ROTATIONAL STIFFNESS OF SEMI-RIGID JOINTS

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Summary: *In this paper the methodology for determination of rotational stiffness of semi-rigid connections in steel constructions according to EC 3 componential method has been analyzed. By application of this concept the determination of rotational connection response comes down to determination of geometrical characteristics of different connection components, thus this method can be applied to a larger number of different types and configurations of connections at steel constructions. The paper also shows the comparison of this method with research results.*

Keywords: *steel connections, semi-rigid connections, rotational stiffness, design of joints.*

1. INTRODUCTION

The procedure for determination of dependence among bending moments (M_j) and relative rotational angle at section of connection spot (Φ_j) as one of the main characteristics of beam-to-column connections is given within Eurocode 1993-1-8 [1], [2]. Determination of calculational *M-Φ* characteristics of beam-to-column connection is based on theory which has been confirmed by experimental results.
In general, moment-rotation characteristics of connection are presented as a non-linear function. For calculational characteristic of moment-rotation relation can be adopted:

- Approximated curve consists of more than one linear segment (linear, bilinear, trilinear, etc.) or
- non-linear curve.

Approximated curve in form of polygonal line should entirely lie under *M-Φ* characteristic which has been determined experimentally. It doesn't entirely correspond to real connection characteristics but, in most cases, provides good enough data for practical application.

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2. GENERAL MODEL OF MOMENT-ROTATION CONNECTIONS

In order to determine characteristics of moment-rotation connection the combined line [1], which consists of three segments (Figure 1) can be adopted. The first segment of the curve is a linear function up to 2/3 of calculational moment connection resistance ($M_{j,Rd}$) while the tension field and connection deformation belong to the elastic area. The corresponding stiffness is so called initial stiffness-$S_{j,ini}$. The second curve segment is non-linear and placed among values of $\frac{2}{3}M_{j,Rd}$ and $M_{j,Rd}$. After achieving the moment $M_{j,Rd}$ the curve is parallel to the Abscissa and that is the third segment of *M-Φ* chart. The end of *M-Φ* chart is determined by rotational capacity of connection (Φ_{Cd}).

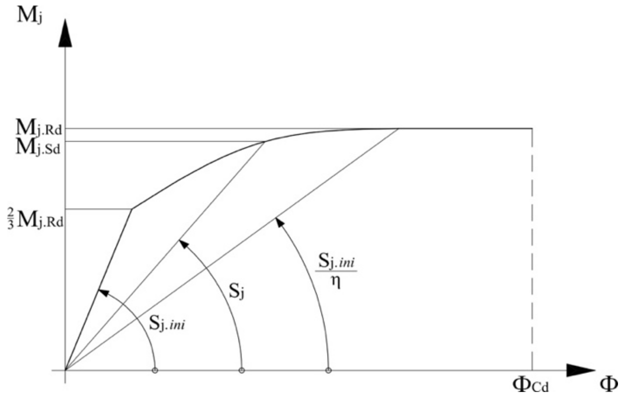

Figure 1. Non-linear M-Φ curve

This model adopts the constant relation (η) among initial stiffness $S_{j,ini}$ and secant rigidity which corresponds to the ending part of non-linear curve segment $\left(S_j = \frac{S_{j,ini}}{\eta}\right)$, and which values vary $\eta = 3{,}0 - 3{,}5$ depending on connection type. These values are obtained as a result of parametric studies and analysis of many experimental researches of connections [3]. Secant rigidity which is defined at non-linear part of the moment-rotation curve, for the value of calculational bending moment at the connection spot ($M_{j,Ed} = M_{j,Sd}$), when the moment belongs within interval of $\frac{2}{3}M_{j,Rd} < M_{j,Ed} \leq M_{j,Rd}$, can be determined by following interpolation formula:

$$S_j = \frac{S_{j.ini}}{\left(\frac{1{,}5\,M_{j.Ed}}{M_{j.Rd}}\right)^{\psi}} \qquad (1)$$

where:

- $\psi = 2{,}7$ – for following connection types: welded, frontal plate with bolts, with bearing plate;
- $\psi = 3{,}1$ – for connections with angles on legs, which are connected by bolts.

The values of secant rigidity S_j can be directly calculated for the concrete value of calculational bending moment at the connection spot-$M_{j,Ed}$from interpolational formula (1).

3. DETERMINATION OF INITIAL STIFFNESS OF CONNECTION

In order to determine the initial stiffness of connection the componential method can be applied [4]. The essence of this method lies in decomposition of connection to relevant components. Determination of rotational response of the connection in this manner comes up to determination of geometrical characteristics of different connection components. Hence, this method can be applied on a larger number of connection types and configurations. It can also be easily spread on new connection types, such as composite connections like coupled concrete-steel connections, with the condition of being familiar with all connection components' deformations, i.e. stiffness coefficients (k_i) and that the connection behavior can be presented by spring set model (Figure 2). Within Eurocode 1993-1-8 there are direct instructions on how to calculate stiffness coefficients for basic connection components: with frontal plate, for welded connections and for connections with connecting angles at beam legs.
The model assumes that both deformations of legs and beam ribs that are exposed to pressure and ribs and beam legs that are exposed to straining are included in deformations of the beam that is exposed to bending, hence, have no effect on deformation of the connection.
The initial stiffness of connection $S_{j,ini}$ is calculated for tension field and deformations of connection components within elastic area. Behavior of each component of the connection is represented by elastic springs. The expression for calculation of rotational stiffness of the connection can be obtained starting from the stiffness definition itself:

$$S_{j,ini} = \frac{M_j}{\Phi_j}. \tag{2}$$

Since connection rotation is a consequence of deformation sum of all of its components, the dependence among connection moment (M_j) and connection rotation (Φ_j) is equal to the dependance among force which affects a certain component:

$$F_i = \frac{M_j}{z}, \tag{3}$$

and deformation of the component itself in direction of force effect - Δ_i. Rotation of connection due to deformation Δ_i equals:

$$\Phi_i = \frac{\Delta_i}{z}, \tag{4}$$

where z is a distance among load center of tightened zone and load center of corresponding pressed zone. When it comes to welded beam-to-column connections this distance is equal to the distance among load centers of beam legs. Having this in mind, the expression (2) can be modified: (3), (4) $\longrightarrow$ (2) ⊔

$$S_{j.ini} = \frac{F_i \cdot z}{\frac{\Delta_i}{z}} = \frac{F_i \cdot z^2}{\Delta_i}. \tag{5}$$

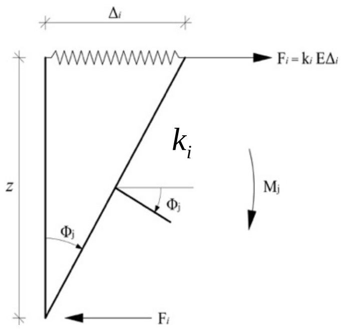

Figure 2. Mathematical model of connection component

The connection among elastic deformation of *i* th connection component (Δ_i) and the force which induces this deformation (F_i) can be expressed as:

$$F_i = K_i \cdot \Delta_i = k_i \cdot E \cdot \Delta_i \tag{6}$$

where:

- K_i – stiffness of *i* th connection component at deformation Δ_i in direction of effect of force F_i, and
- $k_i = \frac{K_i}{E}$ - stiffness coefficient of the *i* th component.

Ultimately, when the equation (5) is replaced with equation (6) the initial stiffness of connection is defined by:

$$S_{j.ini} = \frac{E \cdot z^2}{\frac{1}{k_i}}. \tag{7}$$

As it can be seen from equation (7), in order to determine the initial rigidity of connection it is necessary to be familiar with stiffness coefficients of all connection components as well as with arms of each equivalent force couples.

3.1 Welded beam-to-column connections

Welded beam-to-column connections that are described within Eurocode 1993-1-8 are connections at which the beam is directly welded to the column leg. The column rib can be either with or without stiffening at beam legs area. Figure 3 shows the calculational model of welded beam-to-column connection without stiffness at column rib. Each component of the connection is presented by springs and combined into a spring model.

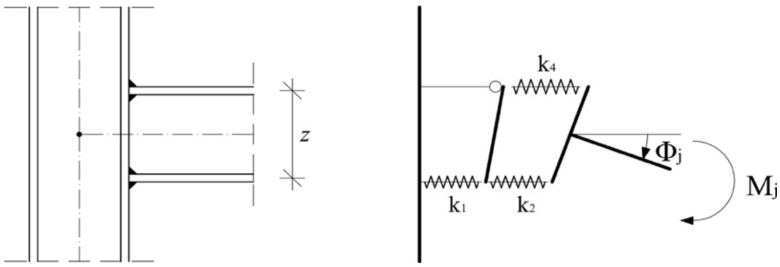

Figure 3. Calculational model of welded beam-to-column connection

Welded beam-to-column connections can be divided to three basic components, i.e. characteristic tightening areas that have different rigidity. At equal connection moments M_j the force F which occurs by division of connection moments to the equivalent couple is equal for all three tightening areas and amounts: $F = \frac{M_j}{z}$, where z – distance among load centers of beam legs.

Total relative rotation of connection Φ_j equals:

$$\Phi_j = \frac{\Delta_1 + \Delta_2 + \Delta_4}{z}. \tag{8}$$

Considering equations (2) and (8), the initial stiffness of connection can be expressed as:

$$S_{j,ini} = \frac{M_j}{\Phi_j} = \frac{F \cdot z}{\frac{\sum_{i=1,2,4} \Delta_i}{z}}, \tag{9}$$

(6) $\rightarrow$ (9) $\Rightarrow$

$$S_{j,ini} = \frac{E \cdot z^2}{\sum_{i=1,2,4} \frac{1}{k_i}}. \tag{10}$$

Deformation of each stiff component of connection should be neglected, i.e. for each adequately stiffed component of connection, the coefficient of stiffness is $k_i \to \infty$.
The same expression can apply to end-plate beam-to-column connection with only one bolt row at the tightened area (10), with careful consideration of components that differ. In this case the distance z is distance among load center of pressed beam leg and load center of tightened bolts.

3.2 Bolted beam-to-column joints

Eurocode 1993-1-8 gives detailed instructions for calculation for connections with end-plate and bolts for both end-plates with and without extension. However, what should be kept in mind is that the laws that define behavior of these kinds of connections are based on calculations with inclusion of certain simplifications, due to which the application of this procedure of calculation is limited by following assumptions:

- it is assumed that each beam-to-column connection has only two bolts at each bolt row;
- it is assumed that when it comes to end-plates with extension, at extension part there is only one row of bolts;
- it is assumed that, when it comes to end-plates with extension, the extension is not tightened in line with the rib.

Application limitations that come from compliance with above mentioned assumptions aren’t very strict, especially when it comes to common constructions.

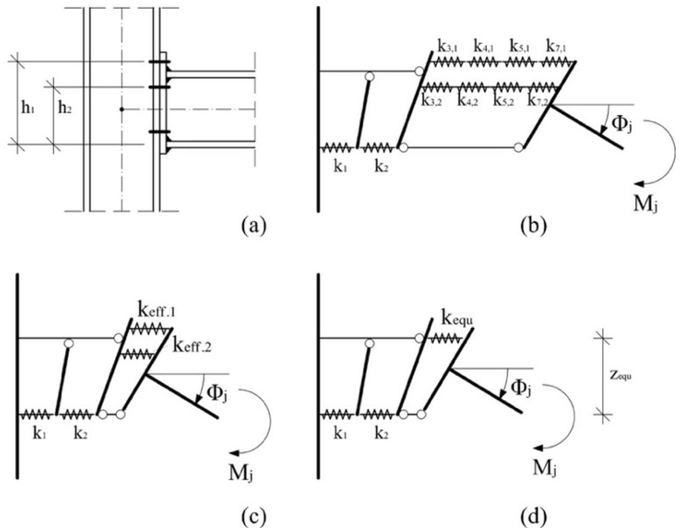

Figure 4. Calculational model of bolted end-plate beam-to-column connection

The expression for initial rotational stiffness of welded end-plate beam-to-column connection can be obtained the same as when it comes to welded joints. However, since the behavior of these connections is more complex, they can be divided to six characteristic components [1]. Having this in mind, the total rotational rigidity of this connection type can be expressed as a sum of each rotation that occur due to six specified characteristic connection components, as given in Figure 4.

Figure 4 presents a model with springs for end-plate connections with two rows of bolts at the area of tightening. Bolt deformations at the tightened area are proportional to the bolt distance from neutral line. Forces at each bolt row, for linear elastic tension state depend on the rigidity of corresponding connection components (k_i). Figure 4 (b) shows exchange of deformations per bolt row of components 3, 4, 5 and 7 with effective spring per bolt row, with effective coefficient of stiffness $k_{eff,r}$(r – index of bolt row number). Effective springs per bolt row are then changed with equivalent spring (Figure 4 (c)) which is set at load center of these connections at distance z_{equ}. Coefficient of rigidity of this equivalent spring is k_{eq}. Coefficient of effective rigidity k_{eq} can be directly applied to expression (10). The equations for determination of $k_{eff,r}$ and k_{equ}, as it is given in [1] can be directly derived from calculational model given in Figure 4. It should be mentioned that the moment-rotation relations of each system given in Figure 4: (a), (b), (c) and (d) are equal. Effective stiffness coefficients of bolts r - $k_{eff,r}$, is expressed by equation:

$$k_{eff,r} = \frac{1}{\sum\limits_{i=3,4,5,7} \frac{1}{k_{i,r}}}. \tag{11}$$

Distance among position of equivalent spring (Figure 4 (d)) and load center of pressed beam leg is:

$$z_{equ} = \frac{\sum\limits_r k_{eff,r} \cdot h_r^2}{\sum\limits_r k_{eff,r} \cdot h_r}, \tag{12}$$

where h is the distance of r^{th} row of tightened bolts from load center of pressed latch leg (Figure 4 (a)).

Stiffness coefficient of equivalent spring is defined by expression:

$$k_{equ} = \frac{\sum\limits_r k_{eff,r} \cdot h_r}{z_{equ}}, \tag{13}$$

and ultimately, the initial connection stiffness is given in expression:

$$S_{j,ini} = \frac{M_j}{\Phi_j} = \frac{E \cdot z_{equ}^2}{\sum_{i=1,2,equ} \frac{1}{k_i}} = \frac{E \cdot z_{equ}^2}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_{equ}}}. \tag{14}$$

In this case, the deformation of each tightened component should also be neglected, i.e. for each tightened component of connection the stiffness coefficient is $k_i \rightarrow \infty$.

4. COMPARISON WITH RESEARCH RESULTS

Comparative analysis of given analytical model for determination of connection rigidity with research results is given through charts in Figures 5 and 6.a)-6.d) Continual non-linear curves are obtained by appliance of rules given in this paper and in [1]. Research results are obtained from database SERICON [5].
Measured data of material and geometry obtained during research have been used in order to determine connection characteristics, i.e. initial stiffness of projected beam-to-column connection. The value of moment resistance $M_{j,Rd}$ has been calculated with partial safety coefficients 1,0. Moment resistance is determined by EC 3 Annex J [6]. Rotational stiffness and moment resistance have been calculated by taking in consideration real forces at panel shearing area of column rib through precise values of coefficients β.

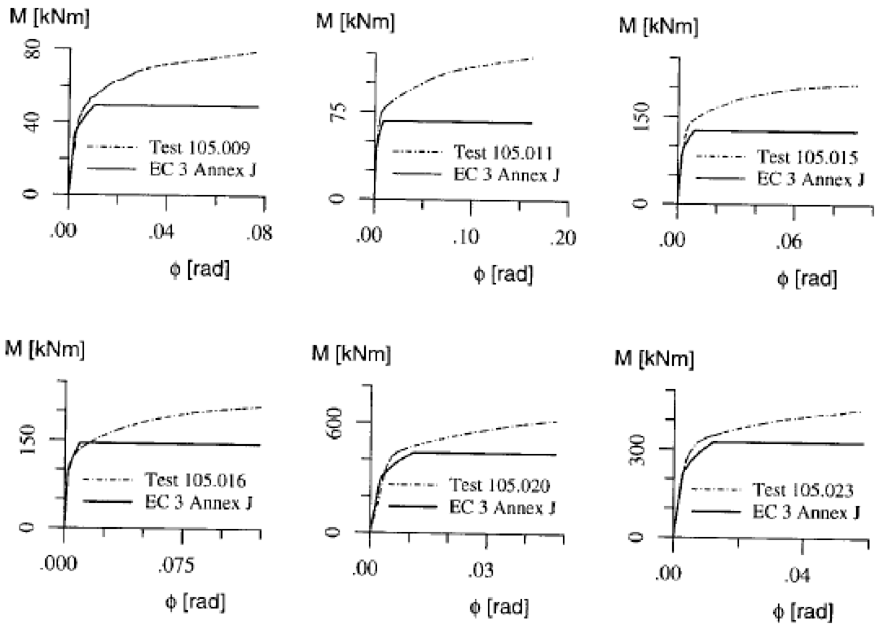

Figure 5. Single sided welded connections, Innsbruck

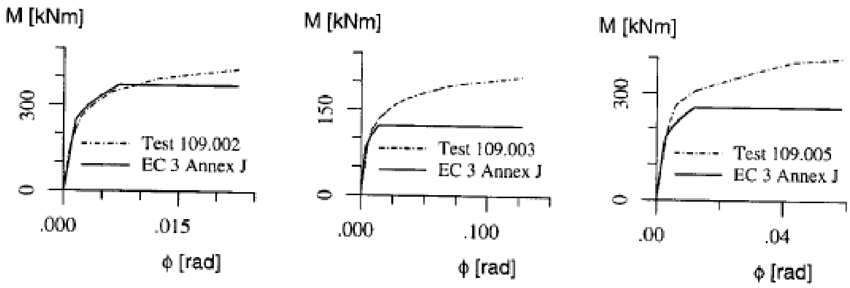

Figure 6. a) Double-sided/single sided end-plate connections

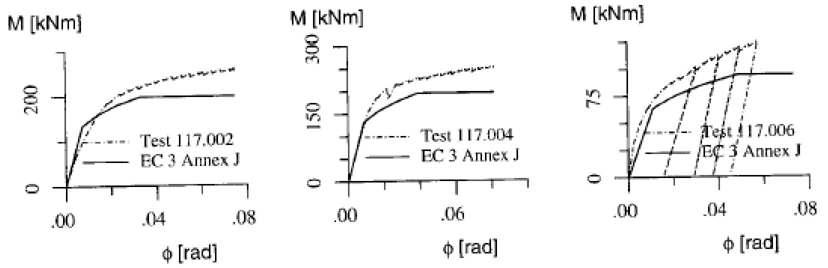

Figure 6. b) Single sided end-plate connections, Leipzig/Aachen,

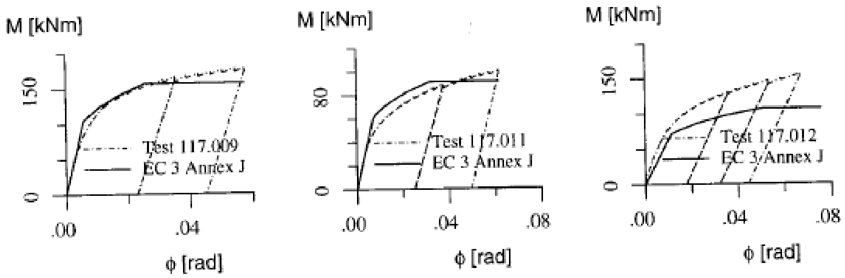

Figure 6. c) Double-sided end-plate connections, Leipzig/Aachen,

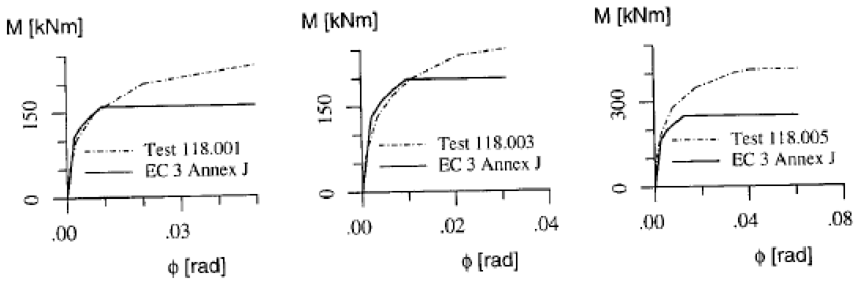

Figure 6. d) Double-sided/single sided end-plate connections, Dundee,

5. CONCLUSION

Chart attachments, Figures 5 and 6 a)- 6 d) show that the prediction of connection stiffness and resistance is in good compliance with realistic connection behavior. The differences in resistance are present due to post-elastic strengthening of material and membrane effect which are not taken in consideration in projecting rules given in [1], [7]. Models for determination of beam-to-column connection stiffness assume that the connection remains elastic up to $\frac{2}{3}M_{j,Rd}$ level. This assumption has been confirmed by research results.

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ROTACIONA KRUTOST POLU-KRUTIH VEZA

Rezime: *U ovom radu analizirana je metodologija za određivanje rotacione krutosti polu-krutih veza u čeličnim konstrukcija prema EC 3 primenom komponentalne metode. Primenom ovakvog koncepta određivanje rotacionog odgovora veze svodi se na određivanju geometrijskih karakteristika različitih komponenata u vezi, tako da se ova metoda može primeniti na veći broj različitih tipova i konfiguracija veza u čeličnim konstrukcijama. U radu je prikazano i poređenje ove metode sa rezultatima ispitivanja.*

Ključne reči: *čelične konstrukcije, polu-krute veze, rotaciona krutost, proračun veza.*