OVERVIEW OF THE METHODS FOR CALCULATING ELASTIC CRITICAL LTB MOMENT

Petar Subotić [1]
Duško Lučić [2]
UDK: 539.385:519.6
DOI: 10.14415/konferencijaGFS2019.034
Summary: *For designing unrestrained steel beams in accordance with EN 1993-1-1 it is necessary to know the value of elastic critical lateral torsional buckling moment . The aim of this paper is to present different methods for calculating elastic critical lateral torsional buckling moment and analysed them. Methods proposed by Trahair, Clark and Hill and Balaz and Kolekova are discussed and examined through numerical example. The obtained results have shown no significant differences between analysed methods in the case of simple beam under uniform distributed load and concentrated point load at mid span. Further more detailed investigation is needed for more complex cases.*

Keywords: *Elastic critical LTB moment, calculation methods, numerical example*

1. INTRODUCTION

When designing long steel beams which are not continuously lateral restrained the possibility of lateral torsional buckling (LTB) should be checked.
In Eurocode 3 (EN 1993-1-1) [6] section 6.3.2.1 rules are given on how this verification should be carried out. In clause 6.3.2.1(1) it is stated that the verification against lateral torsional buckling should be performed using equation 6.54 as follows:

$$\frac{M_{\mathrm{Ed}}}{M_{\mathrm{b,Rd}}} \leq 1,0 \tag{1}$$

where M_{Ed} is the design bending moment
$M_{\mathrm{b,Rd}}$ is the design buckling resistance moment.

Design buckling resistance moment $M_{\mathrm{b,Rd}}$ of laterally unrestrained beam should be calculated using equation 6.55 from [6].

[1] Petar Subotić, Spec.Sci. građ., University of Montenegro, Faculty of Civil Engineering Podgorica, Bulevar Džordža Vašingtona b.b. Podgorica, Montenegro, tel: +382 20 244 905, e – mail: petarsub@ucg.ac.me
[2] Prof. dr Duško Lučić, University of Montenegro, Faculty of Civil Engineering Podgorica, Bulevar Džordža Vašingtona b.b. Podgorica, Montenegro, tel: +382 20 244 905, e – mail: dlucic@ucg.ac.me

$$M_{\mathrm{b,Rd}} = \chi_{\mathrm{LT}} W_{\mathrm{y}} \frac{f_{\mathrm{y}}}{\gamma_{\mathrm{M1}}} \tag{2}$$

χ_{LT} is a reduction factor which allows the effect of lateral torsional buckling. Reduction factor χ_{LT} is given by equation 6.56 in [6].

$$\chi_{\mathrm{LT}} = \frac{1}{\Phi_{\mathrm{LT}} + \sqrt{\Phi_{\mathrm{LT}}^2 - \bar{\lambda}_{\mathrm{LT}}^2}} \tag{3}$$

The value of χ_{LT} depends on the factor Φ_{LT} that is a function of the imperfection factor α_{LT} and the non-dimensional lateral torsional slenderness $\bar{\lambda}_{\mathrm{LT}}$. The values for imperfection factor α_{LT} are given in Table 6.3 in [6] or in the relevant National annex. Non-dimensional lateral torsional slenderness $\bar{\lambda}_{\mathrm{LT}}$ is given by the following equation:

$$\bar{\lambda}_{\mathrm{LT}} = \sqrt{\frac{W_{\mathrm{y}} f_{\mathrm{y}}}{M_{\mathrm{cr}}}} \tag{4}$$

where M_{cr} is the elastic critical lateral torsional buckling moment.

Therefore for the calculation of the non-dimensional lateral torsional slenderness $\bar{\lambda}_{\mathrm{LT}}$ and thereby for the design buckling resistance moment $M_{\mathrm{b,Rd}}$ the value for M_{cr} must be known.
However Eurocode 3 does not give equation for calculating M_{cr} it is only stated that the M_{cr} should be calculated on the basis of gross cross sectional properties and that the actual loading conditions and the lateral restraints should be taken into account. Thus it is up to the designer to decide how to calculate M_{cr}.
The aim of this paper is to present an overview of the existing methods for calculating elastic critical lateral torsional buckling moment and analysed them.

2. SIMPLE BEAM UNDER UNIFORM BENDING

The equation for elastic critical lateral torsional buckling moment for simple beam under uniform bending was first derived by Timoshenko [1].

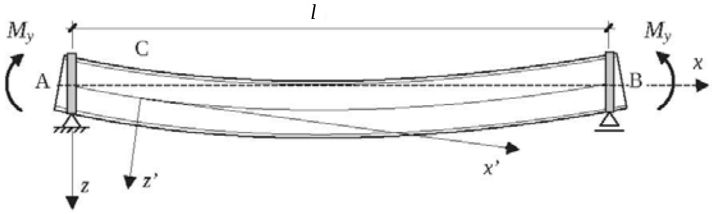


Figure 1. Simple beam under uniform bending

He analysed equilibrium equations of sligtly lateraly torsional buckled beam and calculated which value of bending moment will cause such form of deformation. He derived the folowing equation for elastic critical lateral torsional buckling moment.

$$M_{\mathrm{cr}} = \frac{\pi^2}{l^2} E I_{\mathrm{z}} \sqrt{\frac{I_{\mathrm{w}}}{I_{\mathrm{z}}} + \frac{l^2 G I_{\mathrm{t}}}{\pi^2 E I_{\mathrm{z}}}} \tag{5}$$

where I_z is the second moment of area about z-z axis
I_w is the warping constant
I_t is the St. Venant torsional constant.

This equation applies only to simple beams under uniform bending.

3. BEAMS UNDER VARIOUS LOADING CONDITIONS

The case of simple beam under uniform bending is not very common in practice but deriving exact equations for M_{cr} for different loading conditions is coupled with mathematical difficulties.

Therefor Trahair [2] proposed approximate solutions for different loading conditions in form of the following equation:

$$M_{\mathrm{cr}} = \alpha_{\mathrm{M}} M_{cr}^{simple\ beam} \tag{6}$$

Various loading conditions are taken into account by the factor α_M. The values for the α_M factor are presented in the Table 1.

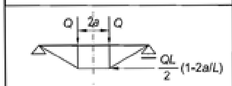
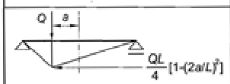
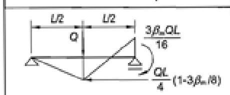
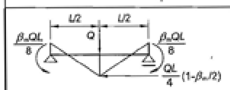
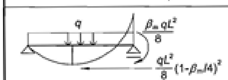
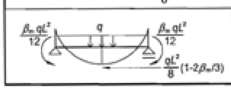
Various loading conditions	Factor α_m	
	$\frac{1{,}75 + 1{,}05\beta_m + 0{,}3\beta_m^2}{2{,}56}$	za $-1 \le \beta_m \le 0{,}6$ za $0{,}6 < \beta_m \le 1$
	$1{,}0 + 0{,}35(1 - 2a/L)^2$	za $0 < 2a/L \le 1$
	$1{,}35 + 0{,}4(2a/L)^2$	za $0 < 2a/L \le 1$
	$1{,}35 + 0{,}15\beta_m$ $-1{,}2 + 3{,}0\beta_m$	za $0 \le \beta_m \le 0{,}9$ za $0{,}9 < \beta_m \le 1$
	$1{,}35 + 0{,}36\beta_m$	za $0 \le \beta_m \le 1$
	$1{,}13 + 0{,}10\beta_m$ $-1{,}25 + 3{,}5\beta_m$	za $0 \le \beta_m \le 0{,}7$ za $0{,}7 < \beta_m \le 1$
	$1{,}13 + 0{,}12\beta_m$ $-2{,}38 + 4{,}8\beta_m$	za $0 \le \beta_m \le 0{,}75$ za $0{,}75 < \beta_m \le 1$

Table 1. Values for factor α_M

The equation (6) does not take into account the point of load application (it is assumed that the loading acts at the shear center) and it is only valid for doubly symmetric cross sections.
The effect of the point of the load application can be taken into account by multiplying equation (6) with the factor γ given by the following equation:

$$\gamma = \sqrt{1 + \left(\frac{0,4\alpha_{\mathrm{M}} y_{\mathrm{q}}}{M_{\mathrm{cr}}^{\mathrm{E}} / N_{\mathrm{cr,z}}} \right)^2} + \frac{0,4\alpha_{\mathrm{M}} y_{\mathrm{q}}}{M_{\mathrm{cr}}^{\mathrm{E}} / N_{\mathrm{cr,z}}} \qquad (7)$$

where y_{q} is the distance from the shear center of the cross section to the point of the load application (negative if the load is acting above the shear center, positive otherwise)

$$N_{\mathrm{cr,z}} = \frac{\pi^2 E I_{\mathrm{z}}}{l^2}$$

General equation for the elastic critical lateral torsional buckling moment of doubly symmetric as well as mono-symmetric cross sections under arbitrary loading conditions was first derived by Clark and Hill [4]. They also analysed fixed end restraints that was not the case with the before mentioned methods. The equation is as follows:

$$M_{\mathrm{cr}} = C_1 \frac{\pi^2 E I_{\mathrm{z}}}{(kl)^2} \left\{ \left[\left(\frac{k}{k_{\mathrm{w}}} \right)^2 \frac{I_{\mathrm{w}}}{I_{\mathrm{z}}} + \frac{(kL)^2 G I_{\mathrm{t}}}{\pi^2 E I_{\mathrm{z}}} + (C_2 z_{\mathrm{g}} - C_3 z_{\mathrm{j}})^2 \right]^{0,5} - (C_2 z_{\mathrm{g}} - C_3 z_{\mathrm{j}}) \right\} \qquad (8)$$

where C_1 is the coefficient that takes into account the shape of the bending moment diagram
C_2 is the coefficient that takes into account the point of the load application
C_3 is coefficient that takes into account the type of the cross section
z_{j} is the factor of the cross section asymmetry
z_{g} is the distance between the shear center and point of load application
k, k_{w} are the effective length factors (k refers to rotation restrictions and k_{w} refers to warping restrictions at end sections).

The values for C_1, C_2, and C_3 are given in Table 2.
The equation proposed by Clark and Hill together with the values for coefficients C_1, C_2, and C_3 was included in the draft version of EN 1993-1-1 but not in the final one.
Possible reasons for excluding the equation (8) from the final version of EN 1993-1-1 could be the works of Balaz and Kolekova [5], [7]. They argued that the use of coefficients C_1, C_2, and C_3 given in the Table 2 may lead in many cases to wrong values of the elastic critical lateral torsional buckling moment.
Their claims have been supported by Fruchtengarten [8] who in his master thesis compared different equations for calculating the elastic critical lateral torsional buckling moment with the results of the finite element program PEFSYS. The proposal of Balaz and Kolekova given in equation (9) provided the closest results to PEFSYS.

Loading conditions	Bending moment diagram	Factor k	Coefficients C1	C2	C3
	ψ = + 1	1,0	1,000	—	1,000
		0,7	1,000		1,113
		0,5	1,000		1,144
	ψ = + 3/4	1,0	1,141	—	0,998
		0,7	1,270		1,565
		0,5	1,305		2,283
	ψ = + 1/2	1,0	1,323	—	0,992
		0,7	1,473		1,556
		0,5	1,514		2,271
	ψ = + 1/4	1,0	1,563	—	0,977
		0,7	1,739		1,531
		0,5	1,788		2,235
M ψM	ψ = 0	1,0	1,879	—	0,939
		0,7	2,092		1,473
		0,5	2,150		2,150
	ψ = − 1/4	1,0	2,281	—	0,855
		0,7	2,538		1,340
		0,5	2,609		1,957
	ψ = − 1/2	1,0	2,704	—	0,676
		0,7	3,009		1,059
		0,5	3,093		1,546
	ψ = − 3/4	1,0	2,927	—	0,366
		0,7	3,009		0,573
		0,5	3,093		0,837
	ψ = − 1	1,0	2,752	—	0,000
		0,7	3,063		0,000
		0,5	3,149		0,000

Loading conditions	Bending moment diagram	Factor k	Coefficients C1	C2	C3
W		1,0	1,132	0,459	0,525
		0,5	0,972	0,304	0,980
W		1,0	1,285	1,562	0,753
		0,5	0,712	0,652	1,070
F		1,0	1,365	0,553	1,730
		0,5	1,010	0,432	3,050
F		1,0	1,565	1,267	2,640
		0,5	0,938	0,715	4,800
F F; a a a a		1,0	1,046	0,430	1,120
		0,5	1,010	0,410	1,890

Table 2. Values for C_1, C_2, and C_3

$$M_{\mathrm{cr}} = \mu_{\mathrm{cr}} \frac{\pi\sqrt{EI_{\mathrm{z}}GI_{\mathrm{t}}}}{l} \quad (9)$$

where $$\mu_{\mathrm{cr}} = \frac{C_1}{k_{\mathrm{z}}}\left[\sqrt{1+k_{\mathrm{wt}}^2+(C_2\xi_{\mathrm{g}}-C_3\xi_{\mathrm{j}})^2}-(C_2\xi_{\mathrm{g}}-C_3\xi_{\mathrm{j}})\right]$$

$$k_{\mathrm{wt}} = \frac{\pi}{k_{\mathrm{w}}l}\sqrt{\frac{EI_{\mathrm{w}}}{GI_{\mathrm{t}}}}$$

$$\xi_g = \frac{\pi z_g}{k_z l}\sqrt{\frac{EI_z}{GI_t}}$$

$$\xi_j = \frac{\pi z_j}{k_z l}\sqrt{\frac{EI_z}{GI_t}}$$

Equations (8) and (9) are identical, the only difference is in the values of coefficients C_1, C_2, and C_3. While calculating C_1 coefficient Balaz and Kolekova have taken also into consideration torsional properties of the cross section. So there are two values of coefficient C_1, $C_{1,0}$ and $C_{1,1}$, that correspond to $k_{wt} = 0$ and $k_{wt} = 1$ respectively.
The values for coefficients $C_{1,0}$, $C_{1,1}$, C_2, and C_3 are given in Table 3. Coefficient C_1 should be calculated using following equation.

$$C_1 = C_{1,0} + (C_{1,1} - C_{1,0})k_{wt} \leq C_{1,1} \text{ but } C_1 = C_{1,0} \text{ for } k_{wt} = 0 \text{ and } C_1 = C_{1,1} \text{ for } k_{wt} \geq 1 \quad (9)$$

Loading and support conditions. Cross-section monosymmetry factor ψ_f	Bending moment diagram. End moment ratio ψ. M-side, ψM-side	k_z 2)	Values of factors C_1 1) $C_{1,0}$	$C_{1,1}$	C_3: $\psi_f = -1$	$-0,9 \leq \psi_f \leq 0$	$0 \leq \psi_f \leq 0,9$	$\psi_f = 1$
M, ψM, L $k_y = 1$, $k_w = 1$ Beam M-side: $\psi_f \geq 0$ $\psi_f \leq 0$	M_{cr} ψ=+1	1,0	1,000	1,000	1,000			
		0,7L	1,016	1,100	1,025		1,000	
		0,7R	1,016	1,100	1,025		1,000	
		0,5	1,000	1,127	1,019			
	M_{cr} ψ=+3/4	1,0	1,139	1,141	1,000			
		0,7L	1,210	1,313	1,050		1,000	
		0,7R	1,109	1,201	1,000			
		0,5	1,139	1,285	1,017			
	M_{cr} ψ=+1/2	1,0	1,312	1,320	1,150	1,000		
		0,7L	1,480	1,616	1,160		1,000	
		0,7R	1,213	1,317	1,000			
		0,5	1,310	1,482	1,150	1,000		
	M_{cr} ψ=+1/4	1,0	1,522	1,551	1,290	1,000		
		0,7L	1,853	2,059	1,600	1,260	1,000	
		0,7R	1,329	1,467	1,000			
		0,5	1,516	1,730	1,350	1,000		
M, ψM, L $k_y = 1$, $k_w = 1$ Beam M-side: $\psi_f \leq 0$ $\psi_f \geq 0$ $\psi_f = \frac{I_{fc} - I_{ft}}{I_{fc} + I_{ft}}$	M_{cr} ψ = 0	1,0	1,770	1,847	1,470	1,000		
		0,7L	2,331	2,683	2,000	1,420	1,000	
		0,7R	1,453	1,592	1,000			
		0,5	1,753	2,027	1,500	1,000		
	M_{cr} ψ = -1/4	1,0	2,047	2,207	1,65	1,000	0,850	
		0,7L	2,827	3,322	2,40	1,550	0,850	-0,30
		0,7R	1,582	1,748	1,38	0,850	0,700	0,20
		0,5	2,004	2,341	1,75	1,000	0,650	-0,25
	M_{cr} ψ = -1/2	1,0	2,331	2,591	1,85	1,000	$1,3 - 1,2\psi_f$	-0,70
		0,7L	3,078	3,399	2,70	1,450	$1 - 1,2\psi_f$	-1,15
		0,7R	1,711	1,897	1,45	0,780	$0,9 - 0,75\psi_f$	-0,53
		0,5	2,230	2,579	2,00	0,950	$0,75 - \psi_f$	-0,85
	M_{cr} ψ = -3/4	1,0	2,547	2,852	2,00	1,000	$0,55 - \psi_f$	-1,45
		0,7L	2,592	2,770	2,00	0,850	$0,23 - 0,9\psi_f$	-1,55
		0,7R	1,829	2,027	1,55	0,700	$0,68 - \psi_f$	-1,07
		0,5	2,352	2,606	2,00	0,850	$0,35 - \psi_f$	-1,45
	M_{cr} ψ = -1	1,0	2,555	2,733	2,00	$-\psi_f$		-2,00
		0,7L	1,921	2,103	1,55	0,380	-0,580	-1,55
		0,7R	1,921	2,103	1,55	0,580	-0,380	-1,55
		0,5	2,223	2,390	1,88	$0,125 - 0,7\psi_f$	$-0,125 - 0,7\psi_f$	-1,88

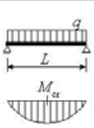
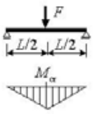
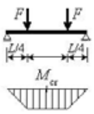
Loading and support conditions	Buckling length factors			Values of factors							
				C_1 [1)]		C_2			C_3		
	k_y	k_z	k_w	$C_{1,0}$	$C_{1,1}$	$\psi_f = -1$	$-0,9 \le \psi_f \le 0,9$	$\psi_f = 1$	$\psi_f = -1$	$-0,9 \le \psi_f \le 0,9$	$\psi_f = 1$
	1	1	1	1,127	1,132	0,33	0,459	0,50	0,93	0,525	0,38
	1	1	0,5	1,128	1,231	0,33	0,391	0,50	0,93	0,806	0,38
	1	0,5	1	0,947	0,997	0,25	0,407	0,40	0,84	0,478	0,44
	1	0,5	0,5	0,947	0,970	0,25	0,310	0,40	0,84	0,674	0,44
	1	1	1	1,348	1,363	0,52	0,553	0,42	1,00	0,411	0,31
	1	1	0,5	1,349	1,452	0,52	0,580	0,42	1,00	0,666	0,31
	1	0,5	1	1,030	1,087	0,40	0,449	0,42	0,80	0,338	0,31
	1	0,5	0,5	1,031	1,067	0,40	0,437	0,42	0,80	0,516	0,31
	1	1	1	1,038	1,040	0,33	0,431	0,39	0,93	0,562	0,39
	1	1	0,5	1,039	1,148	0,33	0,292	0,39	0,93	0,878	0,39
	1	0,5	1	0,922	0,960	0,28	0,404	0,30	0,88	0,539	0,50
	1	0,5	0,5	0,922	0,945	0,28	0,237	0,30	0,88	0,772	0,50

Table 3. Values for C_1, C_2, and C_3 from [5]

The proposal of Balaz and Kolekova was included in National annexes for EN 1993-1-1 of Slovakia, Czech Republic and Austria.
Apart from above mentioned methods for calculating elastic critical lateral torsional buckling moment various computer programs could be used to this end. The most practical and frequently used one is LTBeam developed by CTICM.
In order to make comparison among before mentioned methods a practical numerical example will be examined.

4. NUMERICAL EXAMPLE

Simple beam of length l = 800 cm with IPE 300 cross section is analysed. Elastic critical lateral torsional buckling moment is determined for two most common loading cases in practice, uniform distributed load and concentrated point load at mid span. Also two different load positions are considered, at shear center and at the top of the compression flange.

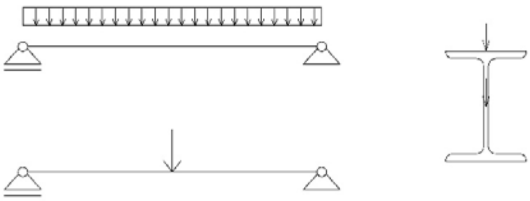


Figure 2. Analysed cases

The values for elastic critical lateral torsional buckling moment (in kNm) using before mentioned methods are presented in Table 4.

Load case	Point of application	Trahair	Clark and Hill	Balaz and Kolekova	LTBeam
Uniform distributed load	Shear center	70.908	71.034	70.879	70.942
	Top of compression flange	57.49	57.411	57.286	57.388
Concentrated point load at mid span	Shear center	84.713	85.655	85.064	85.309
	Top of compression flange	65.995	66.330	65.873	65.448

Table 4. Results of analysis

5. CONCLUSION

In this paper different methods for calculating elastic critical lateral torsional buckling moment are presented. The examined numerical example has shown that there is no significant difference in values of elastic critical lateral torsional buckling moment obtained using methods proposed by Trahair, Clark and Hill and Balaz and Kolekova as well as program LTBeam. The method proposed by Trahir is the least time consuming and therefor it is the easiest to use in comparison to other two methods, but its field of application is limited. Analysed cases are the most common ones in practice whereas for more complex cases (mono-symmetric sections) it could be expected to obtain more significant differences. Further investigation on this subject is of great practical importance.

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PREGLED METODA ZA ODREĐIVANJE ELASTIČNOG KRITIČNOG MOMENTA BTI

Резиме: *Za proračun nepridržanih čeličnih greda shodno EN 1993-1-1 neophodno je poznavati vrijednost elastičnog kritičnog momenta bočno-torzionog izvijanja. Cilj ovog rada je da se prikažu različite metode za određivanje elastičnog kritičnog momenta bočno-torzionog izvijanja. Metode predložene od strane Trahair-a, Clark-a i Hill-a i Balaz-a i Kolekov-e su razmatrane i analizirane kroz numerički primjer. Korišćenjem razmatranih metoda nijesu dobijene značajne razlike u rezultatima u slučajevima grede opterećenje jednako podijeljenim opterećenjem i grede opterećene koncentrisanom silom u sredini raspona.*

Кључне речи: *Elastični kritični moment bočno-torzionog izvijanja, metode proračuna, numerički primjer*