A PREISACH MODEL FOR THE LEAD-RUBBER BEARING HYSTERESIS LOOP

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Summary: *Lead-rubber bearings are often used as isolation/dissipation devices in the bridge structures, as they provide both, period shift and increased damping in the structure, reducing significantly the seismic forces and displacements induced in the bridge. The hysteresis loop of the lead-rubber bearing, usually approximated with bilinear shape, is an important numerical parameter directly used in the analysis and design of this type of the structure. In this paper, an innovative rigorous and closed-form analytical solution of the lead-rubber bearing hysteresis loop was developed applying the Preisach model. This analytical model allows an accurate computation of all points on the hysteresis loop as well as its area.*

Keywords: *Hysteresis loop, Preisach model, lead-rubber bearing, seismic isolation*

1. INTRODUCTION

The concept of the seismic isolation is a softening of the structure by providing an additional flexible element (such as elastomeric bearing) supporting the large percentage of the total structure mass. The effective stiffness of the structure is then the combined stiffness of the regular structure (for example, of the bridge pier) and of the flexible isolation device. Such reduced stiffness increases the period, which in turns means lower acceleration (therefore reduced seismic forces), but also increases displacements obtained from the response spectra. Further, increased displacements can be decreased by the concept of energy dissipation by providing additional sources of the damping in the structure (Figure 1). For the damping obtained through material internal work, energy dissipation in one cycle is presented by the area of the hysteretic loop. Both of these

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features (isolation, i.e. period shift, and energy dissipation) does not need necessarily to be both in a single device.

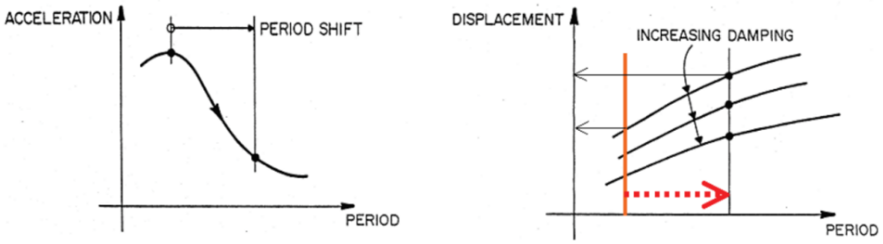


Figure 1. Response spectra showing the effect of the period shift and increased damping[1]

The concepts of isolation and dissipation is often used in the bridge structures throughout the world dominantly for the following two reasons: (1) elastomeric bearings are routinely used in bridges for nonseismic purposes (supporting the superstructure, which is a large percentage of the total mass of the bridge), and (2) bridges are often considered as an "important" structures, designed to remain functional after major earthquakes.

Elastomeric bearings (with steel horizontal shims) can, however, effectively perform only the period shift when used as isolation device (together with its inherent vertical support purpose), but without increased energy dissipation (giving reduced seismic acceleration, but with increased seismic displacement). Their hysteretic loop is narrow, giving around 5% damping, which is the value typically used in the conventional design. If the lead plug, however, is inserted in the middle of elastomeric bearing, then this device, known as lead-rubber bearing (Figure 2), performs successfully (in addition to their inherent vertical support purpose) both functions: period shift and energy dissipation. Since it serves all these functions and it is relatively simple to be manufactured from the regular elastomeric bearing, it is an economical solution for the seismic base isolation.

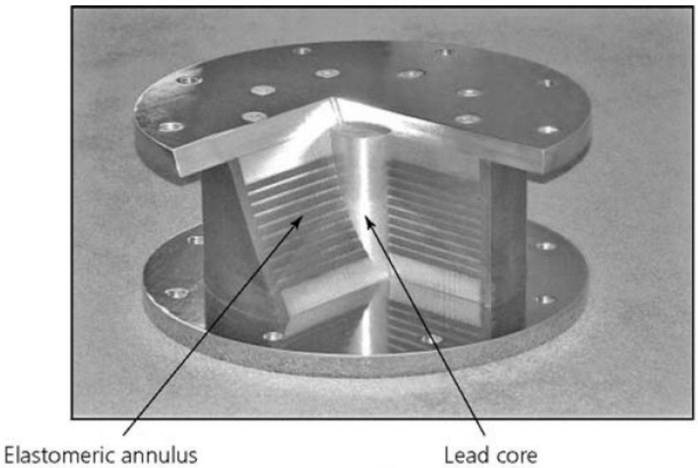


Figure 2. Lead Rubber Bearing [2]

So, the behavior of the bearing is changed with the lead plug insertion, and increased energy dissipation exists, shown by the significantly enlarged area of the hysteretic loop (resulting in the increased damping and smaller seismic displacement). The hysteresis loop of the lead-rubber bearing, usually approximated with bilinear shape[1], [2], is an important numerical parameter used in the analysis and design of this type of the structure. In this paper, the innovative analytical rigorous and closed-form solution of the hysteretic loop of the lead-rubber bearing was developed applying the Preisach model. This allows an accurate computation of all points of the hysteresis loop as well as its area (which is equal to the energy dissipated in one loading cycle). The energy dissipation is directly used in the calculation of the damping ratio of the structure necessary for the determination of the appropriate response spectrum. Further, the residual displacement of the bearing under zero force (a point on the loop) is also a parameter directly used in the design to evaluate the lateral restoring capabilities.

2. BEHAVIOUR OF THE LEAD RUBBER BEARING UNDER CYCLIC LOADING

The reasons why lead is an appropriate material for this purpose are related to its mechanical properties, which allow a good combination with the characteristics of the laminated elastomeric bearing: low yield shear strength (about 10 MPa), sufficiently high initial shear stiffness (shear modulus G approximately equal to 130MPa), behavior essentially elastic-perfectly plastic and good fatigue properties for plastic cycles[3][4]. Considering the characteristics of the rubber it is easy to check that for a lead plug with diameter equal to one-fourth of the diameter of the circular bearing, the initial horizontal stiffness (combined of the elastomer and the lead plug) is increased by about 10 times. After yielding of the lead plug, the stiffness is equal to the stiffness of the rubber bearing alone (due to lead essentially elastic-perfectly plastic behavior).
At room temperature, when lead is plastically deformed, it is being “hot worked” and the mechanical properties of the lead are being continuously restored by the interrelated processes of recovery, recrystallization and grain growth which are occurring simultaneously. In fact, plastically deforming lead at 20C is equivalent to plastically deforming iron or steel at a temperature of greater than 450C. Therefore, lead has good fatigue properties during cycling at plastic strains [3].
Experimental results [3] of a lead-rubber bearing are presented below for later comparison with the herein developed Preisach analytical model of the hysteresis loop. The experimental hysteresis loop is shown on the Figure 3; the dotted line is the behavior of the elastomeric bearing (without lead plug).
The geometry of the lead-rubber bearing whose hysteresis loop is shown on the Figure 3 is as follows: bearing diameter is 650mm, lead plug diameter is 170mm, and the total thickness of the elastomer is 197 mm. Its mechanical properties are estimated as follows: shear modulus of the rubber is G_{rubber}=0.68MPa (back-calculated from numerical values given in [3]); shear modulus of the G_{lead}=100MPa (back-calculated from numerical values given in [3]); and the yield stress of the lead in shear is σ_{lead}=10.5MPa. The loading setup was as follows: vertical reaction is 3.15MN, and horizontal displacement of the bearing is ±91mm.

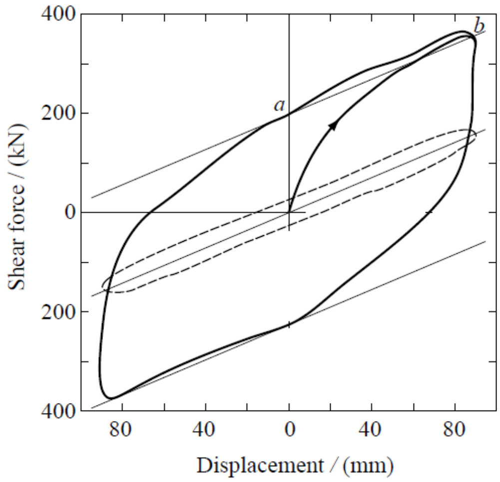


Figure 3. Experimental results of lead-rubber bearing hysteresis loop [3]

Force at point “a” in Figure 3 is known to be dependent on the vertical compressive stress applied to the bearing – it increases with the increase in the vertical stress on the bearing [3]. However, it is considered appropriate to neglect the vertical compressive stress in the hysteresis modeling, as it is expected to be always at the similar level, since larger reaction from the superstructure will usually require larger bearing area, which in turn means relatively constant normal stress. Often used bilinear model also neglects the vertical compressive stress contribution.

3. CURRENT EUROCODE DESIGN APPLICATION OF THE LEAD RUBBER BEARING HYSTERESIS LOOP

The typical approach of constructing an elastic spectrum for damping greater than 5 percent (a value typically used in the conventional design) is to multiply the 5%-damped spectrum by a damping correction factor. The Eurocode response spectrum (EN 1998-1 Article 3.2.2.2 [5]) is multiplied by the damping correction factor, η:

$$\eta = \sqrt{\frac{10}{5+\xi}} \geq 0.55 \tag{1}$$

where ξ is the viscous damping ratio of the structure, expressed as a percentage.
Damping in the structure can be estimated as the following ((EN 1998-2 Article 7.5.4.[5]):

$$\xi_{eff} = \frac{1}{2\pi}\left[\frac{\Sigma E_{d,i}}{K_{eff} d_d^2}\right] \tag{2}$$

where K_{eff} is the effective stiffness of the isolator unit; d_d is the design displacement of the isolating system obtained from the response spectrum and EN 1998-2 Table 7.1[6]; and

$\sum E_d$ is the sum of dissipated energies of all isolators in a full deformation cycle at the design displacement d_d,. As stated in EN 1998-2 Article 7.5.4 Note 2 [6], the damping in the bridge structure depends primarily on the sum of the dissipated energies of the isolators, since the isolators have much larger flexibility and relative displacement than the other components of the structure (relative displacement of the isolator being practically equal to the displacement of the superstructure at this point). Therefore, $\sum E_d$ can be calculated as the area of the lead-rubber bearing hysteresis loop.
Usually the hysteretic loop for cyclic loading of the lead-rubber bearing is approximated by the bilinear shape, shown on the Figure 4. The area of the hysteresis loop can be then estimated as the following:

$$E_d = 4F_0(d_d - d_y) \tag{3}$$

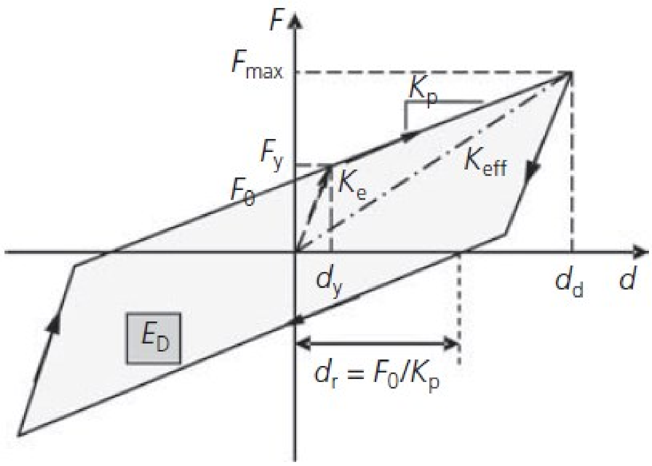


Figure 4. Bilinear model of the hysteretic loop[2]

According to EN 1998-2[6], the parameters of the bilinear approximation of the lead rubber bearing are defined in the following manner:

- d_d is the design displacement;
- d_y is the yield displacement;
- F_y is the yield force under monotonic loading:
- F_0 is the force at zero displacement under cyclic loading, which can be estimated from the product of the lead yield stress in shear (around 10MPa), and the lead plug area[1];
- F_{max} is the maximum force, corresponding to the design displacement d_d;
- K_e is the elastic stiffness at monotonic loading, equal also to the unloading stiffness in cyclic loading; for the lead rubber bearing, EN 1998-2 suggest that the lead plug stiffness may be used for K_e, since usually stiffness of the rubber is much smaller than the stiffness of the lead plug;
- K_p is the post-elastic stiffness; for the lead rubber bearing, EN 1998-2 suggest that the rubber stiffness may be used for K_p.

Further, EN 1998-2 Article 7.7.1 [6] states that the properties of the bilinear system should be determined with maintaining the force value at zero displacement F_0 and estimated value of the design displacement d_d. The straight lines for the loading and unloading branches are defined as to approximate the corresponding branches of the actual experimental loop on an equal area basis. This might lead, however, to the values of elastic stiffness and post-elastic stiffness different than the lead stiffness and rubber stiffness, respectively. It is also worth noting that determined damping depends on the given design displacement obtained by analysis.
Another parameter directly used in the design of this type of the structure is the maximum residual displacement for which the isolating system can be in static equilibrium, denoted as d_r shown in Figure 4. EN 1998-2 recommends limiting the ratio of the design displacement to maximum residual displacement to 0.5. EN 1998-2 recommends the following expression to calculate the maximum residual displacement, d_r:

$$d_r = \frac{F_0}{K_p} \tag{4}$$

It should be noted that the design displacement d_d includes the effect of the given design earthqake excitation for specific case, whereas d_r is solely a parameter of the isolation system, independent from the design earthquake excitation [7].
Therefore, the points on the lead-rubber bearing hysteretic loop (such as residual displacement under zero force) as well as its area are the important numerical parameter directly used in the analysis and design of this type of the structure. The Preisach model is used in this paper for the development of a new, rigorous and closed-form mathematical solution of the hysteresis loop, suitable for programming and automatic computation.

4. PREISACH MODEL OVERVIEW IN GENERAL

Analytical modeling of the hysteresis (the dependence of the state of a system on its history) is of great importance for engineers and physicists in many fields. Preisach model is one of the hysteresis operators than can be used for the mathematical description of the of hysteretic behavior, obtained in the experiments. It may be applied to numerous disciplines of engineering and science such as, for example, porous media, ferromagnetic effects, and continuum mechanic. Application of the Preisach model to cyclic behavior of axialy loaded elasto-plastic material was developed originally by Lubarda et al [8]. Model is extended also to cyclic bending in [12] and [13].
As shown in [9], the Preisach model implies the mapping of an input (shear strain γ (t) in the case of lead-rubber bearing) to the output (shear stress τ (t) in the case of lead-rubber bearing) in the integral form:

$$\tau_t = \iint P(\alpha,\beta) G_{\alpha,\beta} \tau_t d\alpha d\beta \tag{5}$$

where $G_{\alpha,\beta}$ is an elementary hysteresis operator (Fig. 5). Parameters α and β are up and down switching values of the input (Fig. 5), while P (α, β) is the Preisach function. The domain of integration of integral (5) is right triangle in the α, β plane, with α=β being the hypotenuse and (α_0,β_0= - α_0) being the triangular vertex (Fig. 6). Further, the history of loading is represented by staircase line L (t) which divides triangle into positive and negative area. Maxima or minima of loading history are showed by the vertices with coordinates (α,β) on staircase line L(t) such that if the input is increased from the previous time instant, the final position of L(t) graph is horizontal, and vise versa, if the input is decreased, final position of L(t) graph is vertical. It is clear that for unloaded material graph L(t) should be straigt line [10]. Consequently, the triangle is divided in two parts, the positive and negative values of $G_{\alpha,\beta}$, by the staircase line L(t). From the expression (5), it is obtained:

$$\tau_t = \iint_{A+} P(\alpha,\beta)\, G_{\alpha,\beta}\tau_t d\alpha d\beta - \iint_{A-} P(\alpha,\beta)\, G_{\alpha,\beta}\tau_t d\alpha d\beta \quad (6)$$

Connection between Preisach function and output value can be computed by denoting the output value at ε = β by $f_{\alpha,\beta}$ from the limiting triangle, resulting in the following equation:

$$f_{\alpha,\beta} - f_{\alpha} = -2\int_{\beta}^{\alpha}\left(\int_{\beta'}^{\alpha} P(\alpha',\beta')\, d\alpha'\right) d\beta' \quad (7)$$

By differentiating expression (7) twice, with respect to α and β, the Preisach weight function is derived in the following form:

$$P(\alpha,\beta) = \frac{1}{2}\frac{\partial^2 f_{\alpha,\beta}}{\partial\alpha\partial\beta} \quad (8)$$

In general, the Preisach model possesses two important properties: wiping out and congruency properties. Further information about the Preisach model for the cyclic loading of the ductile materials can be found in [8] and [11].

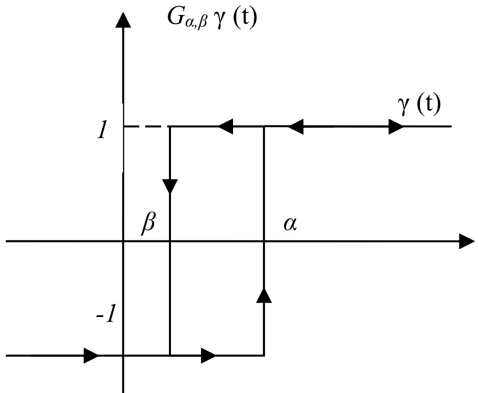


Figure 5. $G_{\alpha,\beta}$ hysteresis operator

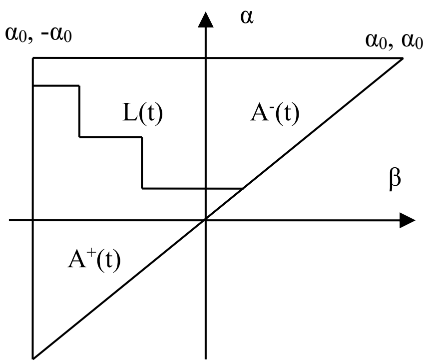


Figure 6. Preisach triangle

5. APPLICATION OF THE PREISACH MODEL ON THE LEAD-RUBBER BEARING HYSTERETIC LOOP

Strictly speaking, the lead-rubber bearing is in a 3D stress field due to vertical reaction from the superstructure and horizontal seizmic force. As explained above, it seems reasonable to neglect the contribution of the vertical reaction, and the pure shear stress state will be assumed herein – one-dimensional hysteretic behavior of elasto-plastic material. Further, the composite material consisting of lead and rubber is considered in this hysteretic analytical modeling as an isotropic and homogenous material, with appropriate material properties in elastic and post-elastic zone. For the lead-rubber bearing applied as isolation/dissipation device, the appropriate Preisach model is the one with the parallel connections of elastic (spring) and plastic (slip) element developed in [8]. A three-element unit will be considered (shown in Fig. 7) representing elastic-linearly-hardening material behavior, shown in Fig. 8. The G and G_h are elastic and hardening moduli; the lead plug properties were taken for elastic part and the rubber properties were used for the hardening part of the behaviour. In this model, the elastic element with length l and modulus G_0 is connected in a series with a parallel connection of elastic and slip element, with length L, modulus h_0 and yield strength Y. Further, the elastic and hardening moduli are computed as $G = G_0\left(l + L\right)/l$ and $G_h = Gh\left(G + h\right)$, where $h = h_0\left(l + L\right)/L$.

The Preisach function can be determined from the hysteresis nonlinearity shown in Fig. 9, which relates the strain input to stress output. The Preisach function in this case has support along the lines $\alpha - \beta = 0$ and $\alpha - \beta = 2Y/G$, i.e. it is given by the following expression:

$$P(\alpha,\beta) = \frac{G}{2}\left[\delta(\alpha - \beta) + (G - G_h)\delta(\alpha - \beta - 2Y/G)\right]. \tag{9}$$

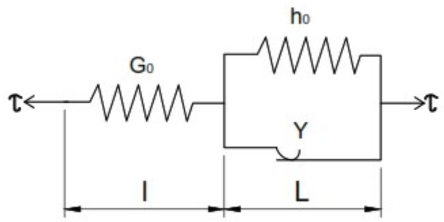
Consequently, the expression for stress as a function of applied strain is the following:

$$\tau_t = \frac{G}{2}\left[\int_{-\gamma_0}^{\gamma_0} G_{\alpha,\alpha}\gamma_t d\alpha - \int_{\frac{2Y}{G}-\gamma 0}^{\gamma_0} G_{\alpha,\alpha-2Y/G}\gamma_t d\alpha\right] \tag{10}$$

The first and second term in equation (10) are elastic and plastic stress, respectively. For a system consisting of infinitely many three-element units, connected in parallel and with uniform yield strength distribution within the range $Y_{min} \leq Y \leq Y_{max}$, the total stress is computed as follows:

$$\tau_t = \frac{G}{2}\left[\int_{-\gamma_0}^{\gamma_0} G_{\alpha,\alpha}\gamma_t d\alpha - \frac{G-G_h}{2}\frac{1}{Y_{max}-Y_{min}}\iint_A G_{\alpha,\beta}\gamma_t d\alpha d\beta\right] \tag{11}$$

In equation (11), the integration domain A is the area between the lines $\alpha - \beta = 2Y_{min}/G$ and $\alpha - \beta = 2Y_{max}/G$ in the limiting triangle.



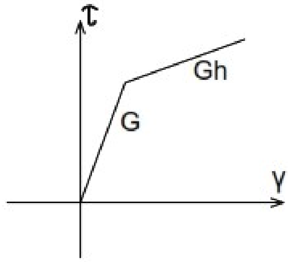


Figure 7. Three-element unit *Figure 8. Elastic-linearly hardening behaviour*

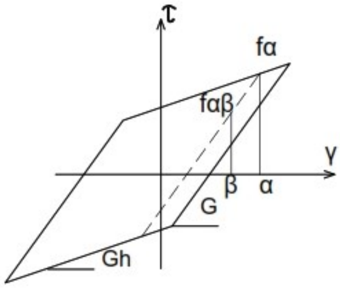


Figure 9. Major hysteresis loop

6. COMPARISON OF THE PREISACH ANALYTICAL MODEL WITH EXPERIMENTAL RESULTS AND BILINEAR APPROXIMATION

By solving the equation (11), the Preisach closed-form solution of the hysteresis loop was obtained. It is shown in Figure 10 and compared with the experimental results[3]. In order to compare the results, the Preisach model was transformed from the stress-strain setup to force-displacement setup. Consequently, the stiffness ($K = \frac{shear\ modulus * area}{thickness}$) of the lead plug and the rubber is used instead of the shear modulus of these materials. The actual stiffnesses from the experiment [3] were the following values: $K_{rubber} = 11.5\ \frac{kN}{mm}$ $K_{lead} = 1.15\ \frac{kN}{mm}$.

Further, it can be concluded from the experiment [3], that the plastic behaviour of the lead plug starts with the relatively small values of the loading. Therefore, the lower plastic limit in the Preisach model is assumed $\mathrm{Y_{min}} = 0$, while the upper plastic limit is $\mathrm{Y_{max}} = 360\ \mathrm{kN}$.

In order to adequately describe the hysteresis loop of the lead-rubber bearing with the Preisach model, the actual bearing properties needs to be first multiplied with the herein-developed constants and then the Preisach model can be applied. These constants were obtained as the ratio of the actual bearing stiffness and the stiffness used in the calibration of the analytical model:

- constant for the stiffness of the lead plug: $\frac{11.5}{8.5} = 1.35$

- constant for the stiffness of the elastomer: $\frac{1.15}{4.30} = 0.27$

On the other hand, the bilinear approximation (also shown on Figure 10) was obtained with the usual assumptions: the elastic stiffness is equal to the lead plug stiffness, the post-elastic stiffness is equal to elastomer stiffness, and the force at zero displacement was computed as the product of the lead yield stress in shear and the lead plug area.

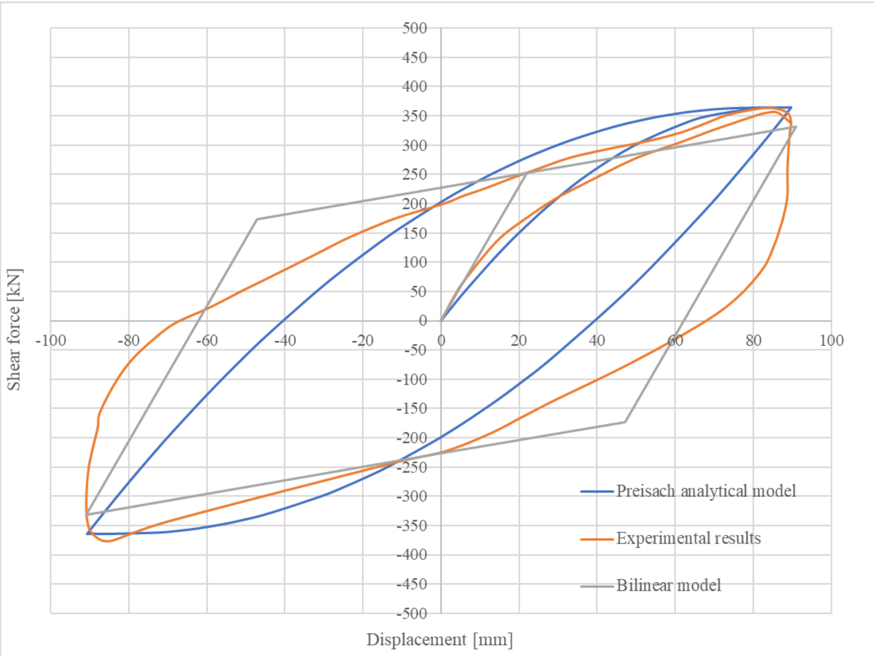


Figure 10. Comparison of the Preisach analytical model with bilinear model and experimental results

The obtained Preisach model gave smaller loop area (for approximately 25%) then the experimental loop, and better mimicked the shape of the experimental loop than the bilinear model (especially in the initial branch of the loop). Therefore, a rigorous, closed-form and conservative solution of the lead-rubber bearing hysteresis loop is developed.

7. CONCLUSION

The herein developed Preisach model presents an innovative, rigorous and closed-form mathematical solution of the lead-rubber bearing hysteresis loop for cycling loading. It was calibrated and compared with the experimental results [3] to verify its adequacy. It can be seen that the Preisach model follows the shape of the hysteresis loop much better than the bilinear approximation (which is often used in the practice). Further, the Preisach model gives smaller area of the loop than the experimental results, which means that global analysis of the bridge will be conservatively performed (due to smaller energy dissipation and damping computed). It is concluded that the Preisach model of the lead-rubber bearing hysteresis loop can be successfully and accurately used for the analysis and design of the bridge with the lead-rubber bearing used as an isolation/dissipation device.

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ПРАЈЗАХОВ МОДЕЛ ХИСТЕРЕЗИСНЕ ПЕТЉЕ ЕЛАСТОМЕРНОГ ЛЕЖИШТА СА ОЛОВНОМ ШИПКОМ

Резиме: *Еластомерна лежишта са оловним језгром се често користе као сеизмички изолатори и дисипатори енергије код мостова, пошто обезбеђују и повећање периоде осциловања и повећање пригушења конструкције, значајно смањујући сеизмичке силе и сеизмичка померања настале у конструкцији. Хистерезисна петља еластомерног лежишта са оловним језгром, која се обично апроксимира са билинеарним моделом, је важан нумерички параметар који се директно користи у анализи и димензионисању овог типа конструкције. У овом раду, иновативан и математички ригорозан аналитички модел хистерезисне петље еластомерног лежишта са оловним језгром је развијен помоћу Прајзаховог модела. Овај аналитички модел дозвољава тачан прорачун свих тачака на хистерезисној петљи, као и њену површину.*

Кључне речи: *Хистерезисна петља, Прајзахов модел, еластомерна лежишта са оловним језгром, сеизмичка изолација*