A STUDY OF A TENSEGRITY STRUCTURE FOR A CYLINDRICAL ROOF

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Summary: *The structure studied in the paper is an infinitesimal mechanism made from insulated bars linked by cables. The rigidity of the structure is ensured by pretensioning the cables. The form-finding of the equilibrium shape of the pretensioned structure was performed using a geometric non-linear structural analysis method.*

Keywords: *Tensegrity, cylindrical roof, form-finding, structural analysis*

1. INTRODUCTION

Almost one hundred years have passed since a ‘‘proto-tensegrity,’’ was built by a truly constructivist artist, Karl Ioganson, in 1920 and exhibited in Moscow in 1921, under the title of ‘‘Study in Balance.’’[3] In 1947, the first tensegrity structure was invented and built by a young artist named Kennet Snelson. The interest in these fascinating sculptures slowly migrated from the intuitive, inspirational world of art into the systematic and rigorous world of science, to recently blossom in applied areas of science and engineering. The terminology of tensegrity was first used by Fuller (1962) to describe Snelson’s structures in his patent “Tensileintegrity Structures”. He characterizes them as islands of compression in an ocean of tension. Nowadays the Rene Motro’s definition is widely accepted: “*A tensegrity system is a system in a stable self-equilibrated state comprising a discontinuous set of compressed components inside a continuum of tensioned components.*”[5]

The Tensegrity structures, which consist of continuous tension elements and discontinuous compression elements, were also proposed by Fuller. A specific feature of tensegrity structures is that their compression components *do not touch each other* and *do not transfer* each other the compression forces which they are subject to. They can be defined as a subclass of cable structures, but unlike the latter their tensile forces are not anchored. The stability and stiffness of tensegrity structures are ensured by a self-equilibrated and self-stress state among tension and compression members. The tensegrity structures have the structural rigidity only when applying the self-equilibrium

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stress to the cables and struts and is conditioned by its pre-stress state; therefore, calculating the pre-stress is the key step for any cable-struts structure. With respect to the basis of the tensegrity concept, Geiger first invented the cable dome, which includes a compressed ring in the boundary of a tensegrity structure, the so-called Geiger form.[4] The first cable dome was designed by Geiger for the Olympics in Seoul (1986), followed by the Redbrid Arena in Illinois (1988), the Florida Suncoast Dome in St. Petersburg (1988), the Taoyuan Arena in Taiwan (1993), and the oval plan Levy form of the cable dome for theOlympics in Georgia (1996). Subsequently, Levy improved the Geiger form and invented the Levy form, which includes tension-only cables, compression-only struts and a compressed ring. [6]

Examining the well-known tensegrity sculpture of Snelson, we can denot that it is very abstract, geometric, reduced to a set of simple basic elements, bars, and cables. Needless to say that at the time it was built theoretical investigation of tensegrity structures of this complexity was simply missing. Hence it is purely experimental. A major step in designing tensegrity structures is form-finding.[1] Form-finding refers to the process of determining special geometrical configurations that lead to, *at least*, a state of self-stress for tensegrity structures. Fuller (1962) and Snelson (1965) carried out early studies on the form-finding of regular tensegrity structures. The existing form-finding methods can be divided into two broad classes, analytical and numerical.[2] One of the simplest form-finding methods is undoubtedly the analytical method. This method has been used successfully for the form-finding of prismatic and cylindrical tensegrity structures.[1] However, the method is feasible only for structures with high orders of symmetry. As a general method, Pellegrino (1986) developed a nonlinear programming approach to the form-finding of tensegrity structures.[4] As a type of form-active structures, tensegrities need a form-finding process to determine their self equilibrated configurations in the absence of external loads. In general, the analytical form-finding methods are used for regular symmetric structures,e.g. prismatic tensegrities and truncated regular polyhedral tensegrities. For most tensegrities, the form-finding analysis can be made only by numerical methods.[2]

As a powerful tool for structural analysis, *the finite element method* has also been introduced for the form-finding of tensegrities, in which one generally needs to assume an initial shape close to the final solution. Recently inspired by the molecular dynamics method, Li proposed a Monte Carlo form-finding method to search the stable configuration of a tensegrity by employing the stochastic scheme of nodal displacements.[2] Pellegrino and Calladine presented a classic singular value decomposition (SVD) technique to obtain the independent self-stress modes and the independent displacement modes of the cable-struts structures. Considering cable dome with multiple self-stress modes, Yuan proposed a general method, referred to as double singular value decomposition (DSVD). SVD and DSVD are effective in finding the pre-stress modes when we know the reasonable structural geometry, but those methods cannot consider the structural deformation and loads. That is way form-finding is a celtal key for solving the problem of geometrical configuration.[4]

Nowadays, tensegrity structures are emerging as the ‘‘structural systems for the future’’ (Motro, 2003) and are perceived as potential solutions to many practical problems. In civil engineering, tensegrity structures have a relatively long history, having been proposed for various applications including shelters, domes (Fuller, 1962; Motro, 1990; Pellegrino, 1992), or bridges (Micheletti, Nicotra, Podio-Guidugli, & Stucchi, 2005 as well as in other fields e.g. aerospace engineering they are regarded as promising deployable structures, which will enable various applications like adaptive space telescopes, flight simulators, antennas, and robots, as well as in the field of biology as models for the structural mechanisms through which cells are organized and function.[6]

2. DESCRIPTION OF THE STRUCTURE

The main target of the study presented in this paper is to create a tensegrity structure for a cylindrical roof of 15-20 meters diameter. The analyses were performed on a half model of 400 mm diameter and 600 mm/2 = 300 mm length. The starting configuration presented in figure 1. was obtained by extending a cable-strut polygon in a three dimensional structure [7] . The analyses presented in the next section show that the prestressed structure is stable and can support loads. However as it can be observed in the figure 1, the structure cannot be used in practice, because it needs supports which must take the reactions on the direction of the generator of the cylindrical surface. (These reactions in the nodes 1:6 and 25:30 are given by the prestress and the external loads.)

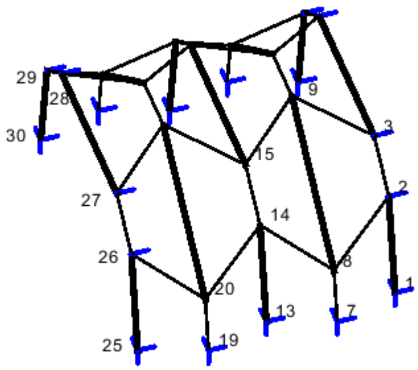

Figure 1. Cylindrical roof structure

In order to avoid the use of these horizontal supports, two longitudinal struts and two additional anchorage cables were added, as it can be seen in the figure 2. The configuration of this new structure was established by a form-finding process which will be described in the next sections.

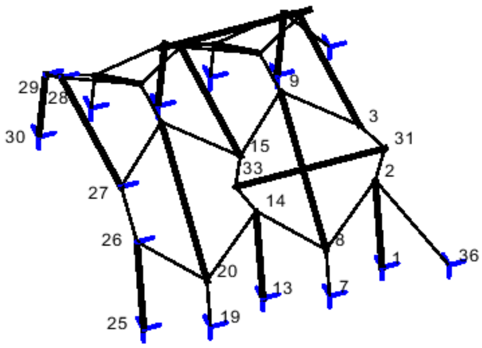

Figure 2. Cylindrical roof structure with longitudinal struts and additional anchorage cables

3. THE ANALYSIS OF THE STRUCTURE

The structure presented in figure 2 has n = 36 nodes, b = 49 truss elements (cables and struts) and a number r = 40 constraints given by the supports.
The equilibrium of a truss structure can be expressed by the equation

$$\mathbf{A\sigma} = \mathbf{F}. \tag{1}$$

Here **A** is the equilibrium matrix. In the case of a structure of n nodes, b trusses and r simple support constraints, matrix A has 3n lines and b+r columns, $\boldsymbol{\sigma} = (N_1\ N_2 \ldots N_b\ R_1\ R_2 \ldots R_r)^T$ is the vector of unknown internal forces in the trusses and in the supports and $\mathbf{F} = (F_{x1}\ F_{y1}\ F_{z1}\ F_{x2} \ldots F_{xn}\ F_{yn}\ F_{zn})^T$ is the vector of external nodal loads.

The deformations $\boldsymbol{\varepsilon}$ (the elongations of the trusses and the displacements of the supports) can be expressed by the equation:

$$\mathbf{A}^T\mathbf{a}=\boldsymbol{\varepsilon}. \tag{2}$$

Here $\mathbf{A}^T = \mathbf{B}$ is the compatibility matrix and $\mathbf{a} = (u_1\ v_1\ w_1\ u_2 \ldots u_n\ v_n\ w_n)^T$ is the vector of displacements of the nodes.
The matrix **A** can be formed by using the direction cosines of the axes of the truss elements.
Since in the case of the structure studied here $q = rang(\mathbf{A}) = 88 < min(3n, b+r) = 89$, the structure has a degree of $s = b+r - q = 1$ statical indeterminacy and has $m = 3n - q = 20$ kinematic degrees of freedom.

To the *m* degrees of freedom correspond a set of *m* independent vectors of displacements of nodes, which are modifying the configuration of the structure without producing defornations of the elements. These displacements can be obtained as the null space $\mathbf{a}_0$ of the matrix $\mathbf{B}=\mathbf{A}^T$. Kinematic systems can support only loads for which the vector **F** is orthogonal with the nullspace $\mathbf{a}_0$.
The statical indeterminacy means that the structure has s = 1 sets of selfstress efforts $\boldsymbol{\sigma}_0$, which satisfy the equilibrium equations without external loads.

$$\mathbf{A}\,\boldsymbol{\sigma}_0=\mathbf{0}. \quad (3)$$

These selfstress efforts can be obtained as the nullspace of the matrix **A.**
Tensegrity structures are prestressable and by prestress they get stiffness. Kinematic degrees of freedom are fixed such that the structure can support external loads.
The stiffening effect of the prestress efforts can be introduced by expressing the equilibrim of the deformed shape of the structure.

$$\mathbf{K}_g\mathbf{a}+\mathbf{A}\boldsymbol{\sigma}=\mathbf{F}. \quad (4)$$

In equation (4) $\mathbf{K}_g$ is the geometric stiffness matrix and takes into account the effect of the initial efforts on the change of the geometry of the structure. Matrix $\mathbf{K}_g$ is formed using the geometric characteristics and the prestress of the structure and does not depends on the mechanical characteristics (area of the section of the elements and elastic properties of the material) .
If the prestress efforts can give stiffness to the structure, matrix

$$\mathbf{Q}=\mathbf{a}_0^T\mathbf{K}_g\,\mathbf{a}_0 \quad (5)$$

is positive definite.

4. THE FORM-FINDING

The final configuration of the structure from figure 2 was obtined in the following manner: First an initial initial configuration was established in which the two longitudinal struts were of 188 mm length. These longitudinal longitudinal struts were removed and replaced by pairs of unit following forces representing the efforts in these elements. The equilibrium configuration of resulted kinematic system with m + 2 = 22 degrees of freedom was determined by solving the equilibrium and continuity equations

$$\begin{gathered}\mathbf{K}_g\mathbf{a}+\mathbf{A}\boldsymbol{\sigma}=\mathbf{F}\\ \mathbf{A}^T\mathbf{a}=\mathbf{0}.\end{gathered} \quad (6)$$

for displacements **a** and efforts **σ**. In equation (6) F contains only the pairs of following forces representing the efforts in the supressed longitudinal struts.

For solving system (6) an incremental-iterative algorithm was used. The geometric stiffness matrice $\mathbf{K}_g$ and the equilibrium matrice **A** were updated after each iteration. On the final converged configuration the length of the longitudinal stuts become 192.87 mm which is maximum for the given kinematic system, such that after reintroducing these elements the rank of the equilibrium matrix **A** becomes less with one unit. In other words the structure becomes once statically indeterminate. Verifiying relation (5) resulted that the prestressed structure is stable.
In table 1 are given the length of the cables and struts and in figure 3 is presented the physical model of the whole structure.

Table 1. Length of the struts and cables of the physical model

Element	example	Length (mm)	Total number
short strut	1-2	130	10
long strut	8-9	200	13
longitudinal strut	31-33	193	4
short cable 1	7-8, 9-10	69	14
long cable	2-8	102	32
short cable 2	2-31	41	8
short cable 3	14-33	38	8
anchorage cable	2-36	156	4

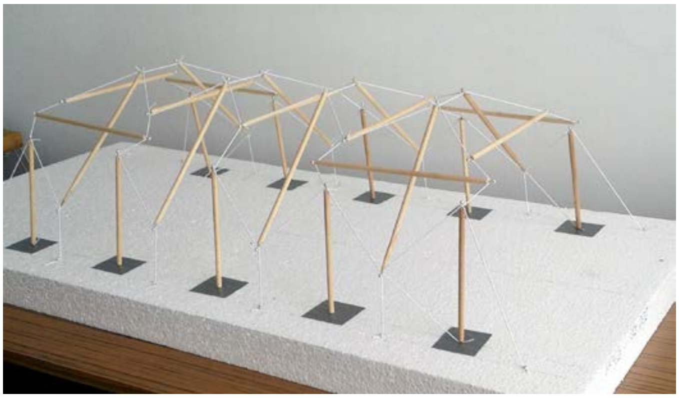

Figure 3. Physical model of the roof structure

Using the analyse and form-finding presented above, it becomes possible to get other more complexe structures. In figure 4 is presented the symmetric half of a cylindrical roof structure with seven elements in its transversal sections.

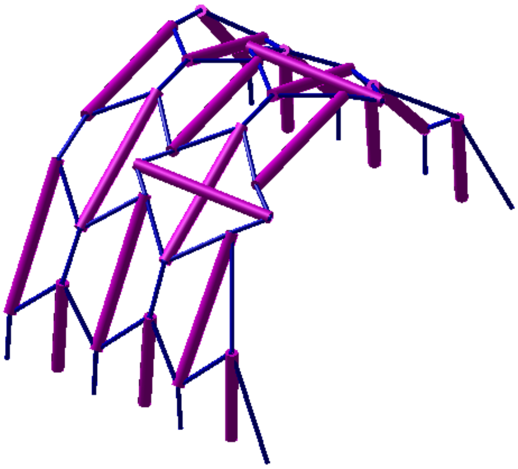

Figure 4. Cylindrical roof structure with 7 elements in the transversal section

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ПРОЈЕКАТ СТРУКТУРЕ ТЕНСЕГРИТИ ЗА ЦИЛИНДРИЈСКО КРОВАЊЕ

Резиме: *Структура студирана у папиру је бесконачно мањи механизам израђен од изолираних шипки повезаних кабловима. Ригидност конструкције обезбеђена је претенцијом каблова. Утврђивање облика равнотежног облика преднапете структуре извршено је коришћењем геометријског нелинеарног метода структуралне анализе.*

***Кључне речи*:** *Тенсегрити, цилиндрични кров, проналажење форми, структурна анализа*