SHORT CRACK PROPAGATION OF CONCRETE USING RHEOLOGICAL-DYNAMICAL THEORY

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Summary: *The Paris law equation can be used only for describing the growth of long cracks. The region of application is defined between threshold energy release rate and fracture energy. The short crack growth occurs under the threshold energy release rate value. Many authors are modified the Paris law for its application on this region. The growth of short cracks of concrete is more complicated than for metals and it is not enough researched. In this paper a novel method for describing a short cracks of concrete is proposed using rheological-dynamical analogy (RDA). This method presents the modification of previously determined RDA method for long crack propagation, where stress range in the crack tip is increased using Kitagawa and Takahashi diagram. Cyclic tensile under three-point bending tests on notched beams are considered. The test results for the verification of RDA method are taken from literature.*

Keywords: *fatigue, RDA method, crack propagation, three-point bending tests, short cracks*

1. INTRODUCTION

Concrete structures can be exposed to cyclic loads which leads to damage of material and sometimes breakage. This phenomenon is known as fatigue. The development of damage in the material can be observed through three stages: crack initiation phase (short cracks), stable crack growth (long cracks) and unstable crack growth which leads to failure. Typical curve of propagation is shown in Fig. 1. The problem of fatigue was at first researched on metals and the problem is mathematically described through the Paris law [1] which can accurately describe the phase of stable crack growth. The Paris law relating the crack growth with the stress intensity factor which is a very important parameter of fracture mechanics. Many modifications of the Paris law have been made for its application on metals and cement materials. Ray S. and Kishen J. M. C [2] proposed analytical model for fatigue crack propagation in plain concrete based on principles of dimensional analysis and self-similarity. Bažant Z. P. and Xu K. [3] modified the Paris

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law with consideration the size effect phenomenon. The first range of crack propagation (short cracks) is still an underexplored problem, especially in concrete. Bažant Z. P. and Hubler M. H. [4] analysed propagation of microcracks. Abraham et al. [5] proposed also the analytical model based on the Paris law for decribing a short cracks by concrete. The aim of this paper is to present a novel method for short crack propagation by concrete based on RDA. The application of RDA for the long crack propagation in concrete have already proposed by Pančić A., Milašinović D.D and Goleš D. [6]. They considered the results from the literature of three point bending tests on notched beams for the verification the results. In this paper is presented the modifcation of proposed method to take into account short craks.

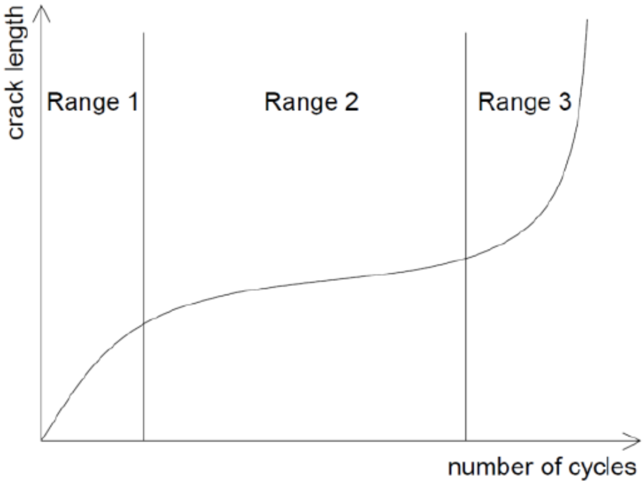

Figure 1. Three ranges of crack propagation [6]

2. LONG CRACK PROPAGATION USING RHEOLOGICAL-DYNAMICAL THEORY

The RDA was developed by Milašinović D. D. [7]. This theory has already been applied on different time-dependent and inelastic problems such as buckling, viscoelastoplastic (VEP) deformation, fatigue of metals etc. The RDA theory take into account mechanical properties of VEP material under cyclic loads. Some of the most important equations and their application on static test of notched concrete beams are shown in [6]. Long crack propagation is analyzed using the equation of the rate of release of energy for viscoelastic (VE) material, which is determined by Milašinović D. D. [8],

$$W_{d,ve}(R) = 2\pi k \Delta\varepsilon^2 \frac{(1-R)^2}{4} \frac{(1+\varphi)^2+\delta^2}{1+\delta^2}\delta \qquad (1)$$

where R represents the load ratio.
The bending stiffness k is given by following equation,

$$k = \frac{\Delta F}{w_e(a)} \tag{2}$$

where ΔF is the force range and $w_e(a)$ is elastic deflection, which is a function of crack length a and it can be found using the finite element method (FEM).
The cyclic strain range $\Delta\varepsilon$ in the crack tip is determined by,

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} \tag{3}$$

where E is Young's modulus and $\Delta\sigma$ is the stress range in crack tip and it is determined using finite element method (FEM).
The creep coefficient can be determined using following Eq. (4),

$$\varphi = K_E \Delta\sigma \tag{4}$$

where coefficient K_E is the structural-material constant at the limit of elasticity, which is defined for the concrete cylinder by Milašinović D. D. [9].
The relative frequency δ represents the ratio between the stress or load frequency ω_σ and frequency of natural vibrations $\omega(a)$.

$$\delta = \frac{\omega_\sigma}{\omega(a)} \tag{5}$$

The load frequency is defined as input parameter, whereas frequency of natural vibrations for notched beams should be determined as a function of crack length using the finite element method (FEM).
The rate of release of energy from Eq. (1) multiplied with number of load cycle gives fracture energy which is assumed as constant value according linear elastic fracture mechanics (LEFM). Hence, the number of load cycles can be defined as a function of crack length with following equation,

$$N(a) = \frac{G_f}{W_{d,ve}(a)} \tag{6}$$

3. SHORT CRACK PROPAGATION USING RHEOLOGICAL-DYNAMICAL THEORY

For the analysis of the short crack propagation in notched concrete beams, the same equations as for long crack propagation but with modification of stress range in the crack tip, are used.
As mentioned, the Paris law describes the propagation of long cracks in the regime between the lower ΔK_{th} and upper K_{max} thresholds of toughness. The lower threshold of

toughness is also called the threshold of cyclic damage and is an important parameter for the short crack propagation.

On different types of metals, some authors experimentally investigated the influence of the load ratio *R* on the toughness threshold [10]. The toughness threshold decreased with an increase in the load ratio. Environmental influence also plays a role, because when testing in a vacuum, the influence of the load ratio can be ignored. Another important parameter, which is crucial for the propagation of the short cracks in metal is the so-called *internal stress*, which depends on the internal structure and irregularities of the metal matrix and which is added to the stress caused by the external load. Determining this stress is very difficult, but it is known that it has an effect on the stress intensity factor, which is of importance for the crack growth. Kitagawa and Takahashi [11] were the first to propose a graphic method that connects short and long cracks with fatigue limit. Vasudevan et al. [10] analyzed critical parameters for the area of short and long cracks in metal using the graphic method from Kitagawa and Takahashi [11] which is shown in Fig. 2.

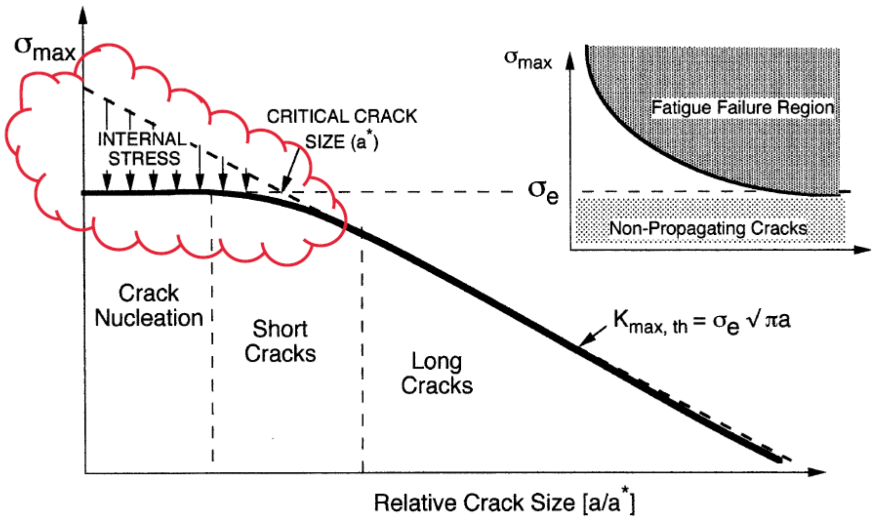

Figure 2. Schematic representation of Kitagawa and Takahashi diagram with marked area for using the RDA theory [10]

According to diagram the stress is shown as a function of crack length. In the nucleation area, the stress shows independent behavior from crack length. In order to adjust the previously described RDA model for application on short cracks a linear function between stress range and short crack length is proposed. This linear function is marked in the previous Fig. 2. Stress σ_{max} is calculated in static test as a maximum value in the crack tip, whereas stress $\sigma_e=\Delta\sigma$ is calculated for stress range in the crack tip by long crack propagation. Both values are determined using the finite element method. The critical value of crack length $a*$ represents the border between short and long cracks and it can be determined from ΔK_{th} but this thresholds of toughness is also difficult to define for concrete. The thresholds of toughness of concrete is not enough researched and therefore, the assumption of critical crack length proved to be better solution by short crack analysis. With this linear interpolation the stress range is increased in the area of short cracks propagation. Other parameters according to RDA remained unchanged, i.e. as with the method for the propagation of long cracks.

4. VERIFICATION

The short cracks propagation using the RDA theory is verified by experimental results from Toumi et al. [12] for notched concrete beams. The fatigue tests were conducted at three different values of the upper boundary of the cyclic load F_u ($0,87F_u$, $0,81F_u$ and $0,76F_u$). The lower boundary load F_{min} was uniformly chosen for all tests, at $0,23F_u$. Maximum and minimum load limits were constant during the fatigue tests. The load ratios were *0,264*, *0,284* and *0,303*. Fracture energy has constant value $G_f=0,0126$ *N/mm* and its determined from Toumi et al. [12].
The used parameters in Eq. (1) are already determined in the paper [6] for the long cracks propagation by the RDA.
The bending stiffness $k(a)$ and relative frequency $\delta(a)$ are functions of crack length and they are numerically determined for different crack lengths using Abaqus software on elastic beam model.
For the static and fatigue stress calculation in the crack tip are used the same finite element mesh as in the Abaqus elastic model. The influence of mesh density on stress calculation results as well as the comparison of static test results with RDA are also shown in [6].
The creep coefficients are calculated using Eq. (4), where $K_E=0,039$ is determined for concrete cylinders by Milašinović D. D. [9]. The creep coefficients are constant during short crack prorpagation and their values are *0,21, 0,19* and *0,17*.
The stress values in the crack tip for short cracks propagation are presented in Table 1, where critical crack length of *10mm* is assumed. These values are used for linear interpolation of stress ranges in the crack tip, Fig. 2.

Table 1: Stress values in the crack tip

a (mm)	Δσ(MPa)			
	ΔP=457,92N	ΔP=501,12N	ΔP=552,96N	
0	8,80	8,80	8,80	Max. stress in the crack tip from static test
10	4,38	4,79	5,29	Stress range for long crack propagation

The resluts of short and long cracks propagation using RDA theory are compared with the experimental results and Paris law resluts from Toumi et al. [12]. The following graphs show the crack length versus number of load cycles.

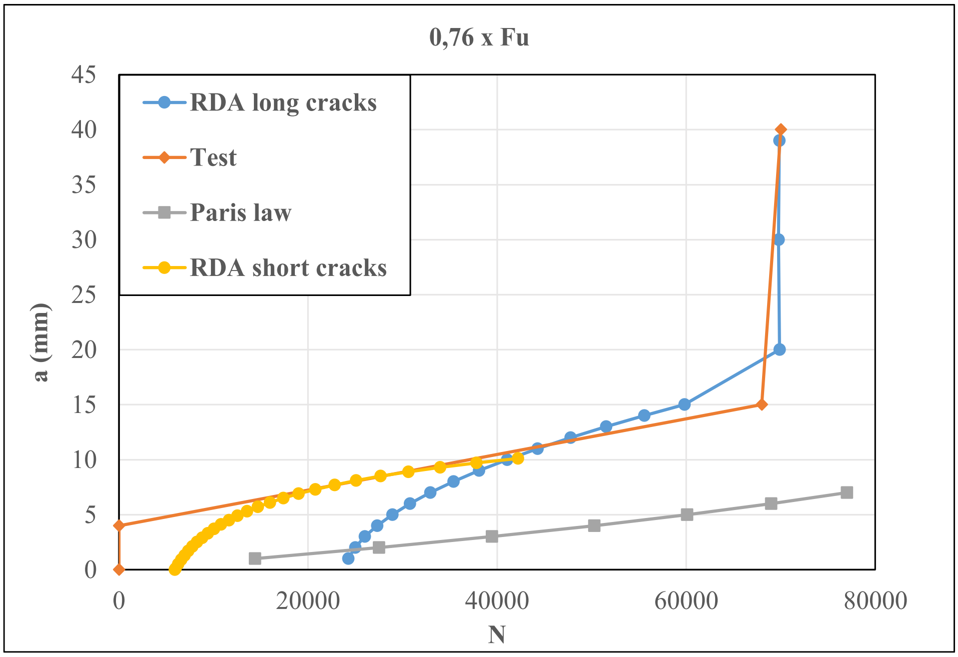

Figure 3. Crack length versus the number of cycles, 0.76Fu

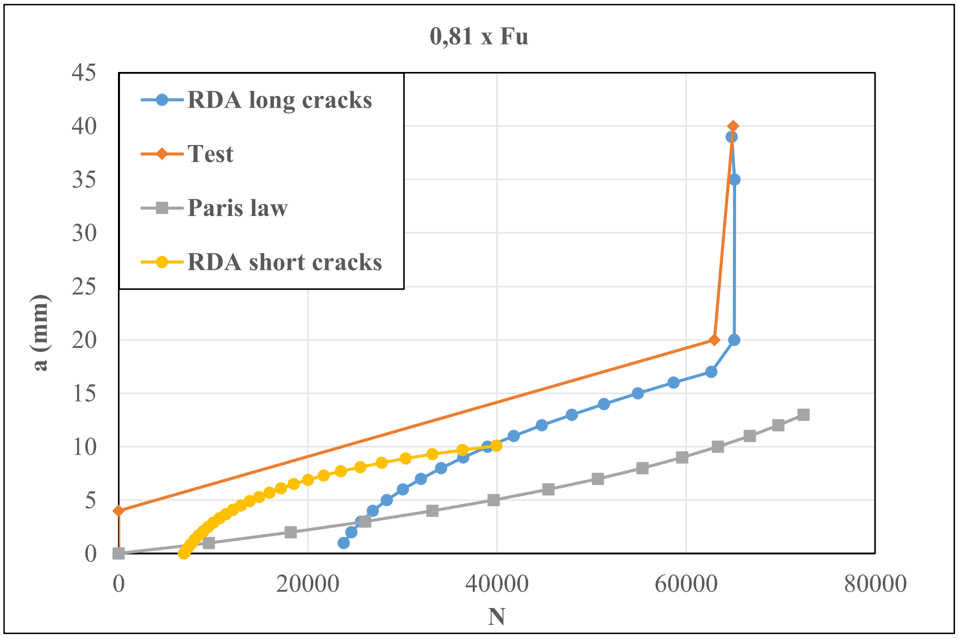

Figure 4. Crack length versus the number of cycles, 0.81Fu

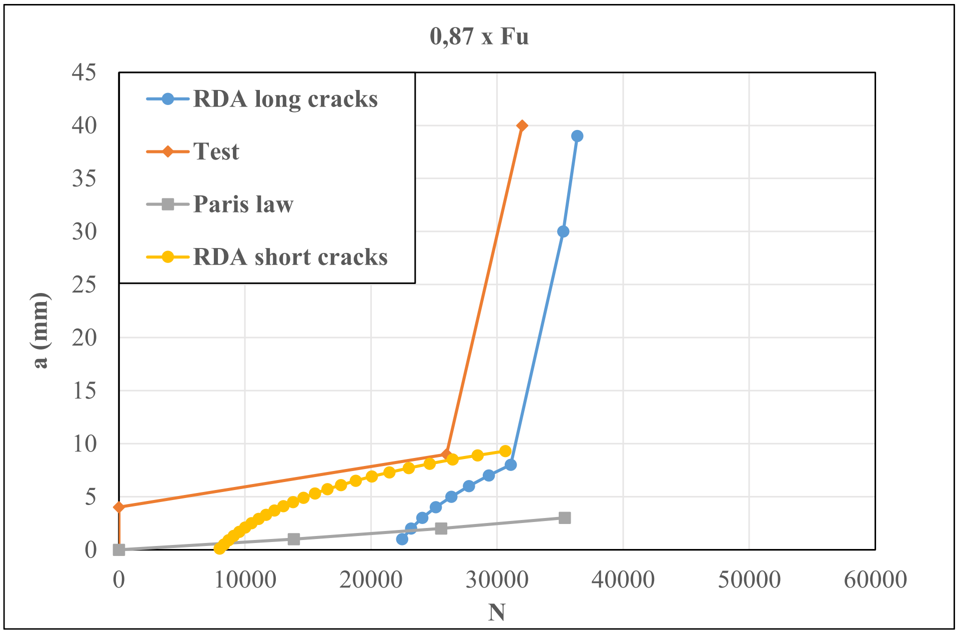

Figure 5. Crack length versus the number of cycles, 0.87Fu

5. CONCLUSIONS

This results showed that the method for long cracks propagation using RDA theory can be modified for the application this theory in the regime of short cracks propagation. Increasing the stress range with linear function is made using Kitagawa and Takahashi diagram. Maximum stress in the crack tip from static test and stress range for long cracks propagation are necessary for this modification. Stress computations are carried out using the finite elemente method with Abaqus software. As with long cracks, the density of finite element mesh on the elastic beam model is very sensitive parameter on results. Thresholds of toughness on critical crack length as a border between ranges is also very important parameter for this analysis. The border is dificult to define in the materials like concrete, and requires more experimental research. The assumption of critical crack length of *10mm* in this example showed good agremment with experimental results. The method presented in this paper is combination of the numerical method with the RDA theory and it can be used for short cracks propagation under condition that input parameters are precisly defined.

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ПРОПАГАЦИЈА КРАТКИХ ПРСЛИНА БЕТОНА ПРИМЈЕНОМ РЕОЛОШКО ДИНАМИЧКЕ ТЕОРИЈЕ

Резиме: *Једначина паризовог закона може се користити за описивање пропагације дугих прслина материјала. Област је дефинисана између доњег прага брзине ослобађања енергије и енергије лома. Прорагација кратких прслина се одвија испод доњег прага брзине ослобађања енергије. Многи аутори су модификовали паризов закон за његову примјену па ову област. Област кратких прслина је доста компликованија код бетона него код метала и није довољно истражена. Главна тема овог рада је описивање нове методе за пропагацију кратких прслина код бетона користећи реолошко-динамичку аналогију (РДА). Ова метода представља модификацију методе за прорпагацију дугих прслина према РДА, при чему се повећава ранг напона у врху прслине помоћу Китагаве и Такахаши дијаграма. Разматрају се циклични тестови савијања греда са иницијалном прслином. Експериментални резултати за верификацију методе РДА metode су преузети из литературе.*

Кључне речи: *Замор, РДА метод, Тестови савијања у три тачке, Кратке прслине*