SEISMIC ANALYSIS OF STEEL FRAMES WITH SEMI-RIGID AND VISCOUS CONNECTIONS

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Summary: Seismic analysis needs to apply a material-geometric non-linear analysis of steel frames with semi-rigid, eccentric and viscous beam-to-column connection for satisfactory steel frame response. Both nonlinearities, the geometric nonlinearity of the structure, and the material non-linearity of the connections are considered simultaneously. The flexibility of the ends of the beam is modeled using rotary springs at its ends with a nonlinear moment-rotation relation defined by a three-parameter model. The eccentricity of the connection is represented by short infinitely rigid elements. The viscous damping related to the relative velocity of the angle of relative rotation is modeled using rotary viscous dampers. For this model of the beam, a complex flexible matrix of stiffness of the element was introduced. Parametric analysis has determined the influence of connection flexibility, eccentricity, viscous damping and second order theory on the seismic response of the structure. The results of examples are illustrated in diagrams and figures. An adopted model for frame calculations can also be used to seismic response control of a structure through specifically added structural elements, such as dissipative joint connections.

Keywords: steel frames, nonlinear dynamic analysis, semi-rigid eccentric connections, dissipative connections

1. INTRODUCTION

Actual seismic design of steel buildings is based on analysis which demands realistic numerical models, needed for control of the behavior of structures due to incidental seismic load or for performance based seismic designs. Based on numerous studies it was concluded that the connections are realistically semi-rigid and that the moment-rotation relationship at the end of the linear elements is non-linear almost in the entire load range generally for all types of connections (Figure 1)[1,2]. In addition to the established flexibility of connections, there is a greater or lesser eccentricity of connections, which can cause significant changes in the impact in the structure. Within the latest requirements in the aseismic design, the control of the behavior of structure, the importance of designing and installing specific structural elements has an important role in reducing or completely eliminating the damages of both constructive and nonconstructive structural elements. These elements largely accomplish significant

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energy dissipation that is passed on to the system by the system in seismic load, and thus protect other structural elements from damage [3].

Figure 1. Relation Moment-Rotation for different type of beam-column connections[4]

The subject of this paper is the seismic material-geometric non-linear analysis of steel frames with semi-rigid, eccentric and viscous beams-column connections. Considerations can also be implemented in systems for controlling the behavior of the structure through specifically added structural elements, such as nodal dissipative connections. Special attention is also paid to the effects of second-order theory, which can be crucial for flexible structures such as steel frame systems. In this way, the work includes nonlinearities, the geometric nonlinearity of the structure, and the material nonlinearity of the connections, which were considered simultaneously. The material non-linearity of the problem occurs only in the connections of the beam elements, while all other parts of the system are considered in the domain of elastic behavior and in this way a substantial rationalization and design efficiency has been achieved, which is applicable as such in standard engineering calculations.

As a result of the presented numerical model of seismic analysis and the type of the structure behavior control, the corresponding parametric analysis carried out.

2. BEAM ELEMENT WITH SEMI-RIGID, ECCENTRIC, VISCOUS, BEAM-TO-COLUMN CONNECTIONS

A beam element with semi-rigid, eccentric and viscous damping beam-to-column connections is shown in (Figure 2). The semi-rigid (flexible) connections are modelled with nonlinear rotational springs at beam ends. The assumption is that the only the influence of bending moment on the connection deformation is considered, while the influences of axial and shear forces are neglected. The spring element is assumed as mass-less and dimensionless. The eccentricity is modelled by short infinitely stiff elements whose lengths are \( e_1 \) and \( e_2 \). The linear viscous damping at nodal connections are represented by dashpots acting at beam ends [5].
Primary unknowns are the joint displacements and rotations, while displacements and rotations of the beam ends are eliminated as has been shown in [5]. Consequence is that the number of degrees of freedom remain the same as for the standard beam element with fully rigid connections. The function describing vertical displacement \( v(x) \) for the element with flexible eccentric connections, is written in the form related to interpolation function matrix and nodal displacements vector:

\[
v(x) = N(x)(I + G)q = \tilde{N}(x)q
\]  

(1)

where \( N(x) \) denoting the matrix of interpolation functions obtained based on the analytical solutions of the second order analysis equations [6], \( q \) element nodal displacement vector, \( \tilde{N} \) corrected matrix of interpolation functions and \( G \) is correction matrix:

\[
G = \frac{1}{\Delta} \begin{bmatrix}
0 & \Delta e_1 & 0 & 0 \\
g_{21} & g_{22} & g_{23} & g_{24} \\
0 & 0 & 0 & -\Delta e_2 \\
g_{41} & g_{42} & g_{43} & g_{44}
\end{bmatrix}
\]  

(2)

Elements of correction matrix \( G \) depends on coefficients \( g_{ij} \) and eccentricities \( e_1 \) and \( e_2 \). Coefficients \( g_{ij} \) are the functions of nondimensional rotational stiffness in node \( i \) and node \( j \) (\( g_i = EI/lk_i \); \( g_j = EI/lk_j \)). Stiffness matrix for the beam element with flexible eccentric connection can be obtained through the total potential energy (Eq. 3), axial, flexural and with additional potential strain energy of the springs \( (U_s) \).

\[
U = U_a + U_f + U_s
\]  

(3)

Based on these considerations of stiffness matrix of beam element can be expressed as a sum of three stiffness matrices:
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\[ k = k_{II} + k_{ef} + k_s \]  \hspace{1cm} (4)

where matrix \( k_{II} \) denoting beam stiffness matrix with the rigid connections according to the second-order analysis, \( k_{ef} \) correction matrix which takes into account the effects of flexibility and eccentricity and \( k_s \) stiffness contribution part which depends on rotational stiffness of nodal springs:

\[ k_{II} = EI \int_0^l \left[ \left( N''(x) \right)^T N''(x) \right] dx \]  \hspace{1cm} (5.1)

\[ k_{ef} = G^T k_{II} + k_{II} G + G^T k_{II} G \]  \hspace{1cm} (5.2)

\[ k_s = \overline{G}^T C \overline{G} \]  \hspace{1cm} (5.3)

The explicit form of matrices \( \overline{G} \) and \( C \) can be found in [5].

3. ELEMENT MASS AND DAMPING MATRICES

Assuming that the mass density \( \rho \) is constant, the element consistent mass matrix \( m \) can be given in a form:

\[ m = \int_{V} \rho \tilde{N}^T(x) \tilde{N}(x) dx \]  \hspace{1cm} (6)

where \( \tilde{N}(x) \) is the matrix of corrected shape functions defined by Eq. (1). After substitution of Eq. (1) into Eq. (6), the consistent element mass matrix, for the proposed element beam with flexible eccentric connections, can be written as sum \( m = m_o + m_{ef} \), where:

\[ m_o = \int_{V} \rho N^T(x) N(x) dx \quad m_{ef} = G^T m_o + m_o G + G^T m_o G \]  \hspace{1cm} (7)

In the above relations, \( m_o \) denotes conventional mass matrix for the beam element with constant cross section and \( m_{ef} \) denotes the mass correction matrix.

Rotational viscous dashpots are attached at beam ends (Figure 2), and as consequence the total moment at each nodal connection \((i=1,2)\) depends on term of relative rotation \( \theta \) between beam end and column and term of relative angular velocity \( \dot{\theta}(t) \) as:

\[ M_i(t) = k_i \theta_i(t) + c_i \dot{\theta}_i(t) , \quad i = 1', 2' \]  \hspace{1cm} (8)

where \( k_i \) and \( c_i \) are rotational spring stiffness and rotational viscous damping coefficients, while dot over the symbols denotes differentiation with respect to time. The tangent or secant form of the above relation may be written if non-linear springs and dashpots are
considered. In the case of periodic response with circular frequency $\omega$ the following relation between the amplitudes may be derived [7]:

$$M_{i(t)} = k^*_i \theta_{i(t)} \quad i = 1,2 \quad k^*_i = \frac{M_{i(t)}}{\theta_{i(t)}} = k_i + i \omega c_i \quad i = \sqrt{-1} \quad \theta_{i(t)} = \theta_{i(t)} e^{i\omega t} \quad (9)$$

where complex flexural stiffness $k^*_i$ of the connection is defined as the ratio between moment and relative rotation amplitudes. After the elimination of relative end rotation vector $\theta(t)$, the relation for end nodal forces $\bar{R}$ transforms to:

$$\bar{R}(t) = \bar{k}^* q(t) \quad (10)$$

where matrix $k^*$ is a complex flexural stiffness matrix of uniform beam with flexible connection according to the linear or second order analysis, including both flexible and viscous phenomena.

Expanding the elements of the dynamic stiffness matrix in series with respect to the circular frequency $\omega$ and neglecting higher terms than the third order, the following expansion is obtained in the decomposed form:

$$k^* = k + i \omega c - \omega^2 m \quad (11)$$

where $k$ is the static stiffness matrix, $c$ the damping matrix and $m$ the mass matrix for the uniform beam with flexible springs and dashpots at its ends[7].

The proposed viscous damping at beam ends causes that viscously damped system does not satisfy Caughey and O’Kelly’s condition [8]. The response of a multi-degree-of-freedom system cannot be expressed as a linear combination of its corresponding modal responses. So, the system is non-classically damped and it generally has complex valued natural modes. It is necessary to explain physical interpretation of solutions represented by complex conjugate pairs of characteristic values. In order to establish the relationship between coefficient $c_i$ of viscous damping in joints and modal relative damping factor $\zeta_i$ for $k$ mode shape, a specific procedure has to be introduced. Based on the parametric study, the relationship between coefficient $c_i$ of viscous damping in joints and modal pseudo relative damping factor $\tilde{\zeta}_i$ for $i$ mode shape can be obtained [4].

4. MODELLING OF NON-LINEAR SEMI-RIGID CONNECTION

A large number of experimental results have shown that the connection moment-rotation relationships are non-linear over the entire range of loading for almost all types of connections [9,10,11]. To describe connection behavior numerous different mathematical models have been proposed during last three decades. In presented procedures the three parameter power, model proposed by Richard and Abbott [12] and
Kishi et al. [13] is adopted to represent moment-rotation behavior of the connection under monotonic loading (Figure 3a). Three parameter power non-linear model can be given in a form:

$$M = \frac{k_o \theta}{1 + \left(\frac{\theta}{\theta_o}\right)^p}$$

(12)

where $k_o$ initial connection stiffness, $p$ shape parameter, $\theta_o = M_u / k_o$ reference plastic rotation and $M_u$ is ultimate moment capacity of the beam-to-column connection.

The independent hardening model was adopted to simulate the inelastic connection behavior under cyclic loading. In this model, the characteristics of connections are assumed to be unchanged through the loading cycles. The moment-rotation curve under the first cycle of loading unloading and reverse loading remain unchanged under the repetition of loading cycles. The skeleton curve used in the model was obtained from three parameter power model. The cyclic moment-rotation curve based on this model is shown in Fig. 3b as a numerical result of connection's hysteresis behaviour. The independent hardening model is simple and easily applicable to all types of steel frames connection models. This model is defined in detail in [14,15].

The advantage of this non-linear model is in a clear formulation with the physical meaning of the parameters that can be simply experimentally obtained.

5. P-DELTA EFFECTS

If the equations of equilibrium are placed on a deformed structure, the P-Delta term appears in the equation. The size of this product, in relation to other members in the equation, determines whether the design should be implemented on the theory of the first or second order theory. The product P-Delta can also be significant if only one of the
factors is large enough, that is, when the movement is small and a large axial force, or when the axial force is small and the movement is large. In such cases, the influence of the axial force on the transverse deformation cannot be neglected, so the calculation must be carried out using the second-order theory, that is, through the P-Delta analysis. It is a form of geometric nonlinear analysis in which it is assumed that the displacements are large, and that the deformations and rotations are of small size.

By introducing axial forces into a design in this way, the structure characteristics change, and the elements of the axial forces are decrees the flexural stiffness and the tensile elements become stiffer. The P-Delta effect is expressed in high-rise buildings, and significantly increases the effective shear force on each floor. This analysis is important in determining the effect of gravitational load in simultaneous action with horizontal forces such as wind forces and seismic forces. Dynamic structural calculation using the second-order theory is required in the case of extremely strong earthquakes, and especially in flexible structures (such as, for example, unbraced frame systems with semi-rigid connections). This is also provided for by certain standards and regulations such as Eurocode 8, AISC (1994) and ACI (1995).

The process of calculation the P-Delta analysis for the given vertical and horizontal load has iterative character, which can significantly increase the calculation time, which is particularly unfavorable in the dynamic analysis. First, it is necessary to carry out a design to estimate the axial force in the structure, and then with these forces calculates the stiffness of the system and determines the deformation and the internal forces. Obtained axial forces differ from those originally estimated, so additional iterations are required, as long as in successive iterations, the difference in force intensity and deformation does not become small enough, or less than the accepted accuracy.

The impact of large displacements can also be considered using another approach. The effects of a significant change in the deformed shape can be taken into account by determining the unbalanced load due to the large displacement of the element nodes at the end of each time step of integration. An unbalanced load is transmitted as a corrective load in the next step of time integration. A more accurate result can be obtained if a change in the geometric stiffness matrix is determined in each change in the axial force in the element. It can also be applied at the same time by changing the geometric stiffness matrix and corrective load [4].

![Figure 4. Effects of static and dynamic loads in seismic analysis](image-url)
For conventional frame systems, which often occur in buildings, there are two types of load: static and dynamic. The static load, which is usually gravitational only, is applied to the structure before the earthquake, and it leads the structure into a deformed equilibrium position. After that, due to the fact an earthquake occurs in the appearance of inertial forces, most of which are usually the most significant horizontal inertial forces. They cause horizontal displacement, which result in the occurrence of the P-Delta effect. In addition, horizontal forces also affect the change of axial forces in the elements. Therefore, the assumption of constant axial forces, strictly taking, there is no justification, since the change can amount to more than 50%. This means that the dynamic load can significantly affect the change in the value of the axial forces determined only on the basis of gravitational (static) load [16].

In order for both types of load to be considered simultaneously, and to apply the real axial force in the design, static loading should be treated as a dynamic one. The static load, in function of time $t$, can be represented in the form, which illustrates Figure 4. Given the fact that the static load does not cause inertial forces, the value of loading time $t_1$ should be such that the effects of force $P_o$, treating it as static or dynamic, are equal in the frames of the accepted accuracy. When the loading time $t_1 \to \infty$, or when the load is applied extremely slowly, only static influences are obtained. Otherwise, when $t_1 \to 0$, there is a effect that encounters influences due to step load. After time $t_2$, for which it can be assumed that $t_2 = 1.10 t_1$, the dynamic load, ie the displacement of the supports starts to appear (Figure 4). In this way, a unique method of numerical integration of the equation of motion for both types of load can be applied.

6. SEISMIC ANALYSIS

Numerical procedures in seismic analysis can be performed using equations of motion of a frame subjected to earthquake as a dynamic loading:

$$\ddot{U} + \dddot{U} + Ku = -M \ddot{U}_g + F$$ (13)

in which $\bar{M}$ is the mass matrix, $\bar{C}$ is viscous damping matrix and $\bar{K}$ is static stiffness matrix for the system of structural elements. Dash above matrices denotes standard correspondent matrices are modified with correction matrix $G$. The time dependent vectors $\ddot{U}, \dddot{U}$ and $U$ are the relative node accelerations, velocities and displacements respectively, while the vectors $\ddot{U}_g$ and $F$ are ground accelerations and externally applied loads.

The equations of motions are integrated using step-by-step integration, with a constant acceleration assumption within each time step. During the numerical integration of the equation of motion, the axial force in the elements changes, i.e. it is not constant. That results, a simple superposition of loads is not applicable in the geometric nonlinear analysis, so the problem needs to be defined in an incremental form.

Secant stiffness method is used to solve the nonlinear equations, that are nonlinear in terms of the displacements as well as the axial force. The solving procedure is very
applicable in computer software algorithm giving the convergent solutions for dynamic loadings. The load increment $\Delta F$ or $\Delta \hat{U}_g$ is divided into a few smaller sub-increments in each time step to obtain faster convergence.

![Figure 5. Response of steel frame due to earthquake loading](image)

The incremental-iterative algorithm is based on evaluating secant stiffness matrix, which depends on the stiffness of connections, represented by slope of its moment-rotation curve at any particular moment value. The convergence is obtained when the differences between two consecutive cycles displacements at all joints reach the prescribed tolerance. The current connection stiffness becomes the starting connection stiffness for
7. CONCLUSIONS

In order to obtain as realistic an quality results in the seismic analysis of steel frames, the effects of flexible nonlinear and eccentric connections, the effects of viscous damping in connections, the effects of second-order theory effects and the effects of dissipative node connections are considered in detail. All of these effects are separately considered through parameter analysis to determine the influence of a particular parameter on the response of the system.

The role of the beam-column connection in steel frames is crucial for seismic analysis, given the fact that the connections were the most critical elements during the earthquake that had already happened. Obviously, it is nonlinear behavior of connections even for lower load levels and is especially evident in stronger seismic load.

In order for nonlinear seismic analysis to be rational, efficient and engineering acceptable, the concept of selective nonlinear behavior of the structure was adopted, i.e. only certain elements of the structure are adopted non-linear behavior, while all other behavior of elements are in the domain of linear elasticity of the material. For steel frame systems it is also sufficient to consider only nodal connections as elements with nonlinear behavior.

It has been also established that it is necessary to take into account the flexibility of connections, because their ignorance will lead to results that are not close to the actual behavior of steel frames.

The eccentricity of the connection also has significance depending on the size and type of connection. It has been shown that even in frame systems with a small eccentricity, circular frequency will differ significantly depending on whether eccentricity is taken into account or not. Two effects, flexibility and eccentricity of the connection, contribute to changing the distribution of internal forces in the structural system.

Energy dissipation in nodes is usually an accompanying, side-by-side and beneficial appearance in the structure response. The paper proposes an approach to the dissipation of seismic energy in connections through dictated increased damping in connections. Numerical calculations have shown that in this way it is easy to control and limit the impacts in the structure, because a high level of reduction of maximum impacts is achieved.

It is known that the effect of geometric nonlinearity increases with increasing load, and it is even more significant in frames with flexible connections than in frames with rigid connections. Flexibility of connections and geometric nonlinearity, simultaneously and individually, have a significant impact on the analysis of frame behavior because they
significantly influence on dynamic characteristics of the system. Comparing the methods of geometric non-linear analysis with constant and variable axial forces in a seismic calculation, it was concluded that in the orthogonal frames the procedure of constant axial forces gives completely satisfactory results, which is not the case in non-orthogonal frames, so it is necessary to apply the procedure to be taken into account the change of axial forces in the elements due to the seismic load effect.

The proposed presented modeling and procedures, based on the developed finite element of the frame structures, significantly enhance the quality of the analysis, and gives good opportunities for modeling the behavior of steel frame structures. The process is based on a numerical calculation that is simply applicable in the design using a computer and contributes to a more efficient and more accurate calculation of structures exposed to earthquake effects. In order to obtain reliable quantitative results in non-linear dynamic analysis it is necessary to have the support of appropriate experimental research (especially for hysteresis behavior for various loading and unloading cases), both in the field of steel beams-column and in the field of node energy dissipation [17].

REFERENCES

7. МЕЂУНАРОДНА КОНФЕРЕНЦИЈА


SEIZMIČKA ANALIZA ČELIČNIH RAMOVA SA POLUKRUTIM I VISKOZNIM VEZAMA

Usvojeni model za proračun ramova može se primeniti i za kontrolu ponašanja konstrukcije preko posebno dodatih konstruktivnih elemenata, kao što su disipativne čvorne veze.

Кључне речи: čelični ramovi, nelinearna dinamička analiza, polukrute ekscentričне везе, viskozno прigušење, disipativne чвorne везе