APPLICATION OF GENETIC ALGORITHM IN THE OPTIMIZATION OF GEODETIC NETWORKS: ANALYSIS ON THE EXAMPLE OF NETWORKS DESIGNED FOR LAND CONSOLIDATION PURPOSES

Zoran Ilić[1*], Darko Kuželka[2], Bogdan Bojović[3], Vladica Krstić[4]

[1]Academy of Technical and Educational Vocational Studies, Niš, Serbia
[2]Geodetic Bureau "Geoinženjering", Bela Crkva, Serbia
[3]University of Novi Sad, Faculty of Civil Engineering, Subotica, Serbia
[4]GTŠ „Neimar", Niš, Serbia
*corresponding author: zoran.ilic.ni@akademijanis.edu.rs

Paper type: Review paper
Received: 2025-10-15
Accepted: 2025-11-28
Published: 2025-12-30

UDK: 528.3
DOI: 10.14415/JFCE-926

ABSTRACT

In this paper, the theoretical foundation of geodetic networks and the methodology of their design are presented, with particular emphasis on the mathematical model of adjustment and the principles of optimization using the Genetic Algorithm (GA). The key criterion of the analysis pertains to the network accuracy, quantified through the standard deviations of point coordinates. The preliminary accuracy assessment was conducted using the Gauss–Markov model, which enables a reliable evaluation of the network precision prior to the optimization process. The practical part of the study includes an example of geodetic network optimization for the purposes of land consolidation surveying in the cadastral municipality of Češko Selo. The optimization was implemented in the MATLAB environment, with carefully tuned parameters of the genetic algorithm. The obtained results indicate that GA effectively identifies the optimal distribution of observation weights and improves the positioning of new points, thereby reducing the total number of observations while maintaining the accuracy and reliability criteria. The standard deviations of the coordinates remained within the predefined limits, confirming that an optimal balance between precision and resource rationalization was achieved. The results demonstrate that the application of second-order optimization represents an efficient approach to the design and planning of modern high-precision geodetic networks.

KEYWORDS:

geodetic networks, second-order optimization, observation weights, accuracy, reliability, genetic algorithm,

1 **INTRODUCTION**

The development of geodetic networks in Serbia has traditionally been oriented toward establishing a reliable reference system necessary for cadastral surveying and property registration. However, modern requirements for higher accuracy, the rapid process of urbanization, and the integration of advanced measurement technologies have emphasized the need for modernization and optimization of existing networks.

The quality of a geodetic network, according to the concept established by Baarda [1], encompasses two key dimensions – precision, which reflects the accuracy of the estimated point coordinates, and reliability, which denotes the system's ability to detect and localize possible systematic (non-random) errors in measurements. Together, these components form the basis for a quantitative assessment of the geometric and functional stability of the network, as well as for its mathematical and statistical evaluation.

In line with this approach, numerous authors have developed theoretical and methodological frameworks for the design and preliminary assessment of geodetic network accuracy and reliability. Among the most significant are the studies of Baarda [1,2], Grafarend [3], Grafarend and Sansò [4], Kuang [5], Oliveira and Dalmolin [7,8], Teixeira and Ferreira [8], and Monika et al. [9].

Building upon their findings, the main objective of this paper is to define an optimal model for geodetic network design that simultaneously meets the criteria of accuracy, reliability, and functionality, while rationalizing the measurement process. This involves minimizing the number of observed directions and distances, optimizing fieldwork duration, and reducing the overall measurement costs.

The research was conducted through a combination of theoretical and empirical approaches, including the following stages:

- **Analysis and synthesis**, involving the decomposition of a complex problem and integration of the obtained results;
- **Verification,** through confirmation of the research hypothesis based on theoretical assumptions, relevant studies, and numerical simulations;
- **Descriptive method,** used to present the main characteristics of geodetic networks, genetic algorithms, and observational data;
- **Modeling,** which includes the development and implementation of a mathematical optimization model in the MATLAB environment;
- **Compilation,** referring to the systematization of relevant scientific literature and sources.

Significant progress in geodetic network optimization has been achieved through the application of metaheuristic algorithms, which have shown high potential in solving complex nonlinear problems. Evolutionary and swarm intelligence algorithms (e.g., GA, SA, PSO, GWO) are increasingly used for the design of first- and second-order networks, the determination of optimal observation weights, and the design of deformation monitoring networks.

The first comprehensive application of the Genetic Algorithm (GA) in geodetic network design was conducted by Al-Shuni [10], who demonstrated that redundant GPS baselines can be eliminated while maintaining the required network accuracy. Subsequent studies further improved this approach. For example, Yetki et al. [11] applied the Shuffled Frog Leaping Algorithm (SFLA) to design a first-order network aimed at maximizing geometric stability, while Baselga [12] and Amiri-Simkooei [13] utilized Simulated Annealing (SA) and other global optimization techniques to optimize measurement weights.

In addition, robust deformation analysis in free geodetic networks using evolutionary optimization algorithms was examined in a doctoral dissertation [18], while study [17] presented an application of the Genetic Algorithm for the optimization of a second-order network using the Liverovići dam as a case study.

The application of swarm intelligence algorithms has also produced notable results: studies have shown that Particle Swarm Optimization (PSO) can reduce the number of baselines in GNSS networks by more than 40%, while simultaneously lowering costs and shortening fieldwork duration [14]. Comparable efficiency has been recorded with the Grey Wolf Optimization (GWO) algorithm, applied for EDM instrument calibration and the optimization of leveling networks [15]. More recent studies have proposed the Butterfly Optimization Algorithm (BOA), which demonstrated higher efficiency compared to Simulated Annealing (SA) and PSO in selecting optimal GPS baselines [16].

Based on the reviewed studies, it can be concluded that the Genetic Algorithm (GA) and related metaheuristic methods represent highly effective tools for addressing geodetic network optimization problems. Their implementation enables the enhancement of accuracy and reliability while reducing costs and improving operational efficiency, thereby representing an important methodological direction in contemporary geodetic science and engineering.

2 MATERIALS AND METHODS

The subject of this study is the optimization of a second-order geodetic network using the Genetic Algorithm (GA). The main objective of the research is to determine the optimal values of the observation weights Pi, such that the standard deviations of the coordinates of the unknown points (σ) are as close as possible to the predefined threshold value of 5 mm. This ensures that the designed criteria for network accuracy and reliability are met, while simultaneously reducing the number of observed directions and distances in the optimal design of the geodetic network intended for land consolidation surveying.

The network is based on four fixed points and fifteen points with approximately known coordinates. A total of 60 direction measurements and 30 distance measurements are planned within the network.

Defined criteria:

- The ratio between the major and minor axes must satisfy: : $1 \leq A \leq 2B$
- Positional accuracy of newly determined points: $\sigma_{POL} \leq 15\ \mathrm{mm}$

- Total station with declared angular accuracy: $\sigma_P = 5''$
- Standard deviation of distance measurements: $\sigma_D \leq 5\ \mathrm{mm} + 5\ \mathrm{ppm}$
- A priori standard deviation: $\sigma_0 = 1$

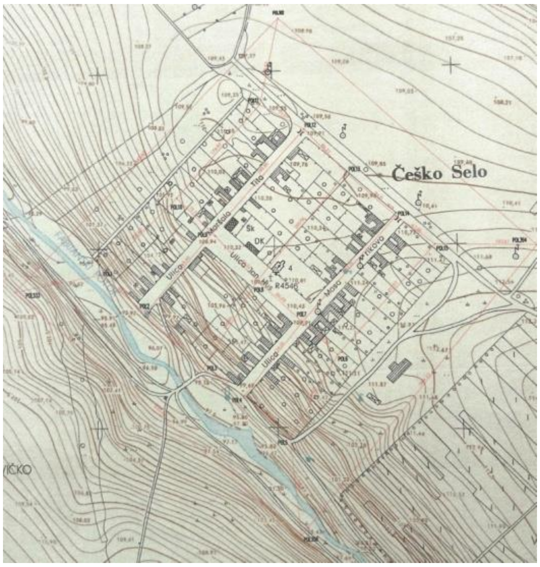

Figure 1. Newly designed geodetic network of the settlement Češko Selo

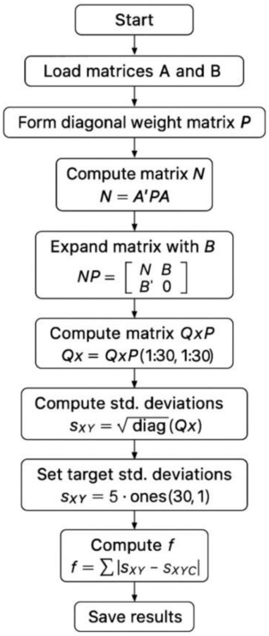

Figure 2. Flowchart structure

In the MATLAB environment, the design matrix A and the datum condition matrix B were taken from the results of the preliminary accuracy assessment. The next step involves the computation of the normal equation matrix N, which in the Gauss–Markov model is calculated as:

$$\mathbf{N} = \mathbf{A}^T * \mathbf{P} * \mathbf{A} \tag{1}$$

Furthermore, after constructing the augmented normal equation matrix (NP),

$$\mathbf{NP} = \begin{bmatrix} \mathbf{N} & \mathbf{B} \\ \mathbf{B}' & \mathbf{0} \end{bmatrix} \tag{2}$$

its inversion yields the cofactor matrix of all unknown and additional parameters:

$$\mathbf{Q}_{xP} = \mathbf{NP}^{-1} \tag{3}$$

where:

- $\mathbf{N} = \mathbf{A}^T\mathbf{PA}$ – the normal matrix,
- $\mathbf{B}$ – the datum condition matrix (fixing coordinates or transformation parameters),
- NP – the augmented normal matrix.
- Inversion of NP→ allows obtaining the covariances of the unknowns (coordinates, orientations).

From $\mathbf{Q}_{xP}$, the "upper-left block" is usually extracted (i.e., the submatrix located in the upper-left corner of the larger matrix), which corresponds to the unknown coordinates:

$$\mathbf{Q}_x = \mathbf{Q}_{xP}(1:n, 1:n) \tag{4}$$

This procedure computes the cofactor matrix of all unknown and additional parameters, $\mathbf{Q}_{xP} = \mathbf{NP}^{-1}$, which contains the cofactors for all unknowns.

Mathematically, if the number of coordinate unknowns is 30 (e.g., 15 points × 2 coordinates), then:

$$\mathbf{Q}_x = \mathbf{Q}_{xP}(1:30, 1:30) \tag{5}$$

From the coordinate cofactor matrix $\mathbf{Q}_x$, the standard deviations are computed using the diagonal elements, as they represent the variances. In this specific case, there are 15×2=30 diagonal elements.

$$\sigma_{XY} = \sqrt{\mathrm{diag}(\mathbf{Q}_x)} \tag{6}$$

The result σ_{XY} is a vector of the form: $[\sigma_{X1}, \sigma_{Y1}, \sigma_{X2}, \sigma_{Y2} \dots\dots\dots\dots\dots. \sigma_{X15}, \sigma_{Y15}]$

As indicated in the flowchart and implemented in MATLAB, the next task is to define the target (desired) standard deviations for each point coordinate in the network. This is usually done by creating a vector of identical values, with a dimension equal to the number of coordinate unknowns.

Mathematically expressed:

For n points with a desired standard deviation σ_C,

$$\mathbf{S_{XYC}} = \begin{bmatrix} \sigma_C \\ \sigma_C \\ \vdots \\ \sigma_C \end{bmatrix} = \sigma_C * \ 1_{nx1} \tag{7}$$

with:

- 1_{nx1}→ a vector of n units.
- σ_C → desired standard deviation (e.g., 5 mm)

Next, the objective function f is calculated. In this case, the objective function (f) measures how much the actual standard deviations of the network (Sxy) deviate from the target standard deviations (Sxyc).

$$\mathbf{S_{XY}} = \begin{bmatrix} s_{xy}, 1 \\ s_{xy}, 2 \\ \vdots \\ s_{xy}, n \end{bmatrix} \tag{8}$$

$$\mathbf{S_{XYC}} = \begin{bmatrix} s_{xyc}, 1 \\ s_{xyc}, 2 \\ \vdots \\ s_{xyc}, n \end{bmatrix} \tag{9}$$

with:

- $\mathbf{S_{XY}}$ → actual standard deviations of the coordinates of the free points after network computation.
- $\mathbf{S_{XYC}}$ → target (desired) standard deviations, predefined, e.g., 5 mm.

Thus, the objective function is the sum of the absolute deviations of the actual standard deviations from the target values:

$$\boldsymbol{f} = \sum_{i=1}^{n} \mathrm{I} s_{xy} - s_{xyc} \mathrm{I} \tag{10}$$

As the final result, we obtain the actual deviations, the target deviations, and the total value.

Saving the results If we also have a vector of observation weights:

$$\mathbf{P} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_m \end{bmatrix} \tag{11}$$

where *m* is the number of observations, the results can then be formally represented as the set:

Results = { $p, \ s_{xy} \ , \ f$ }

Following the previously presented analysis, the next step involves the optimization of the objective function using a Genetic Algorithm. This optimization was performed within the MATLAB software package, which includes the built-in GATool (short for Genetic Algorithm Tool). GATool is a graphical user interface (GUI) that allows the Genetic Algorithm (GA) to be executed, controlled, and analysed without the need for direct coding.

3 RESULTS AND DISCUSSION

For the numerical implementation of the optimization procedure, the MATLAB software environment was used, which proved suitable due to its flexibility in matrix operations, implementation of optimization algorithms, and visualization capabilities. Based on the obtained matrices and results from the preliminary accuracy assessment of the geodetic network design, as well as the defined target standard deviations of the network point coordinates, the optimal observation weights were calculated to meet the specified target standard deviations.

The following results were obtained:

The results of the levelling and adjustment of the geodetic network indicate that satisfactory accuracy and reliability parameters were achieved for the designed network, which includes 4 fixed points (333, 358, 382, 394) and 15 free points. A total of 90 observations (directions and distances) were processed, and the coordinates of 19 points in the plane (X, Y) along with their orientations were determined.

3.1 ANALYSIS OF RESIDUALS AND WEIGHT RATIOS

The observation residuals (column V) range from -3,89 to +3,17, indicating a good adjustment and the absence of gross errors. Larger residual values mostly occur for longer baseline distances (e.g., 333–382, 358–394, 394–382), which is expected due to the increased lengths and the cumulative effects of instrumental and atmospheric influences.

The weighting factors ($Q_{v_{ii}}$ and $Q_{l_{ii}}$) indicate that the directions generally exhibited slightly higher weighted residuals compared to the distances; however, both types of measurements remain within the designed reliability limits.

3.2 ANALYSIS OF INTERNAL RELIABILITY (r_{ii})

The values of the internal reliability coefficient (r_{ii}) range from 0,25 to 1,00, fully complying with the network design requirements ($0{,}2 \leq r_{ii} \leq 1{,}0$). The total sum of $\sum r_{ii} = 41{,}00$ indicates a well-distributed set of measurements and a homogeneous network without dominant observations exerting excessive influence. The highest r_{ii} values occur for the baselines between fixed points, which is desirable, as these distances help to stabilize the network.

3.3 ANALYSIS OF POSITIONAL ACCURACY OF POINTS

The standard deviations of the coordinates of the free points are as follows:

- $m_X = 2{,}39 - 4{,}16$ mm
- $m_Y = 2{,}47 - 4{,}31$ mm
- $m_Z = 4{,}44 - 5{,}99$ mm

These values confirm that the network achieved the designed positional accuracy of $\sigma_{POL} \leq 15$ mm.

The largest deviations in coordinates occur at points 13 and 14 ($m_P \approx 5{,}9$ mm), while the most stable points are 5 and 11 ($m_P \approx 4{,}4 - 4{,}7$ mm).

3.4 ANALYSIS OF THE ERROR ELLIPSES

The error ellipse parameters (A, B, θ) indicate that the major axes (A) ranges from 3,74 to 4,74 mm, while the minor axes (B) ranges from 2,34 to 3,70 mm. The A/B ratio varies between 1,1 and 1,6, which is fully consistent with the standards for geodetic network design ($1 \leq A \leq 2B$).

The orientations of the ellipses (θ) show that the errors are primarily directed along the network's span lines, confirming a good geometric configuration of the points.

3.5 ANALYSIS OF EXTERNAL RELIABILITY AND OVERALL ACCURACY

The values of G_{ii} (external reliability factors) range from 26,3 to 41,7, indicating a moderate transfer of errors to the unknown parameters. The overall adjustment is stable, and the network meets the requirements for high internal and external reliability.

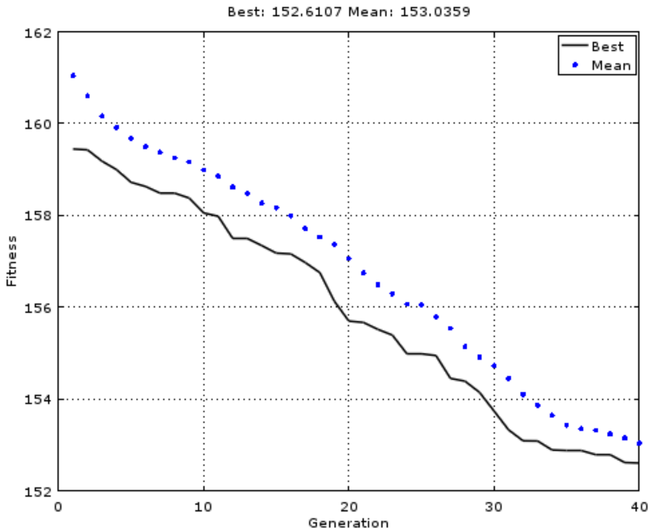

Figure 3. Convergence of the Genetic Algorithm over 40 generations

Analysis of the Genetic Algorithm (GA) Convergence Chart:

- Axes
  - *X*-axis (Generation) – the number of generations, i.e., evolutionary steps (from 0 to 40).
  - *Y*-axis (Fitness) – the value of the objective function being optimized (here, minimization is performed, so lower values indicate better solutions).
- Curves:
  - **Black line (Best)** – shows the best value of the objective function in each generation. This illustrates how the best candidate (optimal solution) improves over the course of evolution.
  - **Blue points (Mean)** – indicate the average fitness of the population in each generation, reflecting the overall quality of all solutions, not just the best one.
- Observations:

- o At the beginning (generation 0), fitness values are higher (~161–162).
- o As the algorithm progresses, both Best and Mean fitness values decrease, indicating that the GA is finding increasingly better solutions.
- o By the end (around generation 40), Best ≈ 152,61 and Mean ≈ 153,04 → the difference is small, showing that the entire population has converged toward a good solution.

The Genetic Algorithm successfully improves the solutions, with the best value ≈ 152.61, and since the average population value approaches the best, the algorithm demonstrates stable convergence.

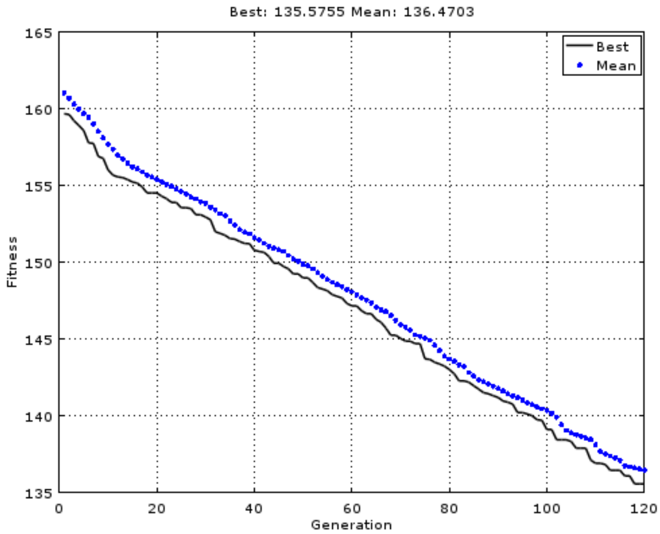

Figure 4. Convergence of the Genetic Algorithm over 120 generations

Analysis of the Genetic Algorithm (GA) Convergence Chart:

- Axes:
  - o ***X*-axis (Generation)** – represents the number of generations in the GA, from 0 to ~120.
  - o **Y-axis (Fitness)** – the value of the objective function being optimized. Here, the goal is minimization, as the values decrease over generations.
- Curves on the Chart:
  - o **Black line (Best)** – shows the best (lowest) fitness value in each generation.
  - o **Blue points (Mean)** – represent the average fitness value of all individuals in the population for a given generation.
- Trend Observations:
  - o Both trends (best and mean fitness) decrease as generations increase, indicating that the GA is converging toward a more optimal solution.
  - o The difference between the black line and blue points decreases over time, showing that the population becomes more homogeneous around a good solution.

- Start: fitness around 160 (Best ~159, Mean ~161)
- End (iteration ~120): fitness around 135–136 (Best ~135,58, Mean ~136,47)
- The GA successfully reduced the fitness by approximately 24–25 units over 120 generations.
- Convergence is gradual, with no sudden jumps, indicating a stable evolutionary process.
- The final difference between the best and mean fitness is small (~0,9), indicating good population homogenization.
- Overall, the obtained result is considered very good.

4 CONCLUSION

Based on the obtained results, the geodetic network can be evaluated as high-precision and stable, suitable for precise geodetic and engineering tasks. All fundamental quality criteria, including measurement accuracy, internal and external reliability, residuals, and positional accuracy of the points, were met or exceeded.

The analysis of observation weights through the application of the Genetic Algorithm (GA) shows that the iteratively selected optimal weights resulted in minimized standard deviations of the coordinates and a stable balance between internal and external reliability. Graphical representations of the weights and algorithm iterations indicate stable convergent behaviour and efficient selection of the best solutions.

The implementation of GA enabled a systematic optimization of observation weights through the iterative process of selection, crossover, and mutation. This approach achieved the minimization of coordinate standard deviations while simultaneously maintaining the network's internal and external reliability. The GA proved to be an effective and flexible tool for solving nonlinear optimization problems in geodesy, as it allows for finding globally optimal solutions and improving existing measurement plans without the need to linearize the objective function.

Based on the results, it can be concluded that the optimized network is geometrically stable, statistically reliable, and fully suitable for both land consolidation surveying and high-precision engineering tasks. The application of GA confirms its potential as a modern method for geodetic network optimization, especially in cases where an optimal balance between accuracy, cost-efficiency, and measurement reliability is required.

Although the applied model produced very satisfactory results, there are several directions in which future research could be further developed:

1. Extension of the Optimization Model – Introducing multi-objective functions that, in addition to accuracy and reliability, incorporate economic aspects of measurements, such as time, costs, and available equipment.
2. Application of Advanced Heuristic Algorithms – Comparing the performance of the Genetic Algorithm with other methods, such as Particle Swarm Optimization (PSO),

Differential Evolution (DE), and Ant Colony Optimization (ACO), to assess the efficiency and stability of the solutions.

3. Development of Adaptive Weight Models – Implementing dynamic weight functions that automatically adjust to changes in measurement accuracy and network geometry.
4. Experimental Field Verification – Validating the optimized model on real geodetic networks of various configurations and sizes to confirm its practical applicability and robustness under real conditions.
5. Further Improvement Opportunities – Future work could involve the use of alternative optimization algorithms and the combination of first- and second-order network optimization problems. This approach would not only reduce the number of measurements, which is the main goal of optimization, but also improve the positions of the points.

REFERENCES:

[1] BAARDA, W. *Measures for the accuracy of geodetic networks*. In: Symposium on Optimization of Design and Computation of Control Networks, p. 419-436, Sopron, Hungary, 1977.

[2] BAARDA, W. *A testing procedure for use in geodetic networks*. Publications on Geodesy, New Series, v. 2, n. 5, Delft: Netherlands Geodetic Commission, 1968

[3] GRAFAREND, E. W. Optimization of geodetic networks. *Boll. Geod. Sci. Aff.*33(4), 351-406, 1974.

[4] GRAFAREND, E. W.; SANSO, F. *Optimization and design of geodetic networks*. Edited by Grafarend and Sanso, Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1985.

[5] KUANG, S. L. *Geodetic network analysis and optimal design: concepts and application*. Ann Arbor Press, Chelsea, MI, 1996.

[6] OLIVEIRA, R.; DALMOLIN, Q. Critérios para a análise da geometria de redes geodésicas por componentes principais. *Boletim de Ciências Geodésicas*, v. 9, n. 1, p. 25-37, 2003.

[7] OLIVEIRA, R.; DALMOLIN, Q. A Influência da Redundância da Observação sobre a Precisão dos Parâmetros. *Boletim de Ciências Geodésicas*, v. 14, n. 3,p. 295-315, 2008.

[8] TEIXEIRA, N. N.; FERREIRA, L. D. D. Análise da Confiabilidade de Redes Geodésicas. *Boletim de Ciências Geodésicas*, v. 9, n. 2, p. 199-216, 2003.

[9] MONICO, J. F. G.; MATSUOKA, M. T.; SAPUCCI, L. F. Confiabilidade interna e externa em Aplicações Geodésicas: Exemplo de uma Rede de Nivelamento. *Geodésia Online*, v. 2, p. 1, 2006.

[10]Ahmed Fuad Al Šuni, DESIGN_OF_DIFFERENT GEODETIC NETWORKS USING GENETIC ALGORITHMS, 2011, https://www.researchgate.net/publication/338820132_DESIGN OF DIFFERENT GEODETICNETWORKS USING GENETIC ALGORITHMS

[11] Mevlut Yetki, Metaheuristic optimisation approach for designing reliable and robust geodetic networks,2013, Survey Review 45(329):136-140, DOI: 10.1179/1752270612Y.0000000021

[12] Baselga, S. (2011). Second order design of geodetic networks by the simulated annealing method. Journal of Surveying Engineering, 137(4):167-173, doi: 10.1061/(ASCE)SU.1943-5428.0000053.

[13] Wlademar Odziemczyk, Comparison of selected reliability optimization methods in application to the second order design of geodetic network,2023, doi.org/10.1515/jag-2023-024, https://www.degruyterbrill.com/document/doi/10.1515/jag-2023-0024/pdf.

[14] Ahmed Al Shouny, Second-order design of GNSS networks with different constraints using particle swarm optimization and genetic algorithms, Open Engineering 2025; 15: 20250109, https://doi.org/10.1515/eng-2025-0109 received October 18, 2024;

[15]M.Yetkin,O.Bilginer, On the application of nature-inspired gray wolf optimizer algorithm in geodesy , DOI: https://doi.org/10.1515/jogs-2020-0107, January 16, 2020; accepted April 3, 2020

[16] Doaa. S. and others. Design of global positioning system (GPS) networks using different artificial intelligence techniques, Journal of Engg. Research Vol. 11 No.(1B) pp.14-25, 2021, DOI:10.36909/ jer.10587

[17] Maksimović J., Kuzmić T, Batilović M, Sušić Z, Bulatović V, Kanović Ž. Projektovanje geodetskih mreža primenom globalnih optimizacionih metoda, 7. Međunarodna konferencija savremena dostignuća u građevinarstvu 2019, 2019.god., str.975-984, DOI: 10.14415/konferencijaGFS2019.090

[18] Mehmed Batilović, Robusna ocena deformacija u slobodnim geodetskim mrežama primenom evolutivnih optimizacionih algoritama, doktorska disertacija, 2020, Fakultet tehničkih nauka Novi Sad. https://nardus.mpn.gov.rs/handle/123456789/18117