

# RHEOLOGICAL CHARACTERISTICS OF MODELS IN APPLICATION OF CONCRETE CREEP THEORIES

## REOLOŠKE KARAKTERISTIKE MODELA U PRIMENI TEORIJA TEČENJA BETONA

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**Summary:** The aim of this paper is to supplement the subject of the paper under the title: 'Generalization of stress-deformation relations for concrete'[1]. The need to better define the modulus of elasticity of concrete that is not sufficiently involved in the practical problems of structural analysis will be considered. The second part of the paper will show how the characteristics of basic rheological models of concrete can be experimentally and theoretically determined

**Keywords:** Concrete, rheological, model, creep, theories

**Rezime:** Cilj rada je da dopuni materiju rada pod naslovom: 'Generalisation of stress-deformation relations for concrete'. Biće razmatrane potrebe boljeg definisanja modula elastičnosti betona koji nisu dovoljno uključeni u praktične probleme teorije konstrukcija. U drugom delu rada biće prikazano kako se mogu eksperimentalno i teorijski odrediti parametri osnovnih reoloških modela betona.

**Ključne reči:** Beton, reološki, model, tečenje (viskoznost), teorije

### 1. INTRODUCTION

Most of the papers in this field have dealt only with basic theories of concrete and its algebraic equations. The subject already presented there, which is based on a mathematical model of concrete creep, will not be repeated. It has been shown that two basic relations, according to the theory of aging and the theory of heritage, can also be obtained through rheological models of concrete.

### 1. UVOD

Većina radova iz ove oblasti bavila se samo osnovnim teorijama betona i njegovim algebarskim jednačinama. Ovde se neće ponavljati već izložena materija u rezimeu navedenog rada, koja je zasnovana na matematičkom modelu veze betona. Pokazano je da se dve osnovne veze, prema teoriji starenja i teoriji nasleđa, mogu dobiti i preko reoloških modela betona.

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PC-Pre-stressed Concrete ; PB-Prednapregnuti Beton

These models (along with the Burges model) were called modified, by the author, because they need to be different from their classical shapes. These are different (concrete) materials that have variable characteristics, because they have aging properties (except for the VE model that received new parameter signs, that can be measured in the same way as for the other two models). That's the reason why it has new signs:

$\tilde{M}$ ,  $\tilde{VE}$  and  $\tilde{B}$ .

Modern building structures require knowledge of their properties over the duration of their life, because we want to know the properties of creep and shrinkage of concrete, and equally his elastic behavior with variable and different modulus of elasticity.

The intensive development of PC technologies and composite PC structures, after the Second Great War, aroused great interest of researchers for their realizations, as well as for theoretical and practical testing of various new types of concrete structures

Experimental and theoretical investigations of the creep and shrinkage of concrete have undertaken by a number of German researchers (Dishinger, Rush, Ross., Trost, et al.), several Russian scientists (Ulickij, Arutjunjan, .Aleksandrovsky, etc.), then French (Courbon, Fressine, et al.). Recently, the work of Bazant and his associates has been noticed in the USA and EU. In addition, intensive research is being carried out in several countries in the world (Hungary, Czech Republic etc.) from this area of PC structures. Many Serbian researchers have also addressed these issues (see: References [1]).

The IMS Institute in Belgrade has very good possibilities for this group of researches (humidity and temperature controls in chambers, samples loading devices with intensity controls, and various units for the parallel monitoring of

Ovi modeli (zajedno sa Burgesovim modelom) autor je nazvao modifikovanim, jer treba da se razlikuju u odnosu na njihove klasične oblike. Radi se o novim materijalima koji imaju promenjive karakteristike zato sto imaju osobine starenja (osim VE modela koji je dobio nove oznake karakteristika koje se mogu meriti na isti način kao za druga dva modela). To je razlog sto oni

nose nove oznake:  $\tilde{M}$ ,  $\tilde{VE}$  i  $\tilde{B}$ .

Savremene gradjevinske konstrukcije zahtevaju poznavanje njihovih osobina tokom vremena njihovog trajanja, jer se želi poznavanje osobina viskoznosti, skupljanja betona i podjednako njihovo elastično ponašanje sa promenjivim i različitim modulima elastičnosti.

Intezivan razvoj tehnologija PB i spregnutih konstrukcija posle drugog svetskog rata, izazvao je veliko interesovanje istraživača za njihovu realizaciju, ali i za teorijska i praktična ispitivanja raznih novih vrsta betona .

Eksperimentalnim i teorijskim istraživanjima tečenja i skupljanja betona bavio se veći broj Nemačkih istraživača (Dishinger, Rush, Ross., Trost i dr.), više Ruskih naučnika (Ulickij, Arutjunjan, .Aleksandrovski i dr.), zatim Francuskih (Courbon, Fressine, i dr.). U novije vreme u USA je zapažen rad Bažanta i njegovih saradnika i dr. Kao dopuna, treba navesti da su intezivna istraživanja u nekoliko država sveta (Mađarska, Češka republika i dr.) iz ove oblasti PB konstrukcija. Mnogi srpski istraživači su se isto bavili ovim problemima (see: References[1]).

Institut IMS u Srbiji ima dosta dobre uslove za ovu grupu istrazivanja (klima komoru sa regulacijom vlažnosti i temperature, prese za optrećivanje uzoraka sa regulacijom intenziteta i različite instrumente za paralelno praćenje deformacija i opterećenja),

deformations and loads), but also permanent laboratory assistants for qualified and safe measurements and meticulous monitoring of a wide range of extensive data. (see [1] [2]).

Experience gained in testing modulus of elasticity, concrete shrinkage and viscous creep of concrete and their comparison with the results of, above all, Ulicky, Aleksandrovsky, Courbon, Trost, Bazant and others. This facilitate to bring decisions about new simply proposals for operating procedures, which has already been presented in part of the first cited paper [1]. This paper should indicate how closely the data of our experiments with selected foreign and our sources match.

ali i stalni laboranti za kvalifikovana i sigurna merenja i pedantno praćenje čitavog niza obimnih podataka.(see: [1] [2] ).

Stečeno iskustvo na ispitivanju modula elastičnosti, skupljanja betona i viskoznog tečenja betona i njihovo poređenje sa rezultatima, pre svega, Ulickog, Aleksandrovskog ,Courbon-a Trosta , Bazanta i dr. olakšalo je donošenje odluka o formiranju novih predloga postupaka rada pri odvijanju eksperimenata i tumačenju rezultata sto je jednim delom vec izloženo u prvom citiranom radu[1]. Ovaj rad treba da ukaže koliko se slažu podaci naših eksperimenata sa odabranim inostranim i našim izvorima.

## 2. ELASTIC MODULE OF CONCRETE AS TIME FUNCTION

(1) Modulus of elasticity as a function of real time (t)

For more significant structures, at the Institute IMS measurements were made of the characteristics of concrete under long-term loading for different environmental conditions.

## 2. MODUL ELASTIČNOSTI BETONA U FUNKCIJI VREMENA

(1) Modul elastičnosti u funkciji realnog vremena (t)

Za više značajnih konstrukcija u Institutu IMS su obavljena merenja karakteristika betona pri dugotrajnim opterećenjima za različite uslove sredine.

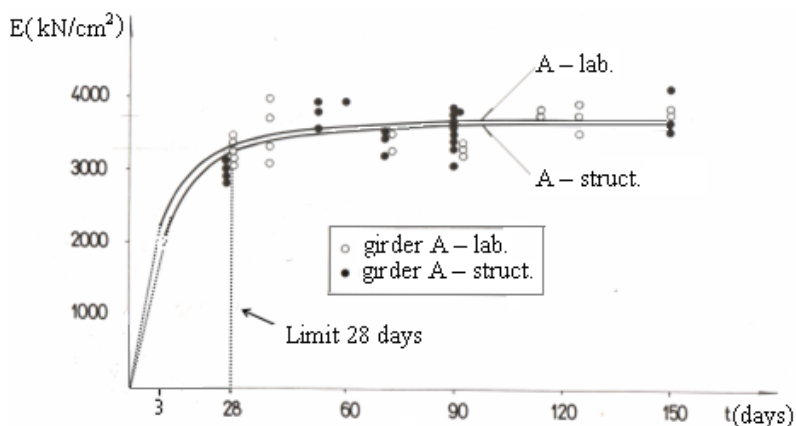


Fig.1 Experimental dependence of modulus of elasticity (E) of concrete age (t) [2]

Only some structures will be listed here: Hangar 2 Belgrade Airport, Railway Bridge over the Sava River in Belgrade, Mounting brackets of 'Gradis' Company from Maribor, et al. The modulus of elasticity of concrete as a function of time was determined on cylinders 15X30cm. The samples were stored in an air-conditioning chamber and at the Hangar construction site. The modulus of elasticity was determined at 28 days, 2 months, 3 months and 6 months. The results are shown in Fig.1.

The results are quite close to the theoretical results indicated by solid curved lines. Larger differences were obtained at the construction site - up to 15%. According to N.Arutjunjan's proposal it is justified, for this type of concrete, to change the modulus of elasticity of concrete by applying the expression(see:[3]):

$$E(t) = E_0 (1 - \xi e^{-\beta t}) \quad (1)$$

Here a more convenient form may be used instead of expression (1):

$$E(t) = E_n (1 - e^{-\alpha t}) \quad (2)$$

where

$E_n = k_n E_0$  - limit value of  $E(t)$   
and  $E_0 = E(\tau_0)$   
 $k_n = 1.15$  - coefficient for modulus increased (for test)  
 $\alpha = 0.073$  - parameter determined for the 'flow' position of the curve

Ovde ce biti navedene samo neke konstrukcije : Hangar 2 aerodroma Beograd , Železnički most preko Save u Beogradu, Montažni nosači kompanije 'Gradis' iz Maribora i dr.

Modul elastičnosti betona u funkciji vremena određen je na cilindrima 15X30cm. Uzorci su čuvani u klima komori i na gradilištu Hangara. Modul elastičnosti je određen pri starosti 28dana, 2 meseca, 3 meseca i 6 meseci. Rezultati su prikazani na Sl.1. Rezultati su dosta bliski teorijskim rezultatima naznačeni punim krivim linijama. Na gradilištu su dobijene veće razlike – do 15%.

Prema predlogu N.Arutjunjana opravdano je, za ovu vrstu betona, promenu modula elastičnosti betona naći primenom izraza(v:[3]):

Umesto izraza (1) može se primeniti nešto pogodniji oblik:

gde je

$E_n = k_n E_0$  - krajnja vrednost  
i  $E_0 = E(\tau_0)$   
 $k_n = 1.15$  – koeficijent povećanja modula (za exper.)  
 $\alpha = 0.073$  –parametar za položaj 'flow' krive u eksperimentu

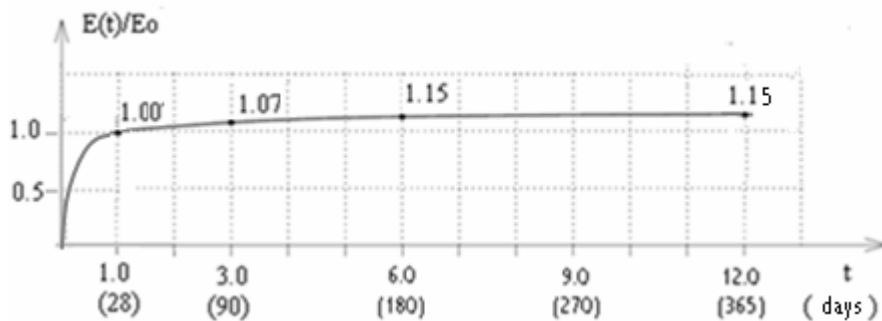


Fig.2 Modulus of elasticity  $E(t)$  as a function of concrete age  $t$ .  
According to the author

The curves of expression (2) are shown in Fig. 1 as medium solid lines. In addition,  $E(t)/E_0$  ratios by expression (2) were found and shown in Fig.2. For the final value  $E_n$ , with  $k_n = 1.158$ , is taken as data from Fig.1. The values are also entered in Table 1. for easier comparison with the Courbone experimental data [1].

Krive prema izrazu (2) su prikazane na Fig.1 kao srednje pune linije. Pored toga, nađeni su odnosi  $E(t)/E_0$  prema izrazu (2) i prikazani na Fig.2. Za krajnju vrednost  $E_n$  uzeto je  $k_n=1.158$  kao za podatke na Fig.1. Vrednosti su upisane i u Tabeli 1. da bi lakše uporedili sa eksperimentalnim podacima Courbon-a [1].

Table 1 Comparative ratio values of  $E(t)/E(28)$  as function of  $t$

Tests by	Measurement time (t) of Modul					
	3	7	28	90	150	365
Courbon	0.63	0.81	1.00	1.10	-	1.16
IMS	0.23	0.47	1.00	1.15	1.15	-

Comparing the previous experimental values with the theoretical values, they do not appear to deviate much from the trend of the curved lines (mean up to 14%). Only major deviations are in the concrete maturation zone for  $t < 28$  days.

Uporedivši prethodne eksperimentalne sa teorijskim vrednostima vidi se da ne odstupaju mnogo od trenda krivih linija (srednja vrednost se razlikuje do 14%). Veća odstupanja su u zoni sazrevanja betona za  $t < 28$  dana.

(2) Modulus of elasticity as function of 'replacing' time ( $\varphi$ ) –Theory of aging

Rather than displaying module as function ( $t$ ), it is often more convenient to represent as the function from 'replacing' time  $\varphi(t)$ :

(2) Modul elastičnosti u funkciji zamenjujućeg vremena ( $\varphi$ ) –Teorija starenja

Umesto prikaza modula  $E$  u funkciji ( $t$ ) često je pogodnije da se prikaže kao funkcija tečenja (zamenjujućeg vremena  $\varphi(t)$ ):

$$E(t) = E_0 (1 + \alpha_n \cdot \varphi(t) / \varphi_n) \quad (3)$$

where

$\alpha_n=0.16$  - coefficient of modulus Increase for expression(3)

gde je

$\alpha_n=0.16$  –koeficijent povećanja modula za izraz (3)

Ulickij suggested the same term in dimensional form in the book (see:[4]). This may be considered to be a more suitable form in the theory of aging, since it uses a 'replacing' time( $\varphi$ ).

Može se smatrati da je u teoriji starenja ovo pogodniji oblik, jer se u njoj koristi zamenjujuće vreme  $\varphi$ . Ulickij je isti izraz predložio u dimenzionalnom obliku u knjizi [4].

Table 2. The ratio  $E(t)/E(\tau_0)$  as function of  $\varphi(t)/\varphi_n$  (For example:  $E(\tau_0)=E(28)$ )

$\varphi(t)/\varphi_n$	0	0.50	1.00	1.50	2.00	3.00
$E(t)/E(\tau_0)$	1	1.026	1.053	1.080	1.106	1.16

In the table is taken  $\varphi_n = 3$ , and to find  $E(t)/E(\tau_0)$  then applies the expression (3).

The coefficient values can be determined on the way as it will be described for the creep functions. Modulus of elasticity, as a function of time  $E(t)$ , is required for proper estimation of elastic deformations and stresses of structures in Structural Mechanics.

(3) Modulus of elasticity as a function of real time ( $t$ ) - The theory of heritage

Modulus of elasticity of old concrete is required in the preparation of previous designs for specific parts or complete structures (for example: prefabricated structures, phase structures, etc.). The work procedure is the same as described above for young concrete under(1).

Vrednosti koeficijenata mogu se odrediti na način kako će biti opisano za funkcije tečenja.

Moduli elastičnosti u funkciji vremena  $E(t)$  potrebni su za pravilnu procenu elastičnih deformacija i napona konstrukcija u Mehanici konstrukcija.

(3) Modul elastičnosti u funkciji realnog vremena ( $t$ ) –Teorija nasleđa

Moduli elastičnosti starih betona potrebni su u izradi prethodnih projekata za određene delove ili kompletne konstrukcije (npr. montažne konstrukcije, fazno građenje i sl.)

Postupak rada je isti kao što je već opisano za mlade betone pod (1)

### 3. RESULTS FOR PARAMETERS IN

#### THEORIES OF CONCRETE

##### a) Young Concrets ( Use of Aging Theory)

Concrete, is said, to be young if the time of initial loading is within limits  $3 = < \tau_0 = < 90$  days , where concrete is a material belonging to an area I, (see: [1]). Here also the concrete has some reversibility properties, but that's the part below 5% of the total creep coefficient and is considered to be neglected, because we want to obtain an idealized form of the theory of aging.

Few authors are engaged in experimental, long-term testing of the behavior of concrete and at the same time to theoretical researchs in real structures. Such problems require greater financial resources and personal engagement, which seeks a lot of willpower and persistence if it is to produce acceptable results.

One, of the most competent researchers for these problems, has been Ulickij, who solved a large number of problems for RC structures in his book (see:[4]).

Figure 3 shows his experimental results of the creep functions, but now with the task that the adopted theoretical curves fulfill the required conditions for young concrete. The two polygonal lines in the figure are for two independent series of concrete samples. Each selected concrete quality, represent a complete macro-rheological material, should fulfill its requirements

### 3. RESULTATI ZA PARAMETRE

#### U TEORIJAMA BETONA

##### a) Mladi betoni ( PrimenaTeorije starenja)

Za beton se kaže da je mlad ukoliko je vreme inicijalnog opterećenja u granicama  $3 = < \tau_0 = < 90$  dana. Beton je tada materijal koji pripada oblasti (I) pri kome se realizuje inicijalno opterećenje [v.[ 1]). Beton u izvesnoj meri ima i osobine reverzibilnosti, ali kako je taj deo ispod 5% totalnog koefijenta tečenja, smatra se da ih treba zanemariti, jer se želi dobijanje idealizovanog oblika teorije starenja.

Mali broj autora se bavi eksperimentalnim, dugotrajnim ispitivanjima ponašanja betona i istovremeno teorijskim istraživanjima u realnim konstrukcijama. Takvi problemi zahtevaju veća finansijska sredstva i lično angažovanje, koje traži dosta volje i upornosti ukoliko se želi da se dobiju prihvatljivi rezultati.

Jedan od najkompetentnijih za navedene probleme je bio Ulickij, koji je za AB konstrukcije rešio veliki broj zadataka u svojoj knjizi ([v:[4]).

Slika 3 pokazuje eksperimentalne rezultate za krivu tečenja, ali sada sa zadatkom da usvojena kriva linija ispunii uslove mladog betona. Dve poligonalne linije na slici važe za dve nezavisne serije betona. Svaki odabrani kvalitet betona kao potpuni makroreološki materijal treba da ispuni svoje uslove.

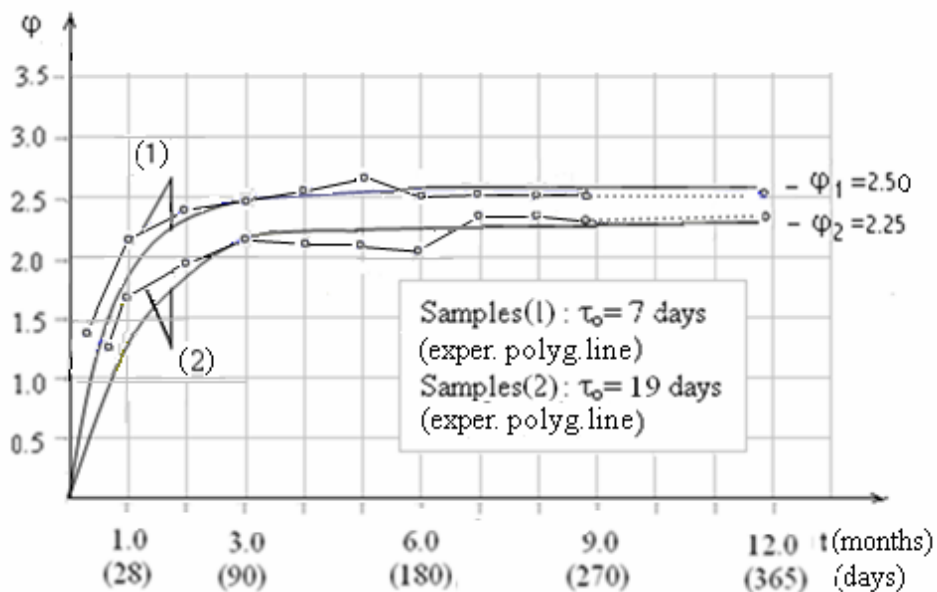


Fig3. Creep functions for two series of concrete samples (Exper.Ulickij[4]) and theoretical (curved lines) - according the author

The theoretical function of concrete creep, for theory of aging, is usually presented in the form of an exponential function:

Teorijska funkcija tečenja betona se, najčešće, prikazuje u obliku eksponencijalne funkcije :

$$\varphi(t) = \varphi_n (1 - e^{-\beta n t}) \quad (4)$$

It is emphasized here, that the form taken in this paper is only for the theory of aging, so the parameters ( $\beta n$  and  $\varphi_n$ ) need to be determined, in advance, if we want to have a correct drawing. Similarly is validity for the curve (2) in Fig. 3. (At the time of the appearance of the book, Ulickij had denoted parameters with  $\gamma$  and  $\varphi^\infty$ ).

Ovde se naglašava da je to oblik koji je uzet u ovom radu samo za teoriju starenja, jer se pojavljuju parametri ( $\beta n$  and  $\varphi_n$ ), koje treba unapred odrediti ukoliko se želi njeno ucrtavanje na slici. Slično opisanom važi i za krivu liniju (2) na sl.3. (U vreme pojave knjige Ulickog konstantu  $\varphi_n$  u izrazu (4) je obeležio sa  $\varphi^\infty$ , a  $\beta n$  sa  $\gamma$ ).



He had used the same term, for the most part, for all concrete theories, but felt that he did not achieve sufficient accuracy (see [4]).

The first condition of the ultimate value of the creep function ( $\varphi_n$ ) is defined by measuring the position of the idealized of the tangent for curve at the point ( $t_n, \varphi_n$ ) (see:  $\varphi_n = \varphi_{tot}$  of area II [1]) based on the estimated mean creep of the idealized function.

The second condition for the 'flow' function  $\varphi(t)$  is that it passes through the point ( $t_1, \varphi(t_1)$ ). The proposal is that the point ordinate is to be chosen, often, in the middle of the two closest points experimental curve.

Solving the expression (4) follow:

$$\beta_n = \frac{1}{t_1} \ln \left[ \frac{\varphi_n}{\varphi_n - \varphi(t_1)} \right] \quad (5a)$$

where

$t_1$  - separation point of area I from area II [1]

$\varphi(t_1)$  - function value of creep for  $t=t_1$

gde je

$t_1$  - vreme razdvajanja oblasti I od oblasti II [1]

$\varphi(t_1)$  - vrednost funkcije za  $t=t_1$

#### Example of usage for curve (1)

Read in :  $\varphi_n = 2.5$  ; For  $t_1 = 90$  [days]  
is  $\varphi(t_1) = 2.43$  ;

Result of (5a) :  $\beta_n = 0.0401$  [ 1/ days ]

#### Example of usage for curve (2)

Read in:  $\varphi_n = 2.25$  ; For  $t_1 = 90$  days  
is  $\varphi(t_1) = 2.10$  ;

Result of (5a):  $\beta_n = 0.0244$  [ 1/ days ]

On je isti izraz koristio za više teorija betona, ali je potvrdio da nije postigao potrebnu tačnost (v.: [4]).

Prvi uslov za krajnje vrednosti funkcije tecenja  $\varphi_n$  je definisan merenjem položaja idealizovane tangente krive u tački ( $t_n, \varphi_n$ ) (videti  $\varphi_n = \varphi_{tot}$  za oblast II [1]) na osnovu procenjenog srednjeg toka idealizovane funkcije.

Drugi uslov za funkciju tečenja  $\varphi(t)$  je da ona prolazi kroz tačku ( $t_1, \varphi(t_1)$ ). Ova tačka se nalazi na mestu razdvajanja između oblasti (I) i oblasti (II) u vremenu  $t_1$ . Ordinata tačke se bira da bude, često, na sredini dve najbliže tačke eksperimentalne krive.

Rešavanjem izraza (4) po  $\beta_n$  sledi :

#### Primer primene za krivu (1)

Očitano :  $\varphi_n = 2.5$  ; Za  $t_1 = 90$  [days]  
je  $\varphi(t_1) = 2.43$

Rezultat (5a):  $\beta_n = 0.0401$  [1/days]

#### Primer primene za krivu (2)

Očitano :  $\varphi_n = 2.25$  ; Za  $t_1 = 90$  [days]  
je  $\varphi(t_1) = 2.10$

Rezultat (5a):  $\beta_n = 0.0244$  [ /days]

Ulickij adopted the same value of coefficient ( $\beta_n$ ) for all the curves and denoted them by  $b=0.04$  [ 1/ days ]. For curve (1) he had obtain the same value, but for curve(2) his proposal was wrong. That is reason becose the deviations between the experimental values and the theoretical are large, which can be seen in his picture.(see: Fig. 1.9 in [ 4]).

Ulicki je usvojio za sve krive isti koeficijent ( $\beta_n$ ) i označio sa  $b=0.04$ . Za krivu (1) to je ista vrednost , ali za seriju uzoraka krive (2) to je pogrešno. Zato su kod njega odstupanja između eksperimentalnih vrednosti i teorijskih velika, što se vidi na njegovoj slici (v.: Sl..1.9 u [ 4]).

Example of usage for Viscosity coefficient  $\eta(t_1)$

Primer primene za Koeficijent viskoznosti  $\eta(t_1)$

$$\eta(t) = \frac{E_0}{\varphi_n \beta_n} e^{\beta_n t} \quad (5b)$$

Data : C25/30 [MPa] ;  $t_1 = 90$  [days] ;  $\varphi_n = 2.5$  ;  $\beta_n = 0.04$ [1/days]  
 Result of expression (5b) :  $\eta(t_1) = 1.009 \cdot 10^{11}$  [ kN · s / cm<sup>2</sup>]

b) Old Concretes (Use of Theory of Heritage)

Concrete is said to be old if the initial load time is within limits  $365 < \tau_0 < t < t_\infty$  days for climatic conditions of Serbia.

Concrete is then a material belonging to area (III) at which the initial load is realized(see: [1]). Concrete also has some characteristics of irreversible plasticity, but since this part is below 5% of the total creep coefficient, it is considered that they should be neglected, because we want to get the form of an idealized theory of heritage, also. Now the expression for the creep function is  $\varphi(t)$  has a similar exponential form as in the theory of aging, but now as a function of the difference between the observation time and the initial loading time ( $t - \tau_0$ ).

b ) Stari betoni (Primena Teorije nasleđa)

Za beton se kaže da je star ukoliko je vreme inicijalnog opterećenja u granicama  $365 < \tau_0 < t < t_\infty$  za klimatske uslove Srbije. Beton je tada materijal koji pripada oblasti (III) pri kome se realizuje inicijalno opterećenje[v.: [1]). Beton u izvesnoj meri ima i osobine nepovratne plastičnosti, ali kako je taj deo ispod 5% ukupnog koefijenta tečenja smatramo da ih treba zanemariti, jer se želi dobijanje oblika idealizovane teorije nasleđa.

Funkcija tečenja  $\varphi(t)$  ima sličan eksponencijalni oblik kao u teoriji starenja, ali sada kao funkcija razlike vremena observacije i vremena inicijalnog opterećivanja ( $t - \tau_0$ ).

The creep function expression is:

Sada je izraz za funkciju tečenja:

$$\varphi(t-\tau_0) = \varphi_\infty (1 - e^{-\beta^\infty (t-\tau_0)}) \quad (6)$$

For old concrete, different values of rheological parameters are valid, than for young concrete. Expressions (4) and (6) should be compared.

To compare the experimental and theoretical values at the Fig. 4 are shown the results obtained during the testing of a series of samples of old concrete for the railway bridge over the Sava River.

Za star beton važe druge vrednosti reoloških parametara nego za mlad beton. Treba uporediti izraze (4) i (6).

Da bi uporedili eksperimentalne i teorijske vrednosti uzeće se sl. 4, koja pokazuje rezultate dobijene tokom ispitivanja serije uzoraka starog betona za železnički most preko Save.

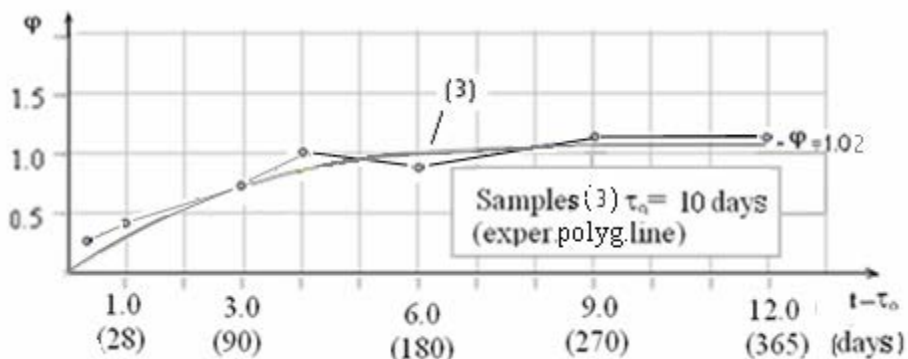


Fig4. Creep functions for one series of concrete samples (Exper.IMS(1)) and theoretical (curved lines) - according the author

Solving expression (6) gives:

Creep function  $\varphi(t-\tau_0)$  should pass through the selected point  $t_1-\tau_0 = 90$  days. Other conditions are the same as for the theory of aging. However, now the values differ significantly.

In this figure is  $\varphi_\infty = 1.02$ , and for theory of aging it was  $\varphi_n = 2.5$ . Many works in this field with error applies the theory of heritage for analysis and young concrete (our opinion) [1].

Sada iz izraza (6) sledi:

Funkcija tečenja  $\varphi(t-\tau_0)$  treba da prođe kroz odabranu tačku  $t_1-\tau_0 = 90$  dana. Ostali uslovi su isti kao za teoriju starenja. Međutim, vrednosti se znatno razlikuju.

Na ovoj slici je  $\varphi_\infty = 1.02$ , a za teoriju starenja bilo je  $\varphi_n = 2.5$ . Mnogi radovi iz ove oblasti sa greškom primenjuju teoriju nasleđa i za analizu mladih betona (samo kao mišljenje).

Example of usage for curve (3)

Read in :  $\varphi_{\infty} := 1.02$  ; For  $t_1 - \tau_0 = 90$   
 is  $\varphi(t_1 - \tau_0) = 0.70$ ;  
 Result of (7) :  $\beta_{\infty} = 0.013$  [ 1/ days].

Primer primene za krivu (3)

Ocitano :  $\varphi_{\infty} = 1.02$  ; Za  $t_1 - \tau_0 = 90$   
 je  $\varphi(t_1 - \tau_0) = 0.70$  ;  
 Rezultat iz (7):  $\beta_{\infty} = 0.013$  [ 1/ days].

c) Design data for concrete creep curves

Design data, based on the measurements described above, of rheological parameters can be recommended for preliminary considerations, for more significant structural designs. The coefficients are quite different for the young or old concrete as described above.

c.1 Young Concrete (Use of the Theory of Aging)

Young concrete, for our Serbia climatic conditions, is considered to be fulfilled if the condition  $\tau_0 \leq 90$  days is valid.

In the given examples it was shown that for young concrete, it is most preferable to choose the abscissa  $t_1$  for the point at which the creep function curve cross. The proposal was  $t_1 = 90$  [days].

The following table will show data for the ratio  $\varphi(t) / \varphi_n$ , for cases in practice, when in time  $t < t_n$  need to calculate the value of  $\varphi(t)$ , based on the results analyzed above for pre-selected values ( $\beta_n$ ).

c ) Projektantski podaci za krive tečenja

Na osnovu gore opisanih obavljenih merenja mogu se preporučiti reološki parametri za prethodna razmatranja kod izrade projekata značajnijih konstrukcija. Koeficijenti se dosta razlikuju za mlade ili stare betone što je već opisano .

c.1 Mladi betoni (Prema teoriji starenja)  
 Mladim betonom se smatra, za naše klimatske uslove Srbije, ako je ispunjen uslov  $\tau_0 \leq 90$  dana .

U datim primerima je pokazano da je za mlade betone najvažnije da se izabere apcisa  $t_1$  tacke kroz koju kriva funkcije tečenja prolazi. Predlog je bio  $t_1 = 90$  [dana].

U sledećoj tabeli biće prikazani odnosi  $\varphi(t) / \varphi_n$ , za slučajeve u praksi, kada u vremenu  $t < t_n$  treba da sračunamo vrednost  $\varphi(t)$ , bazirano na analizi odabranog parametra ( $\beta_n$ ).

Table 3 . Calculation data to find ratio  $\varphi(t) / \varphi_n$

$\beta_n$	Time of Load duration ( t ) [days]				
	3	7	28	90	365
0.02	0.060	0.131	0.429	0.850	0.999
0.03	0.086	0.190	0.568	0.939	0.999
0.04	0.114	0.242	0.674	0.973	0.999

See expression (6) to find  $\varphi(t)$ . Select the value  $\beta_n$  [1/days] and (t) from Table.

c.2 Old Concretes (Use of the theory of heritage)

Old concrete, for our climatic conditions of Serbia, is considered to be fulfilled the condition  $\tau_0 > 365$  [days]. In a given example it is shown, that it is most important, for old concrete to choose

c.2 Stari betoni (Prema teoriji nasleđa)

Starim betonom se smatra, za naše klimatske uslove Srbije, ako je ispunjen uslov  $\tau_0 > 365$  dana .U datom pimeru je pokazano, da je za stare betone najvažije, da se dobro izabere apcisa

well, the point abscissa through which the creep function curve passes. The proposal was  $t_1 - \tau_0 = 90$  [days]. The following table will show the ratios  $\varphi(t - \tau_0) / \varphi_\infty$ , for cases in practice, when in time  $t < t_\infty$  it needs to calculate the value of  $\varphi(t - \tau_0)$ .

tačke kroz koju kriva funkcije tečenja prolazi. Predlog je bio  $t_1 - \tau_0 = 90$  [days]. U sledećoj tabeli biće prikazani odnosi  $\varphi(t) / \varphi_\infty$ , za slučajeve u praksi, kada u vremenu  $t < t_\infty$  treba da sračunamo vrednost  $\varphi(t)$ .

Table 4 . Calculation data to find ratio  $\varphi(t - \tau_0) / \varphi_\infty$

$\beta_\infty$	Time of Load duration (t- $\tau_0$ ) [days]				
	3	7	28	90	365
0.01	0.030	0.068	0.245	0.594	0.975

See expression (4) to find  $\varphi(t)$ . Select the value  $\beta_n$  [1/days] and (t) from Table.

### 3. CONCLUSIONS

Now, the following conclusions can be formed:

1. Testing the creep and shrinkage of concrete is the only sure way to get a better estimation of the stresses and deformations in calculation of RC and PC structures for working load .
2. For young concretes the best results are given by the Dischinger equation(the theory of aging).

The results obtained by unloading the samples are correct, since the reversible deformations are negligible (the errors are below 5%).

3. For old concrete, our conditions for concrete ages over 1 year are valid. The equations according to theory of heritage For concrete of middle age, from 90 days to 365 days, the described solution with is successful applied. The irreversible deformations are negligible for unloading. 4. partial factors can be, also, successful used (see: [1])

### 3. ZAKLJUČCI

Sada se mogu se formirati sledeći zaključci:

2. Za mlade betone, u našim uslovima sredine do 90 dana starosti, najbolje rezultate daje veza prema Dischinger-u (Teorija starenja)

Ispravni su rezultati, koji se dobijaju pri rasterećenju uzoraka, jer je povratna elastična deformacija zanemarljiva (greške su ispod 5%).

3. Za stare betone, za naše uslove sredine i starosti betona preko 1.g. uspešno se primenjuje veza po teoriji nasleđa. Sada su zanemarljive viskoplastične deformacije betona.

4. Za betone srednje starosti, od 90 dana do 365dana, može se uspešno koristiti opisano rešenje sa parcijalnim faktorima izloženo u radu [1].

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Note : The table shows typing errors found in the work of the author under (see:[1])

Page	Row	Old (to Delete)	New (to Print )
2	In Formula (2)	K	K1
2	8	Dish ysinger	Dischinger
2	4	Pseudotime[ ])	Pseudotime[13])
2	In Formula (2a)	$\cdot d\theta\tau$	(to Delete)
2	In Formula (2b)	R	R1
3	In Formula (2c)	$\cdot d\theta\tau$	(to Delete)