ON DYNAMIC VIBRATION ABSORBER MODELS FOR HARMONIC EXCITATION

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Summary: *A primary mass attached through a solid horizontal rod to the wall, able to slide without friction along the horizontal line under a periodic force modeled by a sine function, and an added mass attached to the main one through another horizontal solid rod, also able to slide along the same line without friction, represent a problem encountered in almost all textbooks on mechanical vibrations. However, many of the books consider conditions ensuring zero steady-state amplitude of the primary mass and just several of them consider conditions ensuring either reduction of the primary mass amplitude or cutting one down as much as possible. Once again, in the whole class of books, one can find the rods of either Hookean or the Kelvin-Voigt type, i.e. linear springs or linear springs connected in parallel to dashpots. In this work, the vibration absorbing conditions ensuring the reduction of the primary mass steady-state amplitude will be stated for the Kelvin-Zener model of viscoelastic rod and its fractional generalization. The obtained conditions will be related to the restrictions on coefficients in these models that follow from the Clausius-Duhem inequality. The proposed model could be used for the study of energy dissipation in mechanical systems incorporating polymers, elastomers, living tissues and other real materials.*

Keywords: *Passive vibration control, fractional Kelvin-Zener model, Clausius-Duhem inequality*

1. THE GENERAL MODEL

In order to cut down as much as possible the amplitude of forced vibration, one adds another single degree of freedom system of the same type. One may start with the general case, the fractional Kelvin-Zener model, described by two coupled differential equations of real order, and get all the declared cases as the special ones. Thus, consider motions of the two-degrees of freedom system presented in Fig. 1 with the mass denoted by M as the primary one, and the mass denoted by m as the added one. For simplicity reasons both rods are assumed to be of the same length and cross-sections say L and A, respectively and to deform in uniaxial, isotermal deformation. Let a

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force $f_0 \sin Wt$ acts on the primary mass and let $x(t)$ and $y(t)$ be the coordinates describing the positions of the masses in time instant t respectively, see Fig. 1.

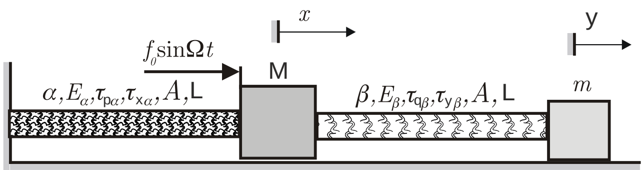

Figure 1 System under consideration

Assuming that there is no sliding friction in the system and that both rods are undeformed in the initial state, the Newton axiom and the Newton-Laplace principle applied, lead to

$$Mx^{(2)} = -p + q + f_0 \sin Wt\,, \qquad My^{(2)} = -q, \tag{1}$$

$$x(0) = 0, x^{(1)}(0) = 0, p(0) = 0, y(0) = 0, y^{(1)}(0) = 0, q(0) = 0, \tag{2}$$

where p and q represent the forces within rods respectively $(\cdot)^{(k)} = d^k(\cdot)/dt^k$, denotes k - th integer order derivative with respect to time t. Introducing the left derivative of real order in the standard Riemann-Liouville form, aka fractional derivative, denoted by $(\cdot)^{(\gamma)} = d^\gamma(\cdot)/dt^\gamma, 0 < \gamma \leq 1$, see [1]

$$\frac{d^\gamma}{dt^\gamma} u(t) = u^{(\gamma)} = \frac{d}{dt} \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{u(\xi) d\xi}{(t-\xi)^\gamma}$$

where Γ is the Euler gamma function $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$, to be used for $\gamma = \alpha, \beta$, the constitutive axioms attached to this system reads

$$p + \tau_{p\alpha} \cdot p^{(\alpha)} = \frac{E_\alpha A}{L}\left(x + \tau_{x\alpha} \cdot x^{(\alpha)}\right), \tag{3}$$

$$q + \tau_{q\beta} \cdot q^{(\beta)} = \frac{E_\beta A}{L}\left[y - x + \tau_{y\beta} \cdot \left(y^{(\beta)} - x^{(\beta)}\right)\right], \tag{4}$$

According to the general theory, see [2,3], the coefficients in axioms (3) and (4) are to follow the consequences of the second law of thermodynamics, i.e. the Clausius-Duhem inequality, that are

$$E_\alpha > 0, \quad \tau_{p\alpha} > 0, \quad \tau_{x\alpha} > \tau_{p\alpha}, \tag{5}$$

$$E_\beta > 0, \quad \tau_{p\beta} > 0, \quad \tau_{y\beta} > \tau_{q\beta}, \tag{6}$$

Since 80s of the previous century it was shown that the chosen axioms is very useful in modelling rheological and structural behavior of many real materials: elastomers, polymers, living tissues, see [5-7].

2. THE ABSORBER CONDITION FOR THE KELVIN-ZENER MODEL

One may declare a desirable state of the system to be the one where x » 0 , p » 0 together will all the derivatives of these variables, leading to

$$\left(1 - \frac{E_2 A}{L}\frac{1}{m\Omega^2}\right) \sin\Omega t + \left(\tau_{q\beta} - \tau_{y\beta}\frac{E_2 A}{L}\frac{1}{m\Omega^2}\right) S_t(-\beta,\Omega) = 0, \tag{7}$$

where $S_t(-\beta,\Omega) = \sum_{j=0}^{\infty}(-1)^j \Omega^{2j+1} t^{2j+1-\beta} \Gamma^{-1}(\beta + 2j + 2)$ stands for the Riemann-Liouville derivative of $\sin\Omega t$ of order b, see [4, p. 355]. Note that $S_t(-1,\Omega)$ coincides with $\Omega\cos(\Omega t)$, see [4, p.318]. Also, due to independence of sine and $S_t(-1,\Omega)$ it should be

$$\left(1 - \frac{E_2 A}{L}\frac{1}{m\Omega^2}\right) = 0, \quad \left(\tau_{q\beta} - \tau_{y\beta}\frac{E_2 A}{L}\frac{1}{m\Omega^2}\right) = 0, \tag{8}$$

what corresponds to zero value of the steady-state amplitude of the primary mass. Note that due to (6) conditions (8) cannot be satisfied for the real materials. Therefore, the amplitude of the primary mass cannot be zero, namely for real materials it can only be reduced.

3. THE SPECIAL CASES PRO ET CONTRA

The special cases of the rods in the problem read:

$$\tau_{p\alpha} = \tau_{x\alpha} = 0,$$

$$\tau_{q\beta} = \tau_{y\beta} = 0, \tag{9}$$

corresponding to the linearly elastic i.e. Hookean material. The vibration absorbing condition reads

$$m\Omega^2 = \frac{E_2 A}{L} \qquad (10)$$

as stated in all the books on mechanical vibrations. Actually, with (9), the axiom (4) reduces to the Hook law established in 1676. Note that in 1690 Leibnitz wrote that the relation between extension and stretching force should be determined by experiment. Also note that in 1973 in the famous Encyclopedia of Physics Vol. VIa/1,Mechanics of Solids I, edited by C. Truesdell ed., Springer-Verlag, Berlin, in the chapter on The Experimental Foundations of Solid Mechanics, J.F. Bell, stated the experiments of 280 years have demonstrated amply for every solid substance examined with sufficient care, that the strain resulting from small applied stress is not a linear function thereof. Besides this model does not satisfy (6) and cannot cover any real material. Thus (10) cannot be used for vibration control of real systems.

Next, one may try with the Kelvin-Voigt viscoelastic model corresponding to

$\alpha = \beta = 1$ and

$$\tau_{p1} = 0,$$

$$\tau_{q1} = 0 \qquad (11)$$

in (3), (4). This model is frequently seen in the books on vibration theory. Note that this model, as the previous one, does not satisfy the restrictions that follow from the entropy inequality (5), (6) so one cannot expect it to work for real materials. Actually, this model does not cover a very simple or real deformation pattern, RDP for short, corresponding to the ramp-and-hold of strain and then determining the stress relaxation in the experiment, see [8] shown in Fig. 2.

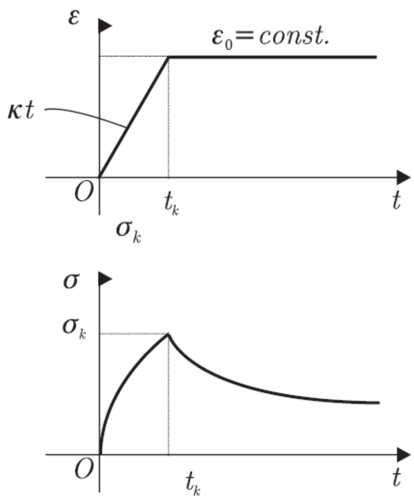

Figure 2 Strain (a) and stress (b) corresponding to real deformation pattern

Recognizing $\sigma = q/A$ and $\varepsilon = y/L$ it is obvious that for the following strain

$$\varepsilon(t) = \begin{cases} \kappa t & 0 \leq t \leq t_k, \\ \varepsilon_0 = const & t > t_k \end{cases}$$

there will be jump of stress in (4), for $\tau_{q1} = 0$ and $x = 0$. However that jump cannot be recognized in any real experiments see [9]. So if the model, cannot cover that very simple deformation pattern how it can be used in more complex situations.

Finally, consider models that follow the pattern presented in Fig. 2 for real materials. One such a model is the standard linear viscoelastic solid aka the Kelvin-Zener model, see [7], obtained form (3), (4) with (5), (6) as the special case for $\alpha = \beta = 1$. For this kind of materials the measure of primary mass amplitude reduction can be obtain from (7) as

$$\left(1 - \frac{E_2 A}{L}\frac{1}{m\Omega^2}\right) \sin \Omega t + \Omega\left(\tau_{q\beta} - \tau_{\gamma\beta}\frac{E_2 A}{L}\frac{1}{m\Omega^2}\right)\cos \Omega t = 0, \qquad (12)$$

as before in case of $x \approx 0$, $p \approx 0$ together will all the derivatives of these variables, this condition reduces to

$$\left(1 - \frac{E_2 A}{L}\frac{1}{m\Omega^2}\right) = 0, \quad \left(\tau_{q\beta} - \tau_{\gamma\beta}\frac{E_2 A}{L}\frac{1}{m\Omega^2}\right) = 0,$$

and due to inequalities that follow from the second law of thermodynamics $\tau_{y1} > \tau_{q1} > 0$ it cannot be satisfied. Thus, as for the fractional case in the case of standard linear viscoelastic body the amplitude of primary mass cannot vanish, it can only be reduced.

In conclusion one may say that the motion of primary mass attached to real viscoelastic rods under the action of harmonic excitation cannot be ceased, just reduced. The conditions (7) and (12) can be used as a measure of this reduction.

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О МОДЕЛИМА ДИНАМИЧКОГ АМОРТИЗЕРА ЗА СЛУЧАЈ ХАРМОНИЈСКЕ ПОБУДЕ

Резиме: *Примарна маса може без трења да клизи по хоризонталном правцу под дејством периодичне силе синусног облика. Маса је преко вискоеластичног штапа везана за непокретни зид. С друге стране примарне масе преко другог вискоеластичног штапа повезана је још једна маса која може да клизи без трења по истом правцу. Овај проблем је део готово свих уџбеника из теорије осцилација. Међутим, многи од тих уџбеника разматрају услове под којима ће амплитуда устаљеног кретања примарне масе бити нула, а само неки од њих говоре о редукцији или смањењу те амплитуде што је могуће више. Опет у готово свим тим уџбеницима користе се Хуков или Келвин-Војтов модел тј. линеарна опруга или линеарна опруга паралелно везана за линеарну пригушницу. У овом раду се услови абсорбовања дејства принудне силе који редукују амплитуду устаљеног кретања примарне масе формулишу за случај Келвин-Зенеровог модела вискоеластичног штапа и његове фракционе генерализације. Добијени услови интерпретирају се у духу ограничења на коефицијенте у моделу који следе из Клаузиус-Диемове неједнакости. Предложени модел се може употребити у анализи дисипације енергије механичких система који укључују еластомере, полимере, биолошка ткива, биоматеријале и друге реалне материјале.*

Кључнеречи: *Пасивно управљање вибрацијама, фракциони Келвин-Зенеров модел вискоеластичног тела, Клаузиус-Диемова неједнакост*