure 4 c)	19.719	19.719	19.707
		48.869	48.869	48.816
	$x_1/a=0.05$ $y_1/b=0.50$	49.118	49.116	49.035
	$x_2/a=0.95$ $y_2/b=0.50$	77.873	77.873	77.722
$a_1/a=0.2=b_1/b$ $a_2/a=0.2=b_2/b$	Figure 4 a)	19.192	19.189	19.066
		47.364	47.359	46.324
	$x_1/a=0.35$ $y_1/b=0.50$	48.103	48.102	47.300
	$x_2/a=0.65$ $y_2/b=0.50$	76.561	76.559	74.957
	Figure 4 b)	19.170	19.169	19.121
		47.784	47.784	47.009
	$x_1/a=0.20$ $y_1/b=0.50$	47.901	47.898	47.778
	$x_2/a=0.80$ $y_2/b=0.50$	77.357	77.355	75.402
	Figure 4 c)	19.337	19.337	19.332
		46.959	46.955	46.972
	$x_1/a=0.10$ $y_1/b=0.50$	47.495	47.495	47.058
	$x_2/a=0.90$ $y_2/b=0.50$	75.427	75.427	74.066

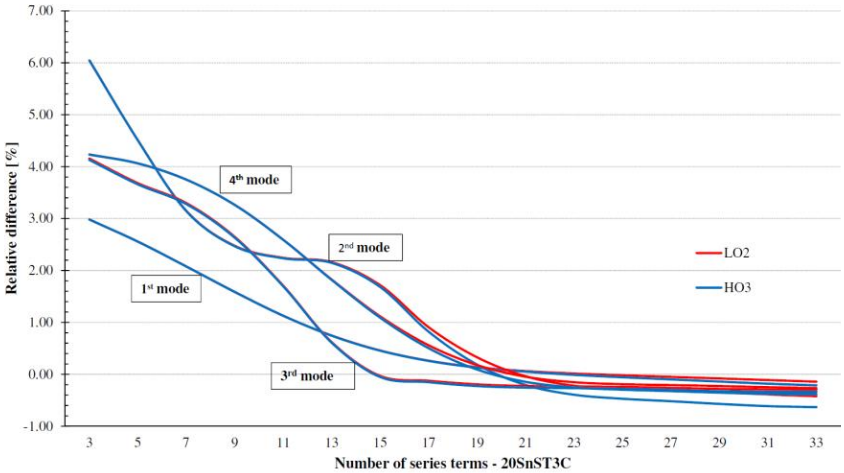


Figure 6. Convergence of relative difference for case C1

Table 2. The first four frequency coefficients for case C2

Size of the cutouts	Cutouts position	FSM		Reference
		LO2	HO3	[
$a_1/a=0.1=b_1/b$ $a_2/a=0.1=b_2/b$	Figure 5 a)	19.542	19.542	19.339
		48.965	48.963	48.605
	$x_1/a=0.40$ $y_1/b=0.40$	49.172	49.171	49.050
	$x_2/a=0.60$ $y_2/b=0.60$	78.373	78.372	78.160
	Figure 5 b)	19.575	19.574	19.527
		49.082	49.081	48.660
	$x_1/a=0.20$ $y_1/b=0.20$	49.192	49.192	49.160
	$x_2/a=0.80$ $y_2/b=0.80$	78.869	78.867	78.074
	Figure 5 c)	19.413	19.413	19.402
		48.365	48.364	48.316
	$x_1/a=0.05$ $y_1/b=0.05$	49.345	49.345	49.347
	$x_2/a=0.95$ $y_2/b=0.95$	77.745	77.745	77.644
$a_1/a=0.2=b_1/b$ $a_2/a=0.2=b_2/b$	Figure 5 a)	19.048	19.047	18.902
		47.308	47.303	46.988
	$x_1/a=0.35$ $y_1/b=0.35$	48.742	48.737	48.308
	$x_2/a=0.65$ $y_2/b=0.65$	78.088	78.076	77.417
	Figure 5 b)	18.936	18.935	18.863
		48.366	48.363	47.980
	$x_1/a=0.20$ $y_1/b=0.20$	48.536	48.535	48.511
	$x_2/a=0.80$ $y_2/b=0.80$	80.094	80.090	79.667
	Figure 5 c)	18.517	18.516	18.503
		45.901	45.895	45.816
	$x_1/a=0.10$ $y_1/b=0.10$	49.188	49.188	49.191
	$x_2/a=0.90$ $y_2/b=0.90$	74.949	74.938	74.792

4. CONCLUSIONS

Main advantage of FSM is its semi-analytical nature which guarantees faster convergence than purely numerical procedures such as Finite element method. Consequently, its main disadvantage is inability to model discontinuities in longitudinal direction, since this part of displacement field is approximated with continuous trigonometric functions. This problem can be solved using negative stiffness method which is based on integration by segments along strip. In this way computational time is increased, but not significantly. On the other hand, effects of cutouts, abrupt changes of thickness and different material properties can be analyzed.

Obtained results show excellent agreement with the ones from literature. Convergence of frequencies by number of series terms is good, while the influence of polynomial order is insignificant for the presented type of problem.

Next step is introduction of membrane effects into strips which will enable modeling of prismatic shells.

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Међународна конференција
Савремена достигнућа у грађевинарству 24. април 2015. Суботица, СРБИЈА

СЛОБОДНЕ ВИБРАЦИЈЕ ПРАВОУГАОНИХ ПЛОЧА СА ОТВОРИМА ПРИМЈЕНОМ МЕТОДА КОНАЧНИХ ТРАКА

Резиме: *Разматране су слободне вибрације правоугаоних плоча константне дебљине са правоугаоним отворима. Плоче су моделиране примјеном метода коначних трака при чему је поље помјерања апроксимирано производом тригонометријских и полиномних функција. У циљу испитивања утицаја реда полинома, примијењена су два типа трака: LO2 и HO3. Траке су у подужном правцу подијељене на сегменте, при чему је сегментима гдје се налази отвор придружена нулта крутост. Представљени приступ је кодиран у пакету Wolfram Mathematica. Кроз нумеричке тестове су упоређени резултати и тип конвергенције LO2 и HO3 трака. Добијени резултати су у сагласности са онима из литературе.*

Кључне речи: *метод коначних трака, слободне вибрације, плоче са отворима*