EXPERIENCES ON THE SHEAR MODEL OF *fib* MODEL CODE 2010

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***Rezime**: International Organization fib has introduced a new standard (Model Code 2010) for shear reinforced concrete and prestressed concrete. The equation that describes the model uses two parameters: θ – angle of the crack that is created under the influence of shearing forces k_v – coefficient of concrete contributions. Standard (MC2010) describes three levels with an approximate determination of these parameters. The first and third level relies on the theory of modified stress field, while the second level of the theory of modified fields of plastic deformation. The paper presents and describes the methods applied in the course of a large number of practical measurements of shear performed in the laboratory for testing of structures and materials. The aim of the experiment was to compare the Hungarian results of the measurements obtained on concrete that were reinforced with polypropylene and steel fiber*

***Keywords:** posmična čvrstoća, razina približavanja*

1. INTRODUCTION

The *fib* Model Code 2010 comprises a mechanical based set of shear design process that is intended to offer for the engineer flexibility in selecting a balance between complexity and accuracy for new structural design and for evaluation or verification of existing structure as well [1][2]. The new code provision contains four "Levels of Approximation" (LoA) [1]. Level I provides the simplest calculation whereas this is the most conventional method among the others [2]. Level II is a balanced model in complexity and accuracy, while Level III is the most accurate and general approximation but needs more complex computation than that of the other Levels [2]. All the first three levels can be used in the everyday engineering practice without using computer. Level IV is a further option which can be used in nonlinear finite element analysis or generalized stress-field approaches [1][2]. This paper presents the basic structural skeleton of the Levels of Approximation I to III. Based on a wide scale experimental procedure aimed to investigate the shear properties of reinforced concrete beams, test

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results are also compared to the results of the multilevel shear approximation of fib MC 2010.

2. SHEAR MODEL OF *fib* MC 2010

The state equation of shear behaviour according to *fib* MC 2010 is the following:

$$V_{Ed} \leq V_{Rd} = V_{Rd,c} + V_{Rd,s} \leq V_{Rd,\max} ,$$

where V_{Rd} is the design shear resistance, which includes both the design shear resistance attributed to the concrete ($V_{Rd,c}$) and the design shear resistance provided by the shear reinforcement ($V_{Rd,s}$) which must not be taken grater than $V_{Rd,\max}$. V_{Ed} is the design shear force. Components of the state equation above can be derived:

$$V_{Rd,c} = 0{,}9 \cdot k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot d \quad , \quad V_{Rd,s} = 0{,}9 \cdot \frac{A_{sw}}{s_w} \cdot f_{ywd} \cdot d \cdot (\cot\theta + \cot\alpha) \cdot \sin\alpha \, .$$

According to the model the design shear resistance of concrete contribution ($V_{Rd,c}$) affected by the characteristic value of concrete compressive strength (f_{ck}), the width of the web (b_w), the effective dept of the section (d), the partial safety factor of the concrete according to the design situation (γ_c), and the *first parameter* of the model k_v, defined by the levels of approximation and indicates the ability of the web to resist aggregate interlock stresses which provide the concrete contribution to shear strength [1][2][3]. However, the value of V_{Rd} cannot exceed the crushing capacity of the concrete calculated as:

$$V_{Rd,\max} = 0{,}9 \cdot k_c \cdot \frac{f_{ck}}{\gamma_c} \cdot b_w \cdot d \cdot \frac{\cot\theta + \cot\alpha}{1 + \cot^2\theta} ,$$

where θ defined by the different level of approximation – *second parameter* of the model – and indicates the angle of principal compressive stress in the web, while α is the angle of the stirrups or bended bars from the beam axis. The concrete strength reduction factor k_c – taking into consideration the effect of cracked concrete – is defined as:

$$k_c = k_\varepsilon \cdot \eta_{fc} = k_\varepsilon \cdot \left(\frac{30}{f_{ck}} \right)^{1/3}$$

where k_ε can be calculated according to the levels of approximation, while $\eta_{fc} \leq 1$. The design shear resistance provided by shear reinforcement ($V_{Rd,s}$) is generally defined by the amount of shear reinforcement (A_{sw}/s_w) and the strength properties of reinforcement used ($f_{ywd} = f_{ywk} / \gamma_s$). Minimum shear reinforcement according to the detailing rules [1] should be applied.

3. LEVEL OF APPROXIMATION I

Level of Approximation I. is suitable for pre-desing or initial sizing of structural elements, where a conservative calculation method is acceptable [2]. In case that $f_{yk} \leq 600\,\mathrm{N/mm^2}$ and $f_{ck} \leq 70\,\mathrm{N/mm^2}$, as well as the maximum aggregate size (d_g) is not less than 10 mm, design shear resistance of reinforced concrete cross section – with no significant axial load – *with no shear reinforcement* can be determined by the following equation [1]:

$$V_{Rd} = V_{Rd,c} = 0{,}9 \cdot k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot d = \frac{162}{1000 + 1{,}125 \cdot d} \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot d\ .$$

Further, the design shear resistance of a reinforced concrete cross-section *with shear reinforcement* can be calculated as [1] – neglecting the concrete contribution:

$$V_{Rd} = V_{Rd,s} \leq V_{Rd,\max}(\theta_{\min}),$$

where – if there is no considerable axial force – the inclination of the fictional compressed trus (θ) according to the proposal to be at least (30°). The crushing capacity of the concrete at that angle ($\theta_{\min} = 30^\circ$) of the principal compressive stress can be calculated by the use of $k_\varepsilon = 0{,}55$. Results of this level of approximation can only be acceptable if the longitudinal strain deformation $\varepsilon_x = \left(\frac{M_{Ed}}{0{,}9 \cdot d} + V_{Ed}\right) / 2 \cdot E_s \cdot A_s$ calculated at the middle of the effective working dept ($\sim d/2$) is not exced of $10^{-3}\,(1‰)$, ie. in case of B500 steel quality the tensioned steel bars are in elastic state ($\varepsilon_s \approx 2 \cdot \varepsilon_x = 2‰ \leq f_{yk}/E_s$):

$$V_{Rd} = V_{Rd,s} = 0{,}9 \cdot \frac{A_{sw}}{s_w} \cdot f_{ywd} \cdot d \cdot (\cot\theta + \cot\alpha) \cdot \sin\alpha \leq V_{Rd,\max}(\theta_{\min}),$$

where after simplification and consolidation we have for the crushing capacity:

$$V_{Rd,\max}(\theta_{\min}) = 0{,}9 \cdot k_c \cdot \frac{f_{ck}}{\gamma_c} \cdot b_w \cdot d \cdot \frac{\cot\theta_{\min} + \cot\alpha}{1 + \cot^2\theta_{\min}} \approx 1{,}55 \cdot \frac{f_{ck}^{2/3}}{\gamma_c} \cdot b_w \cdot d \cdot \frac{\cot\theta_{\min} + \cot\alpha}{1 + \cot^2\theta_{\min}}\ .$$

4. LEVEL OF APPROXIMATION II

In addition to the validity of the material properties specifield at the Level of Approximation I., with a more accurrate determination of the axial strain (ε_x) defined at the middle of the effeective dept the design shear resistance of a reinforced concrete cross-section *with no shear reinforcement* can be written at the Level of Approximation II [1]:

$$V_{Rd} = V_{Rd,c} = 0{,}9 \cdot k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot d = \frac{0{,}36}{1+1500 \cdot \varepsilon_x} \cdot \frac{1300}{1000 + 0{,}9 \cdot k_{dg} \cdot d} \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot d\,,$$

where $k_{dg} = 1$ if $d_g = d_{\max} \geq 16\,\mathrm{mm}$, and $k_v = 520/\left(1000 + 0{,}9 \cdot k_{dg} \cdot d\right)$ if $\varepsilon_x \leq 0$ or $k_v = 95/\left(1000 + 0{,}9 \cdot k_{dg} \cdot d\right)$ if $\varepsilon_x \geq 3‰$. When *shear reinforcement* is applied, the design shear resistance of a reinforced concrete can be calculated – neglecting also the concrete contribution as:

$$V_{Rd} = V_{Rd,s} = 0{,}9 \cdot \frac{A_{sw}}{s_w} \cdot f_{ywd} \cdot d \cdot (\cot\theta + \cot\alpha) \cdot \sin\alpha \leq V_{Rd,\max}(\theta_{\min})\,,$$

where the minimum angel of the principal compressive stress in the web can be calculated on a more precisious way than in the case of LoA I by the use of $\theta_{\min} = 20^\circ + 10^4 \cdot \varepsilon_x$, while the maximum shear capacity defined by the failure of the compressed concrete strut:

$$V_{Rd,\max}(\theta_{\min}) = 0{,}9 \cdot k_c \cdot \frac{f_{ck}}{\gamma_c} \cdot b_w \cdot d \cdot \frac{\cot\theta_{\min} + \cot\alpha}{1+\cot^2\theta_{\min}} \approx \frac{2{,}80}{1{,}2 + 55 \cdot \varepsilon_1} \cdot \frac{f_{ck}^{2/3}}{\gamma_c} \cdot b_w \cdot d \cdot \frac{\cot\theta_{\min} + \cot\alpha}{1+\cot^2\theta_{\min}}$$

where $k_\varepsilon = 1/(1{,}2 + 55 \cdot \varepsilon_1) \leq 0{,}65$ and $\varepsilon_1 = \varepsilon_x + \left(\varepsilon_x + 2 \cdot 10^{-3}\right) \cdot \cot^2\theta$.

5. LEVEL OF APPROXIMATION III.

At LoA III, Model Code 2010 provides an opportunity to take into account the shear force contributed by the concrete [1]. When $V_{Rd} < V_{Rd,\max}\left(\theta_{\min} = 20^\circ + 10^4 \cdot \varepsilon_x\right)$ the design shear capacity of a reinforced concrete cross-section applying also shear reinforcement according to the LoA III can be calculated as:

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \leq V_{Rd,\max}(\theta_{\min}).$$

where:

$$V_{Rd,c} = 0{,}9 \cdot k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot d \approx \frac{0{,}36}{1+1500 \cdot \varepsilon_x} \cdot \left(1 - \frac{V_{Ed}}{V_{Rd,\max}(\theta_{\min})}\right) \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_w \cdot d\,,$$

$$V_{Rd,s} = 0{,}9 \cdot \frac{A_{sw}}{s_w} \cdot f_{ywd} \cdot d \cdot (\cot\theta + \cot\alpha) \cdot \sin\alpha \,,$$

$$V_{Rd,\max}(\theta_{\min}) = 0{,}9 \cdot k_c \cdot \frac{f_{ck}}{\gamma_c} \cdot b_w \cdot d \cdot \frac{\cot\theta_{\min} + \cot\alpha}{1+\cot^2\theta_{\min}} \approx \frac{2{,}80}{1{,}2+55\cdot\varepsilon_1} \cdot \frac{f_{ck}^{2/3}}{\gamma_c} \cdot b_w \cdot d \cdot \frac{\cot\theta_{\min} + \cot\alpha}{1+\cot^2\theta_{\min}} \,.$$

6. EXPERIMENTAL OBSERVATIONS

Some results of earlier hungarian shear tests carried out on RC beams having 100×150 mm cross-section and 2 m length, together with 2Ø16 longitudinal steel bars [4] were compared to the shear model of *fib* Model Code 2010. Experimental variables were the used steel fibre type (Dramix® ZC 30/.5 hooked-end, D&D® ~30/.5 crimped), the applied steel fibre content (0V%, 0,5 V% and 1,0V%) and the amount of stirrup reinforcement (Series A: no strirrups, Ø6/240 and Ø6/120 and for Series B: no stirrups, Ø4/240, Ø4/120). Comparison of beams with only longitudinal steel bars (RC-A1 and RC-B1), as well as beams with different stirrup reinforcement for Series B (RC-B4-Ø4/240, RC-B7-Ø4/120) are considered hereafter in Table 1 and Table 2.

Table 1. Shear test results on RC beams made ***with shear reinforcement*** *and their comparison to the shear design proposal of fib MC 2010*

No.	Experimental							EC2			
	f_{cm}	$V_{R,exp}$	A_{sw}/s_w	ρ_w	$\rho_{w,min}$	k_{dg}	η_{fc}	θ	$V_R=V_{R,s}$	$V_{R,max}$	$V_{R,exp}/V_R$
	N/mm²	kN	mm²/mm	‰	‰	-	-	°	kN	kN	-
RC B4	29,99	27,50	0,104	1,047	0,836	1,33	1,082	21,80°	14,0	54,4	**1,96**
RC B7		35,20	0,208	2,094					28,0		**1,26**

No.	*fib* MC 2010 LoA I.					
	θ	k_ε	k_c	$V_R=V_{R,s}$	$V_{R,max}$	$V_{R,exp}/V_R$
	°	-	-	kN	kN	-
RC B4	30°	0,55	0,595	9,80	83,40	**2,81**
RC B7				19,60		**1,80**

No.	*fib* MC 2010 LoA II.							
	ε_x	θ	ε_1	k_ε	k_c	$V_R=V_{R,s}$	$V_{R,max}$	$V_{R,exp}/V_R$
	‰	°	‰	-	-	kN	kN	-
RC B4	0,171	21,71°	0,0139	0,509	0,551	14,2	61,30	**1,94**
RC B7	0,219	22,19°	0,0136	0,513	0,555	27,8	62,90	**1,27**

No.	*fib* MC 2010 LoA I.					
	k_v	$V_{R,c}$	$V_{R,s}$	$V_R= V_{R,c}+V_{R,s}$	$V_{R,max}$	$V_{R,ex}/V_R$
	°			kN	kN	-
RC B4	30°	10,4	14,2	24,60	61,3	**1,12**
RC B7		7,9	27,8	35,70	62,9	**0,99**

7. CONCLUSIONS

Shear desig model of the *fib* Model Code 2010 is an attempt to combine and harmonize many earlier different approaches [5-11]. For the calculation two parameters θ and k_v must be determined and three different levels of approximation (LoA I...LoA III.) to calculate the shear design force are recommended depending on the combination of the accuracy of prediction and the cost of calculation process as well. Based on some earlier shear test summarised in Table 1. and Table 2 it is found that precision of calculation increases as the level of approximation is increased.

*Table 2. Shear test results on RC beams made **with no shear reinforcement** and their comparison to the shear design proposal of fib MC 2010*

No.	Experimental		*fib* MC 2010 LoA I.				*fib* MC 2010 LoA II.				EC2	
	f_{cm}	$V_{R,exp}$	k_{dg}	k_v	V_R	$V_{R,ex}/V_R$	ε_x	k_v	V_R	$V_{R,exp}/V_R$	V_R	$V_{R,exp}/V_R$
	N/mm^2	kN	.	.	kN	-	$^0/_{00}$	-	kN	-	kN	-
RC A1	29,5	24,2	1,33	0,16	10,1	**2,40**	0,15	0,37	21,7	**1,12**	16,8	**1,44**
RC B1	27,3	21,6			10,0	**2,16**	0,13	0,38	21,3	**1,00**	16,4	**1,32**

REFERENCES

[1] Fédération Internationale du Béton (*fib*): *fib* Model Code for Concrete Structures 2010, **2013**.
[2] Fédération Internationale du Béton (fib): Shear and punching shear in RC and FRC elements, *fib Bulletin 57*, **2010**.
[3] Walraven, J. C.: Fundamental analysis of aggregate interlock, *Journal of the Structural Division, ASCE*, **1981.**, 107(ST11), pp. 2245-2270.
[4] Kovács I.: Effect of steel fibres on the structural behaviour of concrete members, PhD Thesis, *Technical University of Budapest*, **2005.**,
[5] Collins, M. P.: Toward a rational theory for reinforced concrete members in shear, *Journal of the Structural Division, ASCE*, **1978.**, 104(4), pp. 649-666.
[6] Thurlimann, B.: Shear strength of reinforced and prestressed concrete-CEB Approach, *ACI, Special publication,* **1979.**, Vol. 59, pp. 93-116.
[7] Nielsen, M. P.: On the strength of reinforced concrete discs, *Acta Politchnica Scandinavica, Civil Engineering and Building Construction Series*, **1971.**, No. 70, Copenhagen Denmark, 261 p.
[8] Muttoni, A.: The applicability of the theory of plasticity in the design of RC (German: Die Andwendbarkeit der Plastizitätstheorie in der Bemessung von Stahlbeton) IBK 176, *ETH, Zürich Switzerland,* **1989.**, 159 p.
[9] Sigrist, V.: On the deformation capacity of reinforced concrete girders (German: Zum Verformungsvermögen von Stahlbetonträger), *Diss. IBK, ETH, Zürich Switzerland,* **1995.**, 160 p.

[10] Bentz, E. C., Vehicco, F. J., Collins, M. P.: The simplified MCFT for calculating the shear strength of reinforced concrete elements, *ACI Structural Journal*, **2006.**, 103(4), pp. 614-624.
[11] Vehicco, F. J., Collins, M. P.: The modified compression field theory for reinforced concrete element subjected to shear, *ACI Journal*, **1986.**, 83(2), pp. 219-231

ISKUSTVA NA MODELU SMICANJA *fib* MODEL CODE 2010

Rezime:*Međunarodna organizacija fib je uvela novi standard (Model Code 2010) za smicanje armiranog betona i prednapregnutog betona. Jednačina koja opisuje model koristi dva parametra: θ – ugao pukotine koja se stvorila pod uticajem sila smicanja, k_v – koeficijent doprinosa betona. Standard (MC2010) opisuje tri nivoa kod približnog određivanja ovih parametara. Prvi i treći nivo se oslanja na teoriju modificiranog polja napona, dok drugi nivo na teoriju modificiranog polja plastične deformacije. Rad prikazuje i opisuje primenjene metode u toku velikog broja praktičnih merenja smicanja izvršenih u laboratoriji za ispitivanje konstrukcija i materijala. Cilj eksperimenta je bio da se uporede mađarski rezultati merenja dobijeni na betonima koji su bili armirani sa polipropilenom i sa čeličnim vlaknama.*

Keywords: *posmična čvrstoća, razina približavanja*