

## **BENDING RESISTANCE OF COMPOSITE BEAMS WITH NONDUCTILE SHEAR CONNECTORS**

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**Summary:** In this paper, the methods for determination of the bending resistance of composite beams with partial shear connection and nonductile shear connectors, according to Eurocode 4, are studied. The Eurocode 4 predicts the use of two calculation methods: the method based on the nonlinear theory and the simplified method that assume linear relation between elastic and plastic bending moment resistance. The algorithm for the nonlinear section analysis is explained in detail. Using the own computer program for nonlinear section analysis, for one composite beam cross section, the differences in results obtained by these two proposed methods are discussed.

**Keywords:** composite beams, nonductile connectors, Eurocode 4, nonlinear analysis

### **1. INTRODUCTION**

The composite action between steel and concrete part of a steel-concrete composite beam is achieved using shear connectors placed at the interface between steel and concrete. According to Eurocode 4 (EC4) [1], bending resistance of a composite section can be determined in accordance with elastic analysis, rigid plastic analysis and nonlinear analysis.

In comparison with the elastic analysis, the rigid-plastic analysis is easier to apply in practice, while the non-linear analysis is not suitable for hand calculations and usually require computer software [2]. EC4 allows the rigid-plastic analysis to be applied in the section bending resistance calculation only when the considered cross-section is in Class 1 or 2. The section resistance is achieved i.e. the plastic hinge forms, when stresses in all fibers reach the limiting stress values:  $f_{yd}$  in steel part of a composite section,  $0.85f_{cd}$  in concrete part and  $f_{sd}$  in reinforcement (Figure 1(a)). The resulting moment is the plastic resistance moment  $M_{pl,Rd}$ .

The elastic analysis can be used for cross-sections of any class. The elastic resistance to bending ( $M_{el,Rd}$ ) corresponds to the stress distribution where stress in furthest steel, concrete or reinforcement fiber reaches its limiting value ( $f_{yd}$  in steel part of a composite section,  $f_{cd}$  in concrete part and  $f_{sd}$  in reinforcement) (Figure 1(b)). Commonly, it is the stress at the steel bottom flange that governs the  $M_{el,Rd}$  (see Figure 1(b)).

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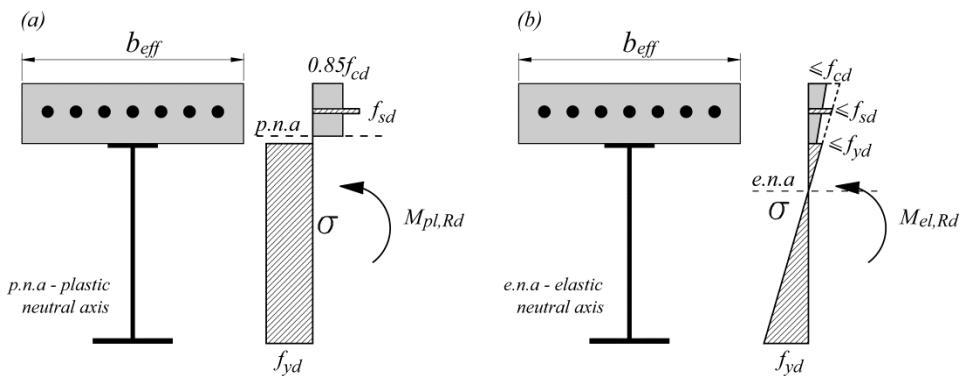


Figure 1. Stress distribution corresponding to:

(a) plastic resistance moment  $M_{pl,Rd}$  (b) elastic resistance moment  $M_{el,Rd}$

The section bending resistance, besides other data such as materials and cross-section properties, depends on the type of the used shear connectors and the design of the connection. EC4 makes distinction between the full and the partial shear connection and ductile and non-ductile connectors.

In full shear connection, the number of shear connectors is sufficient to achieve the full plastic bending resistance of composite section  $M_{pl,Rd}$  and further increase in number of shear connectors does not increase the bending resistance.

In the partial shear connection, the number of shear connectors is lower than required for the full plastic bending resistance  $M_{pl,Rd}$  to be achieved. Therefore, the bending resistance ( $M_{Rd}$ ) is reduced and is smaller than  $M_{pl,Rd}$ . Degree of shear connection  $\eta$  is defined as the ratio between the design value of the compressive force in the concrete slab ( $N_c$ ) and the design value of the compressive force in the concrete with full shear connection ( $N_{c,f}$ ).

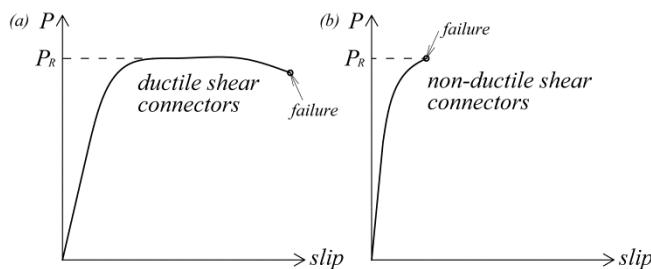


Figure 2. Real load-slip curve for (a) ductile and (b) non-ductile shear connectors

According to EC4, shear connectors are ductile when they possess the sufficient deformation capacity, in slip, to enable redistribution of shear forces in a critical length before fail (Figure 2(a)). Otherwise, shear connectors are non-ductile (rigid) and fail without significant slip when ultimate load is reached (Figure 2(b)). In Figure 2,  $P_R$  is the shear resistance of the connector which is often taken as the connector's

characteristic resistance. The design resistance  $P_{Rd}$  obtains dividing  $P_R$  with partial safety factor for connectors. In the current version of EC4, it is assumed that connectors are ductile if the characteristic slip capacity  $\delta_{uk}$  is at least 6 mm. The most widely used headed stud connectors, with diameter  $d$  between 16 mm and 25 mm and an overall length after welding of at least  $4d$ , can be considered as ductile if satisfy the minimal prescribed degree of shear connection. Otherwise, they are non-ductile. The ductility of other type of shear connectors needs to be proofed or, being on the safe side, they can be considered as non-ductile.

The bending resistance of full shear connection and on partial shear connection with ductile connectors is based on the rigid-plastic analysis and can be found in [3]. This paper is focused on the determination of the bending resistance of non-ductile shear connectors according to EC4.

## 2. NON-DUCTILE SHEAR CONNECTORS

As mentioned above, the ductile shear connectors do not possess significant deformation capacity and fail as soon as they reach its ultimate load  $P_R$ . Among others, the block connectors belong to this group and some of them are shown in Figure 3.

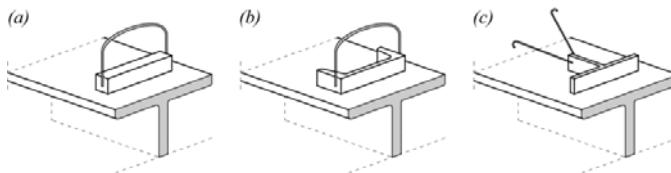


Figure 3. Block connectors:  
(a) hoop block connector, (b) [-channel connector (c) T-connector

The current version of EC4 gives guidelines for calculating the design value of the shear resistance of a single connector only for the headed stud connectors since they are most widely used. However, the previous version of EC4 considered also block connectors and gave expression for  $P_{Rd}$  calculation.

Oppose to the ductile shear connectors that can be uniformly spaced along the critical length since enable redistribution of longitudinal shear force over the length, the optimal design with the non-ductile connectors adopts the distribution of the shear connectors that is based on the distribution of the longitudinal shear force (commonly in practice, this distribution is determined from elastic analysis). For other distributions of shear connectors, the ultimate load is reached as soon as the longitudinal shear force on the heaviest loaded connector equals its resistance.

It should be mentioned that, according to EC4, when headed stud connectors do not satisfy any of the conditions for ductile connectors (for example, degree of shear connection is lower than prescribed), the connection should be designed as with non-ductile connectors [4].

### 3. RESISTANCE OF BEAMS WITH PARTIAL SHEAR CONNECTION AND NONDUCTILE CONNECTORS

When non ductile connectors are used in partial shear connection, EC4 proposes the nonlinear theory to be used. This means that nonlinear constitutive relations for steel, concrete and reinforcement need to be taken into account. For these, the relations given in Eurocode 2 [5] and Eurocode 3 [6] should be used. In addition, the nonlinear analysis can take into account the real load-slip behavior of connectors. However, this relation is not always available and such analysis is not suitable for practice. Therefore, EC4 allows ignoring completely the slip at the steel-concrete interface, which is on the safe side. In other words, the analysis based on the assumption of linear strain distribution over the composite cross-section is proposed. The pre-loading of the steel beam and the effects due to creep and shrinkage need to be taken into account since these factors influenced the ultimate load. This is a different than in beams with ductile shear connectors and full shear/partial shear connection and comes from the fact that behavior of shear connectors do not satisfy assumptions of the rigid-plastic analysis.

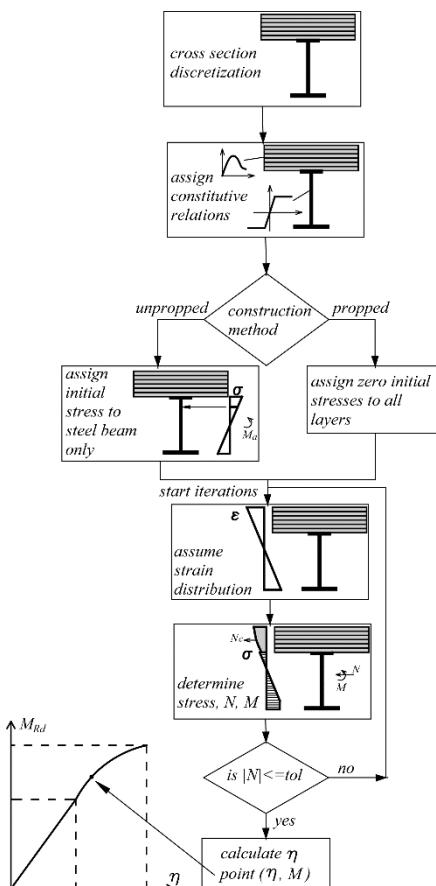


Figure 4. Algorithm for nonlinear section analysis

This approach based on the non-linear theory requires an iterative procedure and can be performed using the fiber-based section analysis. In the first step, the section is discretized into layers and a nonlinear constitutive stress-strain relation is assigned to each layer. In the second step the initialization of stresses is done, depending on the method of construction. For unpropped construction, stresses that correspond to the moment  $M_a$  acting on the steel beam alone, are assigned to steel beam layers. Stresses in other layers, as well as in the case of propped construction, are initialized to zero. In the third step the linear strain distribution over the height of the composite cross section is assumed. For this strain distribution, from known constitutive relations, stresses are determined and, by summation (i.e. integration) over the cross section, the corresponding axial force  $N$  and bending moment  $M$  are determined. Since for the desired ultimate state axial force is zero, the check whether this force is zero (i.e. less than a tolerance) is done. If this condition is fulfilled, the resulting force in the compressed part of the concrete slab is calculated and the corresponding degree of shear connection  $\eta$  is found. The point  $(\eta, M)$  represents one point on the curve shown in Figure 4. If the condition that the axial force  $N$  is smaller than tolerance is not fulfilled, the next iteration starts with new strain distribution. The chart flow for this procedure is shown in Figure 4. It is difficult to apply by hand calculation, and a computer program is required.

In order to calculate the curve for all degrees of shear connection, i.e. from 0 to 1, the strains on the top of the concrete slab need to include all states in the range from 0 to -3.5% (compression). As can be noticed from the Figure 4, the first part of the curve is linear, up to elastic bending resistance  $M_{el,Rd}$ . The following part, from  $M_{el,Rd}$  to  $M_{pl,Rd}$  is highly nonlinear and convex. Because of this convexity, EC4 allows the simplified design procedure that assume linear approximation of states between moments  $M_{el,Rd}$  to  $M_{pl,Rd}$ . In the next example, the differences between the two solutions are discussed.

#### 4. NUMERICAL EXAMPLE

In order to compare the bending resistance of a composite beam with non-ductile shear connectors determined by two methods proposed by EC4 and explained in Section 3, a small computer program for nonlinear section analysis is written.

The studied example includes a composite cross section from Figure 5 that consists of a concrete slab and a wide flange steel section. For simplicity, the reinforcement in the concrete slab is neglected. Data containing dimensions are given in Figure 5. The concrete slab is made of C30/37 concrete while steel section is made of S235 steel.

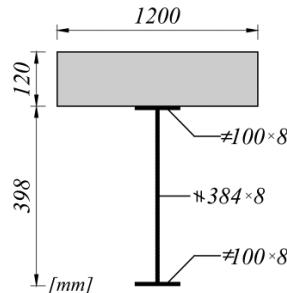


Figure 5. Composite beam cross section

In the nonlinear analysis method, the bilinear stress-strain relation for steel is adopted, as proposed by EC3. Concrete behavior is modeled with “parabola-rectangle” relation in compression and zero strength in tension, as proposed by EC4. The creep effects are taken into account by the modular ratio  $n=2n_0=12.7$ , while shrinkage effects are neglected. The resulting relations between moment resistance and degree of shear connection for nonductile shear connectors and propped construction method are shown in Figure 6. In addition, the nonlinear curve for ductile shear connectors, which is calculated applying the rigid plastic analysis, is plotted. As can be seen, the difference between the linear approximation and the nonlinear curve for the nonductile connectors is the greatest in the mid-range of the curve. For example, according to the nonlinear solution for nonductile connectors, the design bending moment of 270 kNm requires shear connection of 760 KN ( $\eta=0.69$ ) i.e., when linear approximation is used, 845 kN ( $\eta=0.77$ ). For the studied beam, this difference goes up to 12% which is not so significant. On the contrary, when the ductile shear connectors are used, much smaller degree of shear connection is required for the same design bending moment  $M_{Ed}$  ( $\eta=0.49$ ) and the strength of shear connection of 540 kN.

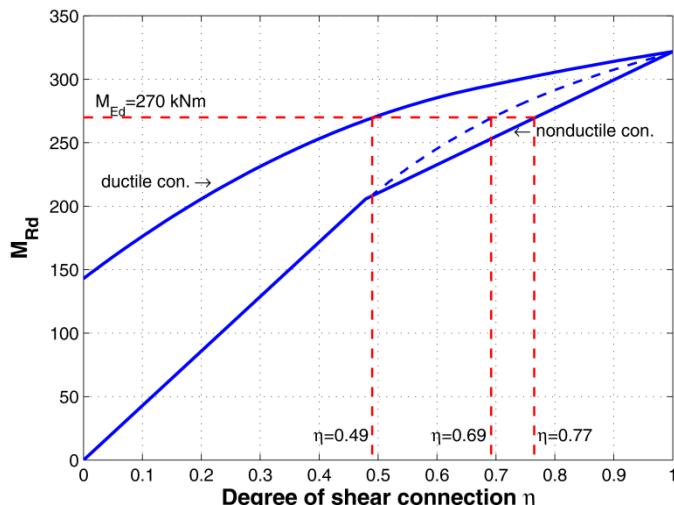
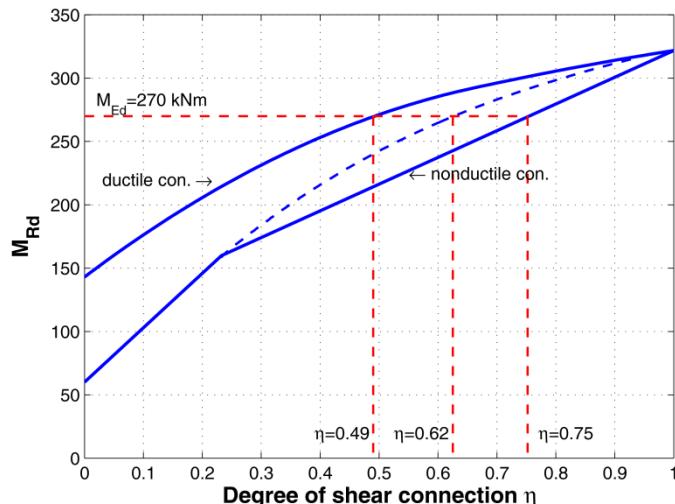


Figure 6. Bending moment – degree of shear connection relation for ductile and nonductile connectors (propped construction)

The results of analysis for nonductile connectors and unpropped construction method are shown in Figure 7. Also, the relation for ductile shear connectors is plotted. It is assumed that moment acting on the steel beam alone has a value of  $M_{a,Ed} = 60$  KNm. As can be seen from Figure 7, the differences between nonlinear solution and the linear approximation for the bending moment resistance – degree of shear connection relation are slightly greater in this case than for propped construction method and go up to 25%. For the marked design bending moment of 270 kNm, this difference is 20%. According to the nonlinear solution for nonductile connectors, this design bending moment requires shear connection of 680 KN ( $\eta=0.62$ ) i.e., when linear approximation is used, 823 kN.

( $\eta=0.75$ ). Since the resistance with ductile connectors, does not depend on the construction method, the required degree of shear connection ( $\eta=0.49$ ) and the strength of shear connection (540 kN) stay the same as before.



*Figure 7. Bending moment – degree of shear connection relation for ductile and nonductile connectors (unpropped construction)*

## 5. CONCLUSION

The determination of the bending resistance of composite beam in partial shear connection when non-ductile shear connectors are used, according to EC4, is explained. The code predicts the use of two different methods for calculation: the method based on the non-linear theory and the simplified method, based on the linear interpolation. The first method requires a computer program and the algorithm for the section nonlinear analysis is explained in detail. The second method is quick, easy and suitable for hand calculation. The numerical example analyses the differences in results obtained by these two methods for one composite beam cross section. In this example, the differences between the required strength of the shear connection for propped construction method are below 15% and, so, are not very significant. In this case, comparing the necessary calculation time and savings, the use of simplified method seems reasonable. However, in the studied example with unpropped construction method, the differences are bigger and go up to 25%. Therefore, in unpropped construction method, there are some benefits from the use of nonlinear analysis method in design.

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## **6. МЕЂУНАРОДНА КОНФЕРЕНЦИЈА**

**Савремена достигнућа у грађевинарству 20. април 2018. Суботица, СРБИЈА**

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## **НОСИВОСТ НА САВИЈАЊЕ СПРЕГНУТИХ ГРЕДА СА НЕДУКТИЛНИМ МОЖДАНИЦИМА**

**Резиме:** У раду су приказане методе прорачуна носивости на савијање спрегнутих греда са недуктилним можданицима према Еврокоду 4. Овај стандард предвиђа две методе прорачуна: методу засновану на примени нелинеарне анализе и поједностављену методу код које се препоставља линеарна релација између степена спрезања и момента носивости, за моменте веће од еластичног момента носивости. Објашњен је детаљно алгоритам прорачуна спрегнутог пресека према нелинеарној анализи. Користећи сопствени компјутерски програм, за један попречни пресек греде, анализоране су разлике у носивости на савијање добијене применом поменуте две методе.

**Кључне речи:** Спрегнуте греде, недуктилни можданици, Еврокод 4, нелинеарна анализа