RHEOLOGICAL-DYNAMICAL PARAMETERS FOR CONCRETE USING NON-DESTRUCTIVE TESTING

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Summary: *Non-destructive test methods are a valuable tool in determining the parameters of materials, because on the same samples can be repeated several times. This is of particular importance in structures made of materials whose properties change over time, such as concrete, reinforced concrete and pre-stressed structures. Determination of the dynamic modulus of elasticity and the resonant frequency of the acoustic response gives adequate data, on which the rheological-dynamical analogy is based. Analogy combines dynamic rheological model and damage mechanics. Within this approach are functionally related key parameters of the continuum, such as Poisson's ratio and the scalar damage variable. Ultimate strain is determined by the secant stress-strain relation from fracture mechanics. The result of this analog modeling is working diagram of concrete, which is in this paper determined as average, based on several cylindrical samples of concrete tested by non-destructive methods. Thus established working diagrams are used to analyze the rotational capacity of reinforced concrete beam in bending.*

Keywords: *Reinforced concrete structures, rheological-dynamical analogy, non-destructive test methods, model of concrete*

1. INTRODUCTION

Reinforced concrete materials have been studied and employed in diverse fields of science and engineering disciplines due to their wide application in infrastructure in many countries. From a practical standpoint, the ultimate limit state design of reinforced concrete elements brought the stress-strain relationship into focus. The compression response of concrete, and in particular the ascending branch, compressive strength, ultimate strain and post-peak regime, have an important role in the design of concrete and concrete-based structures.
Dynamic structural analysis is increasingly important for civil structures. In this context, non-destructive tests are a promising tool, as they allow obtaining integrated and comprehensive information about structure stiffness and damping and, moreover, may be repeated and compared over time. The determination of the modulus of elasticity of

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concrete by way of its acoustic response represents a methodology to obtain a design parameter which, associated with the compressive strength, achieves the guidelines for the design of structural elements in plain, reinforced and prestressed concrete.
The discussion of instantaneous deformations of concrete under load is timed from a theoretical viewpoint because deformations provide indirect information concerning the internal structure as well as the microscopic fracture mechanism. Analytical models of time-dependent stress-strain response of concrete under compression are required. For global failure analysis, the failure mechanism must be treated in a smeared manner, as a continuum with damage. Therefore, an analytical model for the study of concrete under compression was developed by the first author [1]. This approach, referred to as the rheological-dynamical continuum damage model, combines rheological-dynamical analogy and damage mechanics. The model relies on global parameters to connect pre-peak branch with post-peak regime. The analytical stress-strain curve of concrete at the ascending branch can be computed if the compressive strength, elastic modulus, concrete density and Poisson's ratio are experimentally evaluated. The model was verified on experimental results for five concrete compositions using the load-controlled tests.
In the paper [2] a new rheological-dynamical continuum damage model for concrete under compression was further used to research the behaviour of reinforced concrete beams subjected to bending. The numerical predictions regarding moment-curvature and ductility of a reinforced concrete beam was presented for the above mentioned five concrete compositions, demonstrating capabilities of a new analytical model.
In this paper a new rheological-dynamical continuum damage model for concrete under compression was tested using non-descructive experimental results given in [3].

2. AVERAGE RDA STRESS-STRAIN CURVE UNDER COMPRESSION

The propagation of mechanical waves (or stress waves) with transition from the short-time modulus of elasticity (E_D) to the long-time one (E_H) represents a physical basis for the analogy between two different physical phenomena, the rheological and the dynamical. Generally speaking, the RDA is derived in order to solve dynamic problems, but it can be used in the analysis of quasi-static loading considering the corresponding limit values of the derived analytical expressions. For instance, each stress-strain curve of a specimen can be obtained using the RDA modulus function, including the compressive strength σ_{crF} and ultimate strain ε_{crF}. Hence, the RDA modulus function is used in [1] to obtain the quasi-static stress-strain curve, as follows

$$\sigma_{cr} = \frac{1}{2K_E}\left(\sqrt{1+4K_E E(0)\varepsilon} - 1\right), \qquad E(0) = E_H\left(1+\varphi^*\right). \tag{1}$$

Slope $E(0)$ is the elastic modulus of the material in its initial state. K_E is the structural-material constant and φ^* is the structural creep coefficient at the limit of elasticity, as discibed in [1] (μ is Poisson's ratio).

$$\varphi^* = \frac{2\mu}{1-2\mu}. \tag{2}$$

Since the development of micro cracks induces a reduction in the stiffness of materials, the damage state can be characterized by variation in the long-time modulus of elasticity

[4]. Thus, the damage variable D is characterized by variation in Young's modulus $E(D)$, as follows

$$E(D)=(1-D)E_H,\ \ 0\le D\le 1. \tag{3}$$

On the other hand, the RDA modulus function of time also induces a reduction in the stiffness of materials. Because of that we can suppose

$$E(D)=(1-D)E_H=E_R=\frac{1+\varphi_{cr}+\delta^2}{(1+\varphi_{cr})^2+\delta^2}E_H\ ,\quad \delta=T_K^D\omega_\sigma\,. \tag{4}$$

T_K^D is the dynamic time of retardation and ω_σ is the load frequency. In the case of quasi-static loading ($\delta\to 0$) we obtain the critical damage variable

$$D_{cr}=\frac{\varphi_{cr}}{1+\varphi_{cr}}\,,\qquad\qquad \varphi_{cr}=\sigma_{cr}K_E\,. \tag{5}$$

The stress-strain curve under compression, Fig. 1, is really based on a transition from the modulus E_D (dynamic modulus) to the modulus E_H.

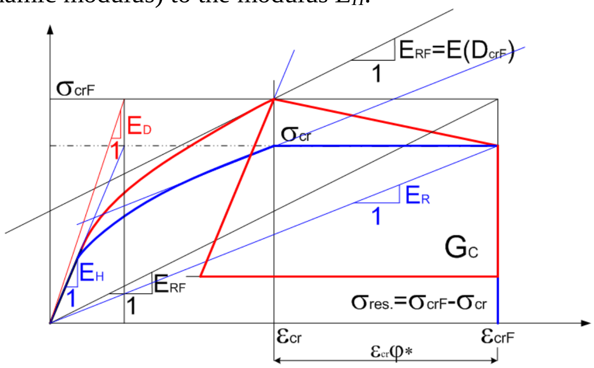

Figure 1. Stress-strain curve of concrete under compression

Dynamic modulus may be found by using a non-destructive test relating the resonant frequency f of the first longitudinal mode, length l_0 of the concrete cylinder with free ends and density ρ of the material, according to the equation

$$E_D=4\rho l_0^2 f^2,\qquad\qquad v_L=2l_0 f=\sqrt{E_D/\rho}\,. \tag{6}$$

The Young's modulus E_H (named also as long-time modulus) is defined as the ratio of the axial stress to axial strain for a material subjected to uni-axial load. It is important that E_H of concrete be known because engineers increasingly use this value in the structural design proces. However, once a structure is erected the in situ elastic properties cannot be measured directly without damaging the structure itself. Most often E_H is inferred from the compressive strength f_c of companion cylinders, rather than being measured directly, through the application of established empirical relations.

Theoretically, the three-dimensional (3D) stress-wave velocity must be used to find the modulus E_D. However, due to the bar-like shape of the specimens (i.e., l_0/□$_0$=2 for cylinders), there is a clue that 3D stress-wave velocity should be equal to the one-dimensional speed v_L. Hence, the measured wave velocity v_L is related to the material elastic constants of concrete (E_c=E_H and □=□$_D$) using an equation based on Hooke's law for 3D isotropic materials

$$E_H = \frac{v_L^2 \rho (1+\mu_D)(1-2\mu_D)}{1-\mu_D}, \tag{7}$$

where μ_D is the dynamic Poisson's ratio. Thus, if we adopt

$$\Psi = \frac{E_H}{E_D}, \qquad 0 \le \Psi \le 1, \tag{8}$$

the following theoretical expression for μ_D yields

$$\mu_D = \frac{(\Psi - 1) + \sqrt{\Psi^2 - 10 \cdot \Psi + 9}}{4}. \tag{9}$$

Consequently, the elastic modulus in initial state $E(0)$, may be defined only according to the mathematical description. Similarly, □$_S$ and □$_S$:

$$E(0) = \frac{v_L^2 \rho (1+\mu_S)(1-2\mu_S)}{1-\mu_S} \approx E_D, \qquad \varphi_S = 1/\Psi - 1, \qquad \mu_S = (1-\Psi)/2 \tag{10}$$

.

where μ_S is the initial Poisson's ratio and φ_S is the initial creep coefficient.

3. RDA ANALYSIS USING PUBLISHED DATA

The RDA parameters of standard concrete cylinders was computed from the information provided by the methodology developed by Diógenes at all. [3]. Table 1 gives measured values as presented in [3].

Table 1. Measured parameters as presented in [3]

specimen	E_c [GPa]	$E_{c,d}$ [GPa]	□□$E_c/E_{c,d}$	□□[kg/m^3]	□$_D$	□$_S$
CP 01	31,53	41,59	0,758	2440,07	0,293	0,121
CP 02	36,51	41,56	0,878	2455,26	0,218	0,061
CP 03	31,42	40,91	0,768	2430,65	0,287	0,116
CP 04	37,16	41,33	0,899	2423,68	0,201	0,050
CP 05	35,69	41,10	0,868	2435,95	0,226	0,066
CP 06	34,72	41,98	0,827	2471,36	0,254	0,086
AVERAGE	**34,51**	**41,565**	**0,830**	**2442,83**	**0,252**	**0,085**
BS	32,95	41,565	0,793	2442,83	0,274	0,104
Lyndon	34,57	41,565	0,832	2442,83	0,251	0,084

Table 2. RDA parameters obtained by the dynamic Poisson's ratios

specimen	f_c =□$_{crF}$	□$_{cr}$	□$_c$=□$_{cr}$	□$_{cu}$=□$_{crF}$	G_C [N/mm]	f_c/G_C
CP 01	51,14	38,77	0,00084	0,00203	18,50	2,76
CP 02	42,57	37,40	0,00114	0,00203	14,67	2,90
CP 03	46,19	35,48	0,00078	0,00184	15,47	2,99
CP 04	49,05	44,10	0,00166	0,00277	21,61	2,27
CP 05	50,84	44,15	0,00144	0,00263	22,44	2,27

CP 06	51,97	42,98	0,00114	0,00232	21,41	2,43
AVERAGE	**48,63**	**40,38**	**0,00108**	**0,00217**	**18,79**	**2,59**
BS	48,63	38,55	0,00091	0,00201	17,65	2,76
Lyndon	48,63	40,45	0,00109	0,002181	18,83	2,58

Table 2 gives the RDA parameters of concrete cylinders calculated by dynamic Poisson's ratio. Similarly, the RDA parameters calculated by initial Poisson's ratio are presented in Table 3.

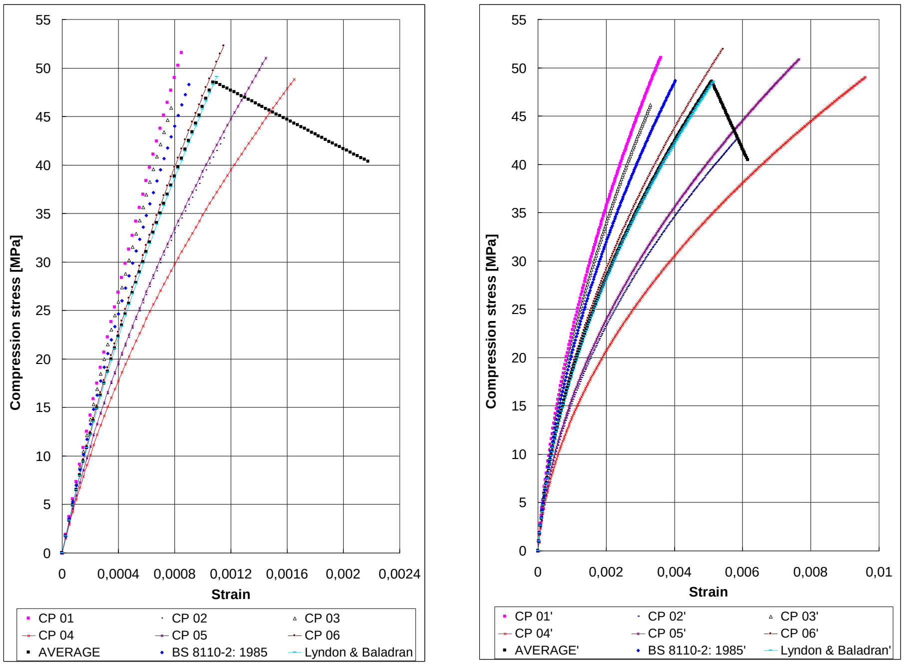

Figure 2. Stress-strain curves calculated by RDA modeling techique: (left) using dynamic Poisson's ratios, (right) using initial Poisson's ratios.

Table 3. RDA parameters obtained by the initial Poisson's ratios

specimen	$f_c = □_{crF}$	$□_c = □_{cr}$	$□_{cu} = □_{crF}$	G_C [N/mm]	f_c/G_C
CP 01	51,14	0,00361	0,00477	18,42	2,78
CP 02	42,57	0,00581	0,00662	14,14	3,01
CP 03	46,19	0,00331	0,00430	15,03	3,07
CP 04	49,05	0,00960	0,01068	21,31	2,30
CP 05	50,84	0,00764	0,00880	22,37	2,27
CP 06	51,97	0,00543	0,00657	21,10	2,46
AVERAGE	**48,63**	**0,00511**	**0,00616**	**18,24**	**2,67**
BS	48,63	0,00402	0,00507	17,14	2,84
Lyndon	48,63	0,00516	0,00621	18,29	2,66

4. ROTATIONAL CAPACITY OF REINFORCED CONCRETE BEAM USING RDA STRESS-STRAIN CURVES

By the methodology explained in [2], using the average RDA stress-strain curves shown in Fig. 2 for concrete, for both dynamic and initial Poisson's ratios, and bilinear elastic-perfectly plastic stress-strain relationship for steel reinforcement with yield strength f_y=400 MPa, and modulus of elasticity E_s=200 GPa, the rotational capacity and the ductility of rectangular reinforced concrete beam in pure bending are calculated for various reinforcement ratios. The calculation is based on assumption that plane cross-sections remain plane. The influence of reinforcement in compression, the effect of confinement due to stirrups and the contribution of the concrete in tension are neglected. Concrete properties used in calculations (dynamic $E_{c,d}$ and initial E_c modulus of elasticity, peak stress □$_{cr}$, strain □$_c$=□$_0$ at the peak compressive stress and the ultimate strain □$_{cu}$ of concrete) are shown in Table 1, and Tables 2 and 3 for dynamic and initial Poisson's ratios respectively. Equilibrium of internal forces and compatibility of deformations at the state of impending failure are shown in Fig 3.

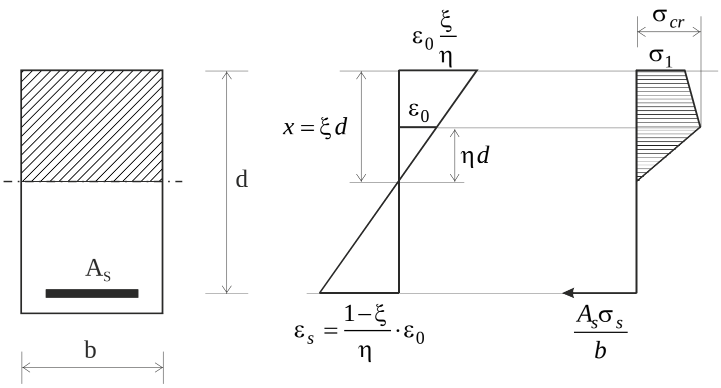

Figure 3. Strain and stress distribution in singly reinforced rectangular cross-section in pure bending

Reinforcement ratio is varied between the minimum and maximum value, according to Eurocode 2, and took values A_s/bd=0.0013, 0.005, 0.01, 0.02, 0.03 and 0.04, including overreinforced value 0.05, where A_s is the cross sectional area of tensile reinforcement, while b and d are the width and the effective depth of the beam, respectively. Dimensionless bending moment-curvature relations for dynamic and initial Poisson's ratios are shown in Fig. 4. Dimensionless curvature 1/□ is obtained from curvature □

$$\kappa = \frac{1}{r} = \frac{\varepsilon_0}{\eta d} \quad \Rightarrow \quad \frac{1}{\eta} = \frac{d}{\varepsilon_0} \cdot \frac{1}{r} \,. \tag{11}$$

Fig. 4 shows that the higher bending moment (or higher load level) is required in order to reach the same curvature if the initial Poisson's ratio instead of the dynamic is used in calculations. Rotational capacity, or the ultimate curvature, is higher in case of using dynamic Poisson's ratio.

From Fig. 4 it can also be seen that at the maximum reinforcement ratio of 0.04 the ultimate strain of concrete in compression is reached before the reinforcement begin to yield, causing the undesirable brittle fracture of cross-section.

The ductility in bending is expressed in two ways: as the ratio between the ultimate curvature $\square_u$ and the curvature $\square_{sy}$ at which reinforcement begin to yield (Fig. 5 left) and as the ratio between $\square_u$ and the curvature $\square_0$ at which the peak compressive stress of concrete is reached (Fig. 5 right). As it was expected, the higher difference between ductility calculated with initial and dynamic Poisson's ratio is find for $\square_u/\square_0$, particularly for lower reinforcement ratios (Fig. 6). In this case "dynamic" ductility is from 7.4 to 15.4% higher then "initial".

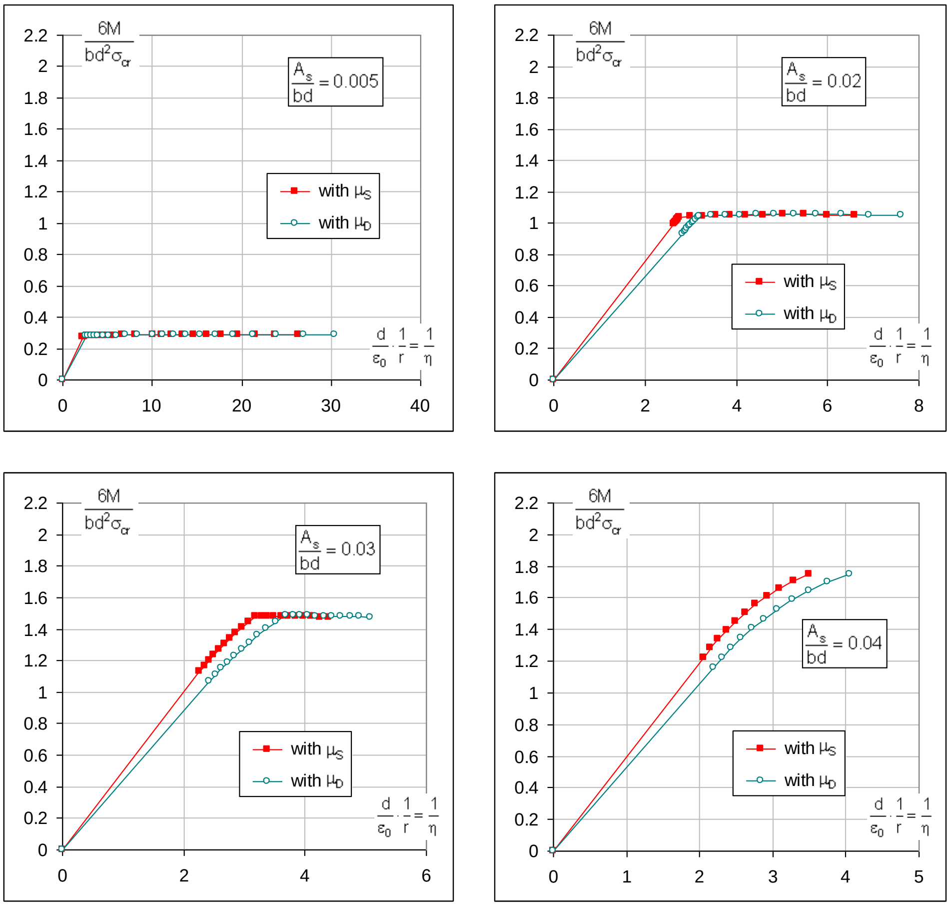

Figure 4. Dimensionless bending moment-curvature relations for various reinforcement ratios

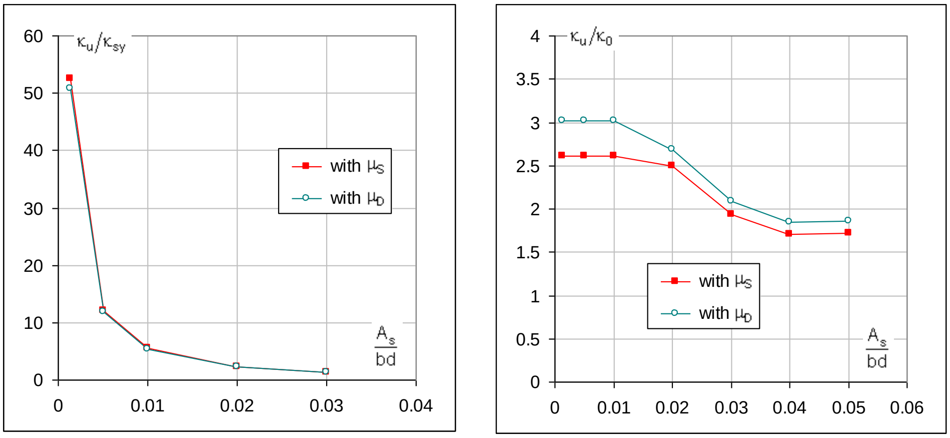

Figure 5. Ductility relative to beginning of reinforcement yielding (left) and relative to reaching the peak compressive stress of concrete (right)

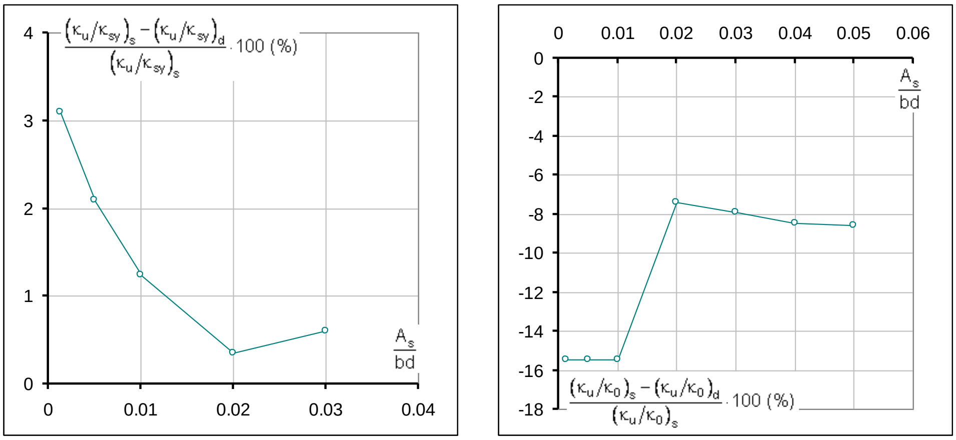

Figure 6. Difference between ductility calculated with initial and dynamic Poisson's ratios: relative to begining of reinforcement yielding (left) and relative to reaching the peak compressive stress of concrete (right)

Fig. 7 shows that ductility of reinforced concrete beam is much more dependent on reinforcement ratio than on adoption of initial or dynamic Poisson's ratio for calculation of stress-strain curve of concrete in compression.

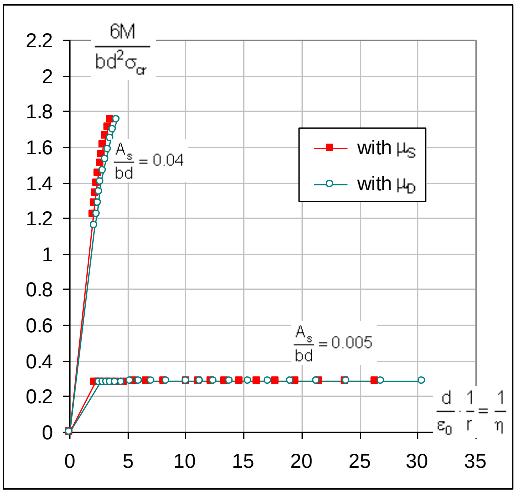

Figure 7. Dependance of bending moment-curvature relation on reinforcement and Poisson's ratios

5. CONCLUSIONS

In the numerical analysis of concrete structures, subjected to dynamic loading conditions, three moduli of elasticity of concrete are utilized. They are static modulus E_c=E_H, dynamic modulus $E_{c,d}$=E_D and sustained modulus which accounts for the long-term creep effect of concrete E_R. Among them, the most commonly used is the static which defines the stress-strain relations for concrete under static loads. It is determined according to the procedures specified in codes. By comparison, the dynamic modulus of elasticity is the ratio of stress to strain under vibratory conditions, and it is a key parameter for the structural analysis of concrete structures under dynamic conditions like seismic loadings. The RDA modelling technique may calculate stress-strain curves for concrete in compression for different input parameters using only the mathematical description between rheological and dynamical models. In this paper, two resulting average curves, one calculated with initial ("initial" curve) and second with dynamic Poisson's ratio ("dynamic" curve), are used to calculate rotational capacity and ductility of steel reinforced concrete beam in bending for various reinforcement ratios.

For all reinforcement ratios, "dynamic" curves give higher ultimate curvature and higher bending moment for the same rotation, than the "initial" curves. Ductility, expressed as a ratio between ultimate curvature and curvature at the beginning of yielding of reinforcement, is slightly higher for "initial" curves, but the difference decreases with the increase of reinforcement ratio.

Dimensionless bending moment-curvature relations for two RDA curves may be considered as limits between which is situated the relation established according to concrete parameters given in Eurocode 2. The influence of adopted modulus of elasticity on ductility and rotational capacity is far smaller than the influence of reinforcement ratio.

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РЕОЛОШКО-ДИНАМИЧКИ ПАРАМЕТРИ БЕТОНА ДОБИЈЕНИ НЕДЕСТРУКТИВНИМ МЕТОДАМА

Резиме: *Недеструктивни поступци испитивања су драгоцено средство за утврђивање параметара материјала, јер се на истим узорцима могу више пута поновити. Ово је од посебног значаја код конструкција изведених од материјала чија се својства мењају током времена, као што су бетонске, армиранобетонске и претходно напрегнуте конструкције. Одређивање динамичког модула еластичности и резонантне фреквенције акустичним одзивом даје адекватне податке, на којима се управо базира реолошко-динамичка аналогија. Аналогија комбинује реолошко динамички модел и механику оштећења. У оквиру овог приступа функционално су повезани кључни параметри континуума, као што су Поасонов коефицијент и скаларна променљива оштећења. Ултимативна деформација се одређује помоћу секантне везе напон-деформација из механике лома. Резултат аналогног моделирања је радни дијаграм бетона, који је у овом раду утврђен као осредњени на основу више цилиндричних узорака бетона испитаних недеструктивним методама. Овако установљен радни дијаграм је употребљен за анализу ротационог капацитета армиранобетонских греда изложених савијању.*

Кључне речи: *Армиранобетонске конструкције, реолошко-динамичка аналогија, недеструктивни поступци испитивања, модел бетона*