LOCAL BUCKLING OF THE WEB OF STEEL GIRDER IN ELASTIC AND PLASTIC DOMAIN

Dragana Turnić [1]
Srđan Živković [2]
Marija Spasojević Šurdilović[3]
Tomislav Igić [4]

UDK: 624.042.072.2
DOI: 10.14415/konferencijaGFS2019.021

Summary: *Starting from the differential equations of elastic surface it is determined the critical buckling load of the rectangular plate. Especially, we consider the case of the local buckling of web due to the load that acts on the plane of the web. The concept of two basic methods is presented: the method of effective width and reduced stress method. These methods are recommended by European standards. An example of the buckling of welded plate girder subjected to partially distributed load with the increase of force until the ultimate load by numerical simulation is done.*

Keywords: *local stability, buckling of web, steel girder, plasticity, Eurocode 3*

1. INTRODUCTION

The problem of stability is very significant in the case of the welded plate girders. When the critical load is reached, the buckling of plates appears. In the framework of the theory of elastic stability, the solutions for critical load were obtained in analytical form, i.e. the accurate solutions for some characteristic plates, certain types of loads and boundary conditions. In the following, we will consider the use of the solutions in the elastic domain. Our analysis will refer to the appearance of the plastic region of the plate.
In particular, we will analyze the effect of high intensity load acting on a small surface. This case has a practical application. The European standards look at these problems and give their recommendations [3]. The occurrence of instability is manifested by the buckling of the local part of the web of the girder and it is followed by the plasticity, i.e. by the non-linearity and leads to the failure. There is a number of factors that influence this phenomenon. The solution of this problem is very complex, among the other things, due to the economic reasons, which puts on the use of girder with slender web.

[1] PhD Dragana Turnić, assistant professor, University of Niš, Faculty of Civil Engineering and Architecture, Aleksandra Medvedeva 14, Niš, Serbia, tel: +381 18 588 200, e – mail: dragana.turnic@gaf.ni.ac.rs
[2] PhD Srđan Živković, assistant professor, University of Niš, Faculty of Civil Engineering and Architecture, Aleksandra Medvedeva 14, Niš, Serbia.
[3] PhD Marija Spasojević Šurdilović, assistant professor, University of Niš, Faculty of Civil Engineering and Architecture, Aleksandra Medvedeva 14, Niš, Serbia.
[4] PhD Tomislav Igić, full professor, University of Niš, Faculty of Civil Engineering and Architecture, Aleksandra Medvedeva 14, Niš, Serbia.

2. DIFFERENTIAL EQUATION OF ELASTIC SURFACE

It is assumed that the transverse load as well as the forces act on the mid-surface of the plate. To perform the differential equation of the elastic surface of the plate, the equilibrium of a small element obtained by cutting out of a plate with two pairs of planes is used. One pair of planes is parallel to the *xz*-plane, and the other pair of planes is parallel to the *yz*-coordinate plane (Figures 1).

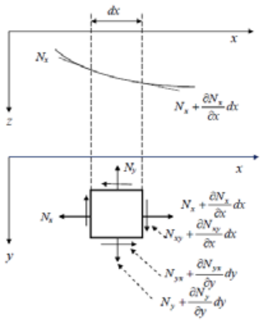

Figure1. Equilibrium of a small element of plate [1]

Beside the transverse load qd_xd_y, projections on the z-axis of the normal force N_x and N_y, as well as transverse forces N_{xy}, appear as the transverse load on the element of the plate, ie.

$$\left(N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \right) dxdy.$$

Thus, from the equilibrium equations, the projection of the force on the z-axis is obtained

$$\frac{\partial^2 M_x}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -\left(q + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \right).$$

Replacing the expressions for the moments (see [1], [2]), we have

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D}\left(q + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \right). \qquad (1)$$

We remark that Saint Venant (in 1883 year) carried out the differential equation of elastic surface including the effects of volume forces. However, before that, the famous French scientist Sophie Germain carried out the equation, and Lagrange finally obtained it (in 1811 year) including only the transverse load q, ie.,

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \qquad (2)$$

or in a short form $\Delta\Delta w = \frac{q}{D}$, were $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is Laplace's differential operator.

3. STABILITY OF PLATE

Considering the problem of stability of plates, at first, we observe a load subjected to the mid-surface of the plate.
Putting q=0 from the equation (1) we have

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D}\left(N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \right). \tag{3}$$

Our goal is to determine the value of a critical load when the buckling of the plate appears. The exact solution of this equation exists only for certain load values, certain plate shapes and given boundary conditions.

3.1 Simply supported plate under uniformly distributed load

We consider the plate in Figure 2, which length is a and the width is b. The plate is subjected to the uniformly distributed load P on the mid-surface (for details, see [1], [7]). The load is N_x=-p, and the other loading is equal to zero, i.e. N_y=N_{xy}=0.

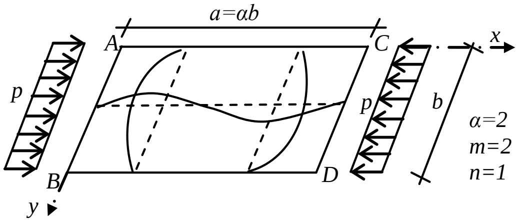

Figure 2. Simply - supported rectangular plate under uniformly distributed load

In this case, the differential equation (3) gives

$$D\Delta\Delta w = -p\frac{\partial^2 w}{\partial x^2}. \tag{4}$$

The boundary conditions of the cosidered plate, which is simply-supported along all edges, are

$$w=0, \quad \Delta w=0.$$

We will find the solution of the equation (4) in the form of a double trigonometric series equation

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} c_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \text{ gde je } c_{mn} = \text{const.}$$

The critical load values are obtained from the equation of stability

$$p_{kr} = \frac{D\pi^2}{b^2}\left(m\frac{b}{a} + \frac{n^2}{m}\frac{a}{b} \right)^2,$$

and the critical stress is

$$\sigma_{kr} = k_{mn,kr}\sigma_E ; \quad k_{mn,kr} = \left(\frac{m}{\alpha} + \alpha \frac{n^2}{m} \right)^2 . \tag{5}$$

The lowest critical stress level will be found for different numbers of half waves (see Figure 2). The lowest critical stress is obtained from the conditions of minimum values of $k_{mn,kr}$ for n=1 (one half wave in the y - direction). The solution is

$$\alpha = m = 2; \qquad k_{m1,kr} = \left(\frac{m}{\alpha} + \alpha \frac{1}{m} \right)^2 = 4,$$

ie., in this case, based on (5), for n=1

$$\sigma_{kr} = k_{mn,kr}\sigma_E = k_\sigma \sigma_E = 4\sigma_E .$$

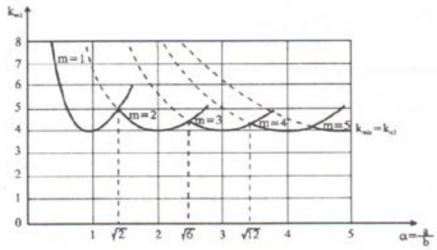

Figure 3. The buckling load coefficient for a simply-supported thin plate

Figure 3 shows relations between k_{m1} and α for values m [7].

4. CALCULATION METHODS

When the observed field is in elastic zone, then according to the theory of elastic stability, the material of the plate is ideally elastic. For that matter, there is a geometric perfection and there is no curvature or residual stresses. The load acts only in mid-surface of the plate and for certain cases we can obtain analytical solutions for buckling. Usually, the boundary conditions are such that the plate is simply supported. Using numerical analysis with advanced computer programs, numerical solutions for different plate geometry, boundary conditions, and stiffness of the plate were developed. The reference slenderness of plate λ is defined in the following way:

$$\lambda = \sqrt{\frac{F_y}{F_{cr}}} \tag{10}$$

where: F_y is characteristic yield resistance, F_{cr} is elastic critical load.
Based on the recommendation for the design according to European standards EN1993-1-5:2006 [3] two basic methods are presented: the effective width concept and reduced stress

method. In particular, it is possible to confirm methodologically solutions based on the finite element method (FEM).

4.1 Effective width concept. Plastic yield and non-linearity.

Many researchs show that the ultimate load of a plate under compression may significantly exceed the critical load level calculated according to the theory of elastic stability of thin plates (unlike thick plates). In linear elastic analysis, the distribution of the load is assumed to remain uniform until the plate buckles. However, when the plate starts to buckle, the stresses are re-distributed in the plate. The plate behaviour under these large deformations, or post critical behaviour, is a complicated area to describe. Some differential equations describing the phenomenon were derived by von Kármán in 1910, but the methods for solving these are complex [8].

1 He introduced a reduction of the plate width to the effective width because, due to the local buckling of the plate, the ability of carring load is reduced. In this way, he approximates two strips, outside the middle of the plate, to take over the load over the effective width b_{eff}. Condition that the plate of the width b_{eff} has a critical stress

$$\sigma_{kr,eff} = f_y, \tag{11}$$

where f_y is yield stress, gives the von Karman's effective width formula

$$b_{eff} = b\sqrt{\frac{\sigma_{kr}}{f_y}}. \tag{12}$$

Previous results were used by Winter [4] to give the proposal in the form of Winter's function

$$\frac{b_{eff}}{b} = \frac{1}{\bar{\lambda}_p}\left(1 - \frac{0.22}{\bar{\lambda}_p}\right); \quad (\bar{\lambda}_p \geq 0.673), \tag{13}$$

where $\bar{\lambda}_p$ is the plate slenderness parameter-local buckling. On the basis of numerous experiments he included inelastic behavior of steel and the influence of material imperfection. This proposal is now used within European norms.

4.2. Reduced stress method

This method introduces a reduced stress σ_{Rd} , which is constant along the width of the plate. The design load F_{Ed} is obtained by the amplification factor α_u. The ultimate resistance F_{Rd} is given in the form:

$$\frac{F_{Ed}}{F_{Rd}} = \frac{F_{Ed}}{\rho\dfrac{\alpha_{ult,k} F_{Ed}}{\gamma_{M1}}} \leq 1, \tag{14}$$

where γ_{M1} is the partial safety factor within this method.

5. LOCALIZED LOAD SUBJECTED TO WELDED I GIRDERS

When loading acts on a short lenght of the flange in the plane of web of welded steel I girder, it can cause many problems and even structural damage or colapse of the girder. Such type of load can be found during assembly of bridges when the bridge is being slide in to designed position. In this case the temporary or permanent supports become a localized loading to which the girder is exposed. Regarding the ultimate load capacity, it depends on the ultimate load capacity just before the colaps of the girder.
This problem is very complex and until now the exact theoretical solution has not been found. Due to the high localized load value acting on a small surface, a high value of the stresses reaches the plasticity in a certain area of the cross-section of the web. However, despite the appearance of plasticity, the buckling of the web begins in the next step. Many parameters affect the complexity of this problem. Thickness of the web, distance between transversal stiffeners, yield stresses of the web, distribution lenght of the load, the existence of longitudinal stiffeners, ets. In particular, the imperfections of the girder both geometrical and material are significant and have been the subject of research during past decade [5].
In the studies carried out in [2], a model was formed on which the numerical simulations were performed and where the ultimate loads for several types of girders were determined. Due to its practical application, several different material models with corresponding stress-strain (σ-ε) curves recommended for design by European standards [6] have been used.
Figure 4 shows a gradual development of the deformations of the web with an increase of force. During the loading of the I girder, from the beginning, the deformations of the web appear. With the further increase in force, the deformation on the upper part of the web continues to increase. The load was increased until ultimate load was reached. The ultimate load manifests as progressive increase of registered deformations of the web without further increase of force. The visible buckling of the web below the zone where load was applied was observed. Figure 5 shows the girder model in the ANSYS program when the ultimate load is reached.
Plastification occurs after reaching 60% of the limit load. Then the deformations do not have to be significant. Plastification develops on the most loaded part of the web, first only on the outside of the web-surface and then spreads along the thickness of the web.

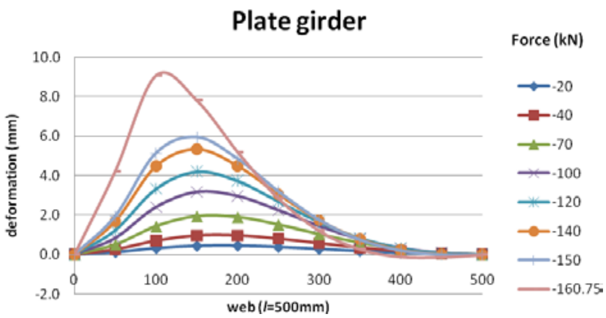

Figure 4. Development of girder deformation with increasing force

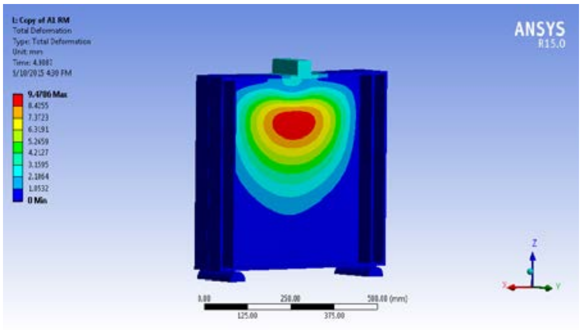

Figure 5. Deformation of the girder

6. CONCLUSION

The problem of stability of a plate was treated starting from the differential equation of the elastic surface. An example of the theoretical solution of the case of a thin rectangular plate was shown. It is determined the critical load value P_{kr}, a critical stress σ_{kr} and the coefficient k_{mn}. However, it was pointed out that, in the case of I girder with real material, the plasticity of certain zones and the yield stresses appear during the process of buckling of web. The stress-strain relations become nonlinear.

The paper consider a case of a localized loading to which the girder was exposed. The ultimate load exceeds the load which manifestes in the occurrence of a local buckling of the web. During the non linear simulation of the tests, it is determined [6] that in the zone where load was applied, the yield stresses in the web were reached without a deformation of the plane web. This problem is very complex and until now, the exact, theoretical solution has not been obtained.

This area is interesting both from the theoretical and practical point of view, especially in some types of bridges where, due to economy, high-slender webs are predicted.

The two basic methods for calculation recommended by EN 1993-1-5: 2006 i.e. effective width concept and reduced stress method are also presented in the paper.

The European standards also recommend a methodological verification of the solution by the finite element method. An example of a numerical model is given, which shows the development of the deformation reaching the ultimate load and the failure of the girder after local buckling. The results of stresses and deformations obtained by numerical simulation agree the test results of the corresponding girder [6].

REFERENCES

[1] Timošenko, S., Vojnovski-Kriger, S.: *Teorija ploča i ljuski*, Građevinska knjiga, Beograd, 1962.

[2] Turnić, D.: *Nelinearno ponašanje i granična nosivost limenih nosača opterećenih lokalizovanim opterećenjem*, doktorska disertacija GAF Niš, 2016.

[3] EN 1993-1-5: 2006 Eurocode 3: Design of steel structures, Part 1-3: Plated structural elements, November 2006, ISBN 0 580 49667 8.

[4] Winter, G.: Strength of Thin Steel Compression Flanges, *Transactions, American Society of Civil Engineers*, 1947, vol. 112, p.p. 527-554.

[5] Graciano, C., Casanova, E., Martínez, J.: Imperfection sensitivity of plate girder webs subjected to patch loading, *Journal of Constructional Steel Research*, 2011, vol. 67, p.p. 1128–1133.

[6] Turnić, D., Marković, N., Igić, T., Nikolić, V., Spasojević Šurdilović, M.: Analysis of material non-linearity of steel girders subjected to patch loading, *Tehnički vjesnik*, 2017, vol.24. No.6, p.p. 1991-1998.

[7] Zdravković, S.: *Teorija površinskih nosača-Ploče i ljuske,* AGM Knjiga, 2015.

[8] Clarin, M.: *Plate Buckling Resistance - Patch Loading of Longitudinally Stiffened Webs and Local Buckling*, PhD thesis, No. 2007:31, Luleå University of Technology, Department of Civil, Mining and Environmental Engineering, Division of Structural Engineering - Steel Structures, 2007.

LOKALNA STABILNOST REBRA ČELIČNOG NOSAČA U ELASTIČNOJ I PLASTIČNOJ OBLASTI

Summary: *Polazeći od diferencijalne jednačine elastične površine određeno je kritično opterećenje izvijanja pravougaone ploče. Posebno je tertian slučaj lokalnog izbočavanja rebra usled opterećenja koje dejstvuje u ravni rebra. Prikazan je koncept dve osnovne metode: metod efektivne širne i metod redukovanog napona. Ove metode su preporučene u evropskim propisima. Dat je primer izbočavanja zavarenog pločastog nosača pod uticajem lokalizovanog opterećenja sa porastom sile do dostizanja graničnog opterećenja uz pomoć numeričke simulacije.*

Keywords: *lokalna stabilnost, izbočavanje rebra, čelični nosač, plastičnost, Eurocode 3*