ULTIMATE STRENGTH OF LONGITUDINALLY STIFFENED PLATE GIRDERS UNDER COMPRESSION

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Summary: *In this paper a unified frame for quasi-static and dynamic inelastic buckling and ultimate strength of uniformly compressed longitudinally stiffened plate girders is presented. The finite strip method is used in structural analysis. The nonlinear behavior of the material is modelled using the rheological-dynamical theory. According to this theory, a very complicated nonlinear problem in the inelastic range of strains is solved as a simple linear dynamic one. The orthotropic constitutive relations for inelastic buckling and a new modulus iterative method for the solution of nonlinear equations are derived in previous papers and the extensive numerical application is presented here.*

Keywords: *Finite strip method, rheological-dynamical theory, ultimate strength*

1. THEORETICAL BACKGROUND

The purpose of this paper is to investigate ultimate limit state (*ultimate strength*) of uniformly compressed plate girders with longitudinally stiffeners under quasi-static and dynamic loading. These are the structures which are generally made by joining flat plates at their edges. Some important subsets of these systems are those composed of structures with essentially prismatic form, with or without stiffeners, such as the ones used in column members, stiffened slabs and box girders. Analysis of the behavior of these structures is here performed using the finite strip method (FSM). The FSM approximation of displacement field is based on beam eigenfunctions, which are derived as the solution of the differential equation of beam transverse vibration, and proved to be an efficient tool for analyzing a great deal of structures for which both geometry and

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material properties can be considered as constant along a main direction. This method was pioneered by Cheung [1], who combined the plane elasticity and the Kirchhoff plate theory. Wang and Dawe [2] have applied the elastic geometrically nonlinear FSM to the large deflection and post-overall-buckling analysis of diaphragm-supported plate structures. Also, the FSM is very rapidly increasing in popularity for the analysis of thin-walled structures. Kwon and Hancock [3] developed the spline FSM to handle local, distortional and overall buckling modes in post-buckling range. Interaction of two types of column failure (buckling) in thin-walled structures, local and global (Euler) column buckling, may generate an unstable coupled mode, rendering the structure highly imperfection sensitive. The geometrically nonlinear harmonic coupled finite strip method (HCFSM) [4, 5, 6] is also one of the many procedures that can be applied to analyze the large deflection of folded-plate structures and buckling-mode interaction in thin-walled structures. For these problems, only geometrically nonlinear terms such as square derivatives of transverse displacement *w* need to be included (von Karman approach). An analysis of the buckling-mode interaction is carried out using the HCFSM in [7], taking into account the visco-elastic behavior of material.

If uniformly compressed plate girders or thin-walled girders undergo inelastic deformation, these structures generally sustain both nonlinearities, geometrically nonlinear effects and a nonlinear behavior of the material caused by inelastic deformation. A mathematical-physical analogy named the rheological-dynamical analogy (RDA) has been proposed in explicit form to predict a range of inelastic and time-dependent problems related to one-dimensional members, such as buckling, fatigue etc. [8, 9]. Consequently, the RDA inelastic theory enables the engineer concerned with materials (*for various quasi-static and dynamic structural problems*) to utilize simple models, expressible in a mathematically closed form, to predict the stress-strain behavior. The main results in the paper [8] are obtained in regard to inelastic buckling in the short to intermediate column range taking into account the governing RDA modulus. However, wide-flange column members or thin-walled girders fail as continua by first developing local or global buckling modes, which may develop into plastic mechanisms and failure, which is why two-dimensional (2D) or three dimensional (3D) analyses must be used.

The proposed approach combines the RDA and damage mechanics [10] to solve the nonlinear problem of plate girders under compression using 2D analysis in the frame of the FSM. The one-dimensional RDA modulus is used to obtain one simple continuous modulus function and a stress-strain curve [11]. When the critical stress exceeds the limit of elasticity, the first iteration of the modulus provides the Hencky loading function and the von Mises yield stress, whereas the next ones involve the strain-hardening of the material through visco-plastic flow. At the end of the iterations the member failure occurs. The key global parameters, such as the creep coefficient, Poisson’s coefficient and the damage variable are functionally related. However, it is a fact that material damage growth is accompanied by an emission of elastic waves which propagate within the bulk of the material [12]. Consequently, a 3D analysis of the propagation of mechanical waves is used in this paper. The elastic properties of steel and aluminum determined on test cylinders and based on longitudinal resonance frequencies [13], are used in the numerical applications. For the analysis of plate girders using the FSM, an inelastic isotropic 2D constitutive matrix is derived starting from the one-dimensional state of stress. Although the quasi-static and dynamic constitutive relations are derived for isotropic materials, different stress components induce orthotropy in the material through the RDA modulus-stress dependence. The nonlinear term is the stiffness matrix, which depends on the inelastic orthotropic constitutive matrix. Because of that, a new modulus iterative method for the solution of nonlinear equations is used. In the case of inelastic buckling of rectangular slabs it has been demonstrated that convergence of the method is fast and that it gives satisfactorily accurate solutions with only several iterations [14]. Presented numerical algorithm is implemented in software package BASS [15] and the exhaustive numerical study is performed. Obtained solutions for known modulus are compared with the ones from CUFSM [16], and they excatly match.

2. NUMERICAL APPLICATION

A theoretical investigation into the effectiveness of a stiffener against the ultimate strength of a stiffened plate girders under thrust is carried out. The transition from the various buckling modes by changing the plate/stiffener proportions for various stiffening configurations is shown. Four models are analyzed for two different materials, steel and aluminum. For each material, plate without stiffeners is analyzed first. Then, plate girders with one, two and three stiffeners are examined and buckling curves are given.
Series of the buckling analyses, the elastic, visco-plastic (VP) and failure (ultimate strength) are performed on the stiffened plate girders under quasi-static and dynamic loading. The panels (*Model 1*, *2* and *3*) were divided into finite strips as shown in Fig. 1.

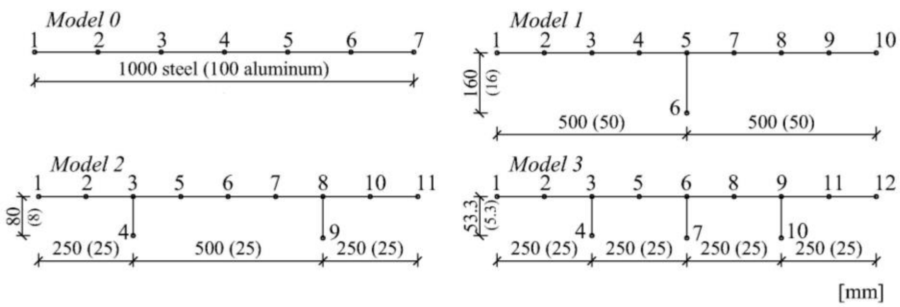


Figure 1. Models of steel and aluminum stiffened panels with nodal lines

Figures 2 and 3 illustrate the buckling curves (critical stress versus length/width ratio, a/b) for steel and aluminum slabs. In order to obtain the inelastic quasi-static critical stresses, the Euler formula for buckling of an isolated plate strip was employed to find the structural-material constant of a plate. The convergence of the failure stresses for all a/b ratios is obtained using only six or seven iterations. The first iteration gives the visco-plastic yield stress. An excellent agreement with the generalized beam theory (GBT) is observed [14], in which the values of both ratios E/E_T and E/E_S (E_T is the unixial tangent modulus and E_S is the secant modulus) depend on the applied stress level and are obtained using the uniaxial stress-strain law which adequatly describes the material behavior along the fundamental path. The GBT theory used the Ramberg-Osgood curve. The dynamic visco-plastic and failure buckling curves of steel and aluminum slabs for relative angular frequencies (RAF) 1 and 10 are also presented. All dynamic stresses are below the elastic critical stresses. The reason for that is the cyclic stress variation in the material under which the visco-plastic effects like viscous damping are developed.

Figures 4 to 9 present the quasi-static elastic, visco-plastic and failure buckling curves for three panels (*Model 1*, *2* and *3*) made of steel and aluminum.

For buckling of a stiffened plate (panel), it is well known that there exists a minimum stiffness ratio of a stiffener to the plate, $(EIs/bD)B_{min}$, which gives the maximum limiting value of the buckling strength. Considering the ultimate strength it was confirmed that there exists a significant stiffness ratio of a stiffener to the plate, $(EIs/bD)U_{min}$, similar to $(EIs/bD)B_{min}$ for the buckling strength.

The effect of various parameters like panel geometry, stiffening scheme, stiffener size and position are considered in quasi-static and dynamic stability analysis of stiffened panels. Figure 10 presents typical buckling modes.

The proposed appoach for quasi-static and dynamic inelastic buckling and for global failure analysis combines the FSM linear stability analysis and the RDA inelastic theory. The RDA inelastic theory is a new theory for the simulation of inelastic material behavior, alternative to other theory based on nonlinear fracture mechanics, plasticicity theory or damage mechanics previously published in the literature.

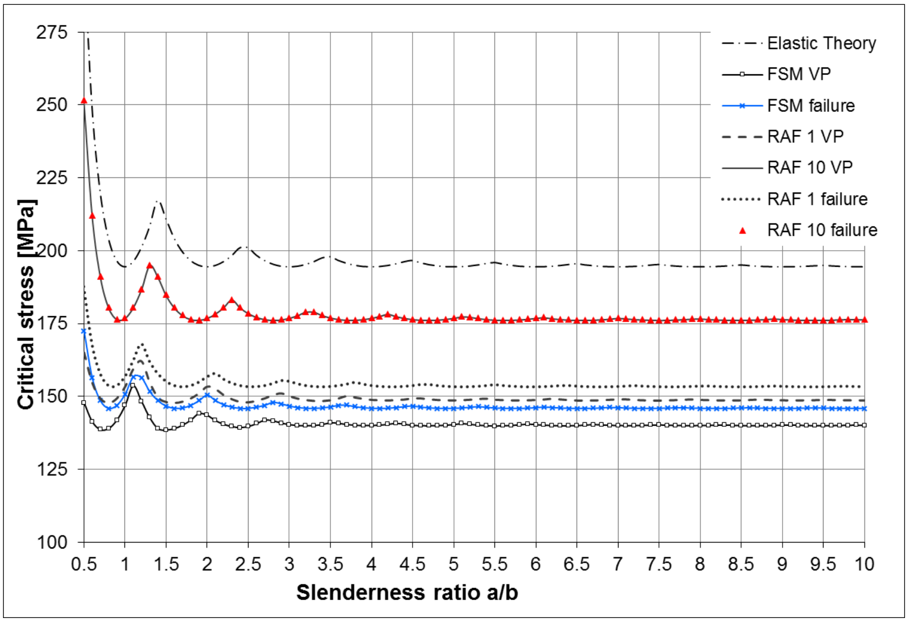


Figure 2. Quasi-static and dynamic elastic, visco-plastic and failure buckling curves for a steel slab - Model 0

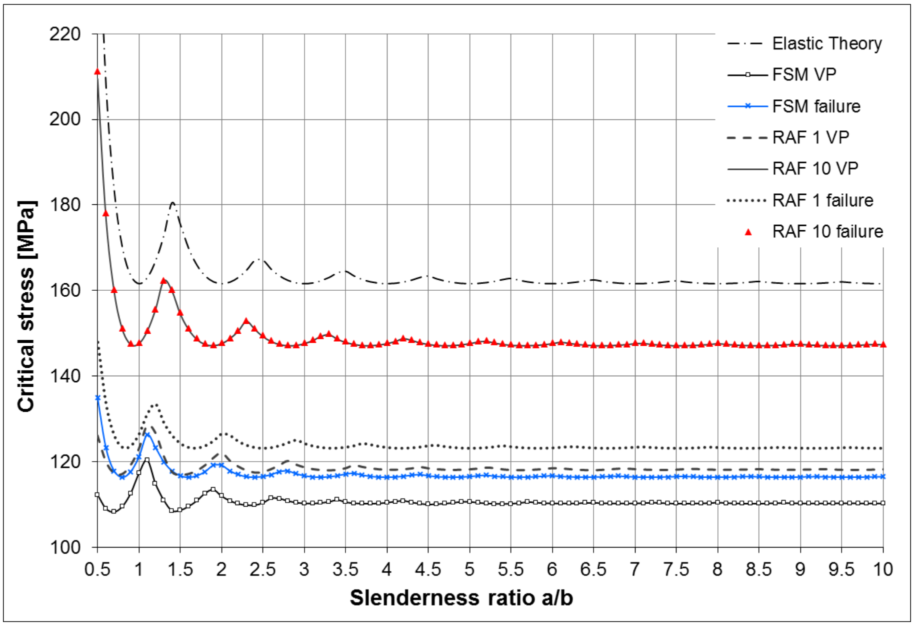


Figure 3. Quasi-static and dynamic buckling curves for a aluminum slab - Model 0

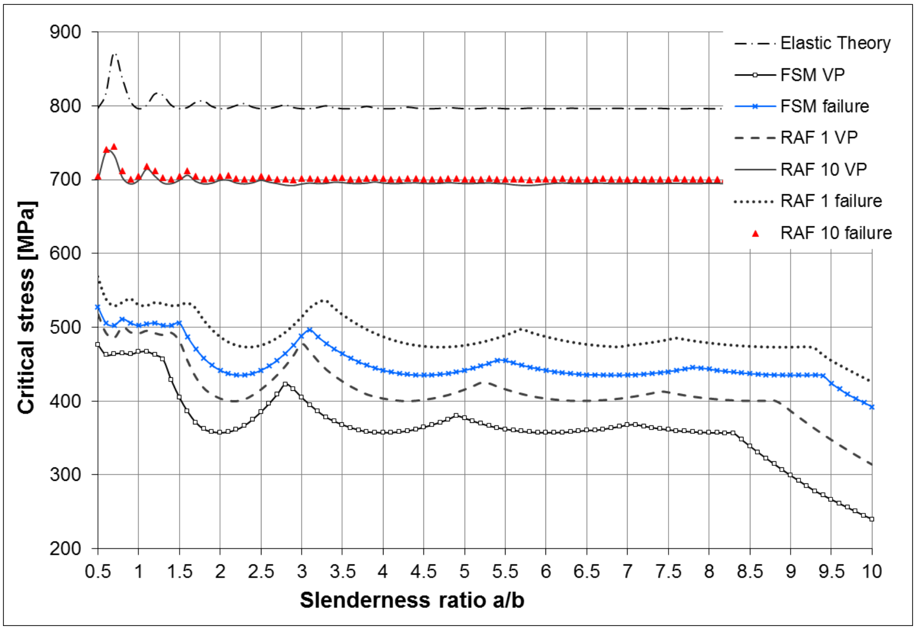


Figure 4. Quasi-static and dynamic buckling curves for a steel panels - Model 1

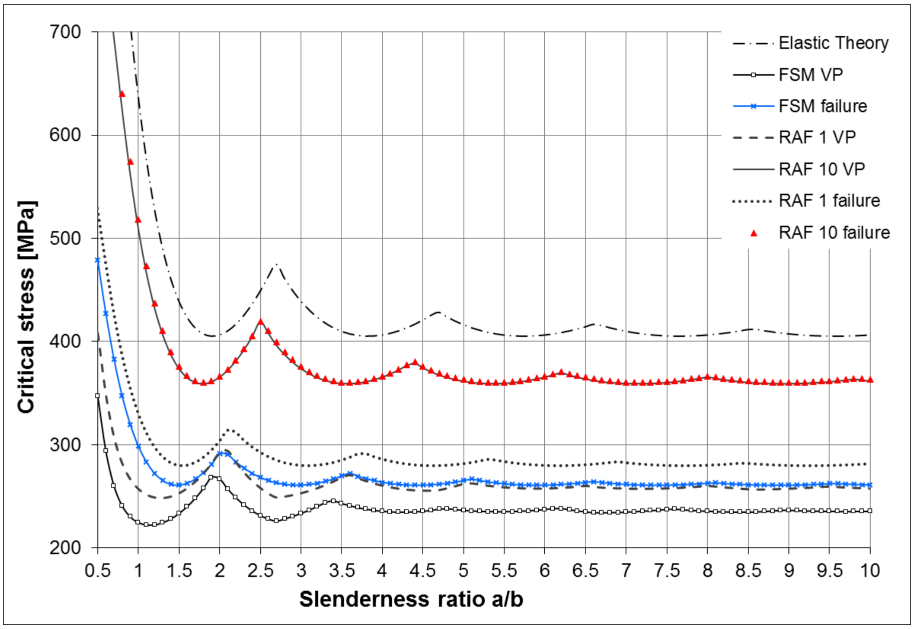


Figure 5. Quasi-static and dynamic buckling curves for a steel panels - Model 2

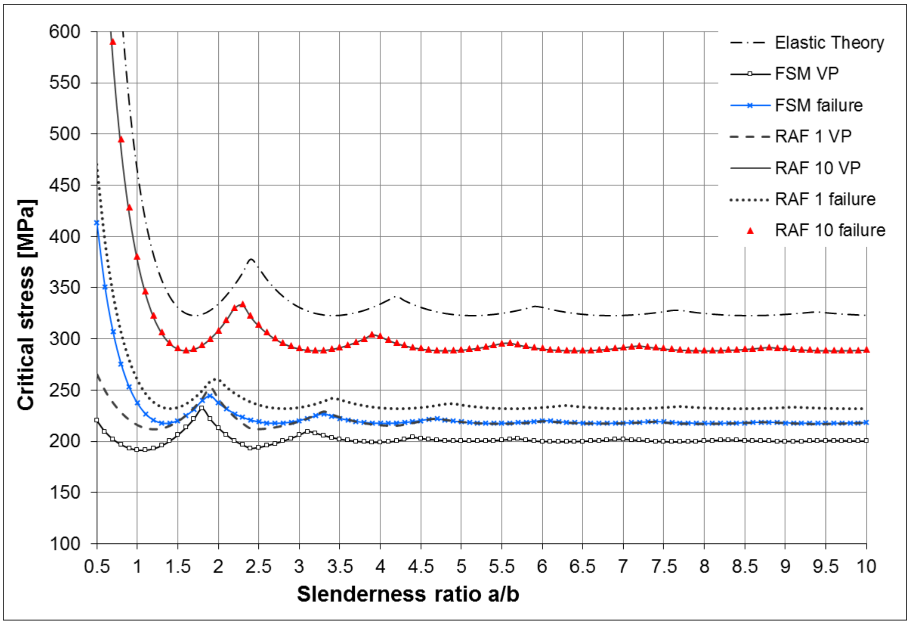


Figure 6. Quasi-static and dynamic buckling curves for a steel panels - Model 3

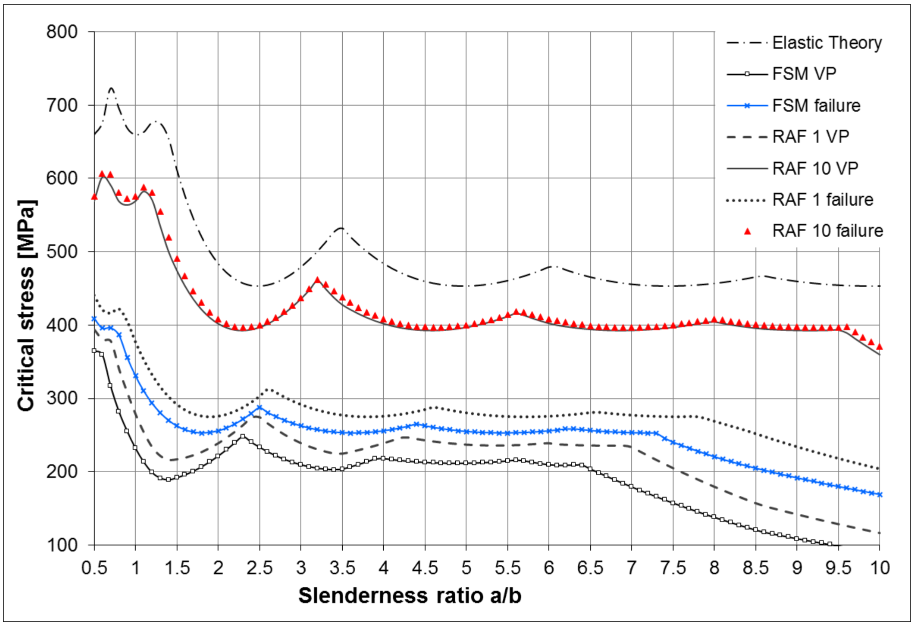


Figure 7. Quasi-static and dynamic buckling curves for a aluminum panels - Model 1

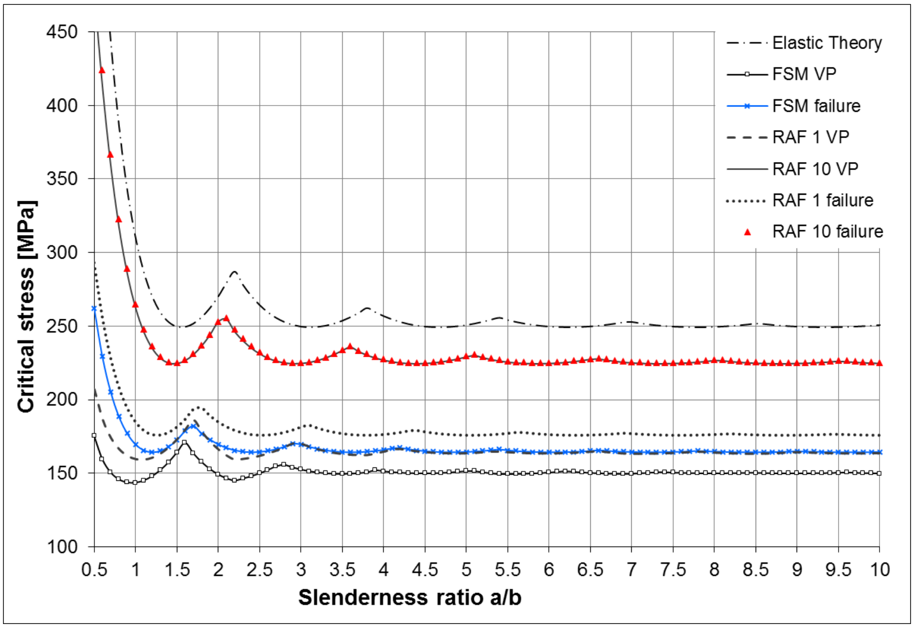


Figure 8. Quasi-static and dynamic buckling curves for a aluminum panels - Model 2

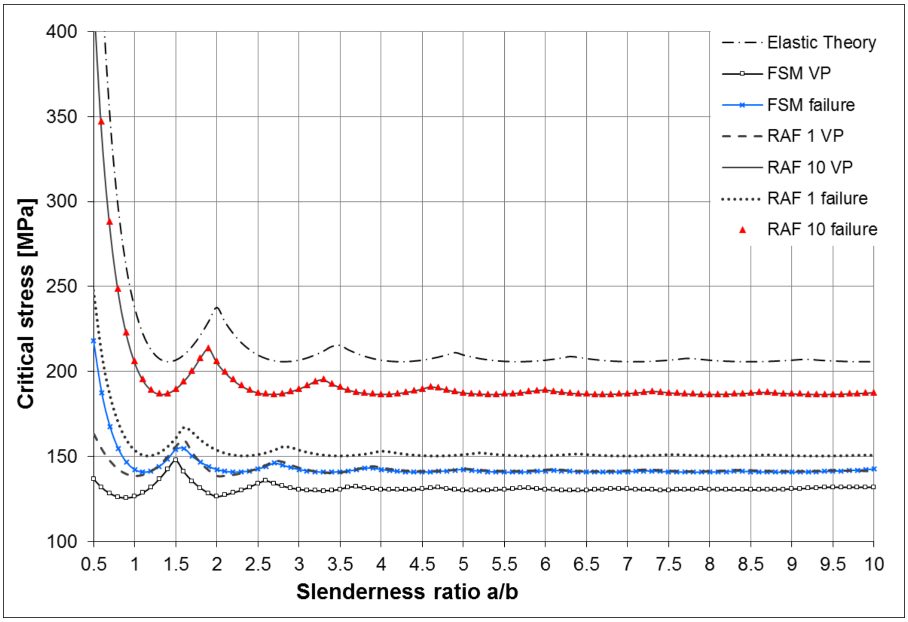
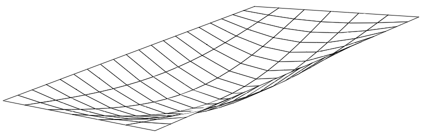
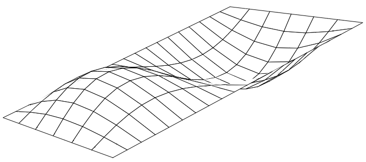


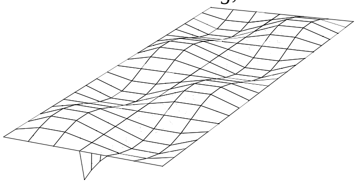
Figure 9. Quasi-static and dynamic buckling curves for a aluminum panels - Model 3



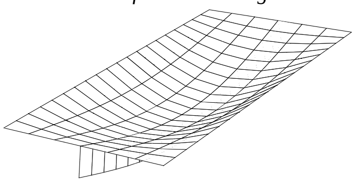
Steel elastic buckling, l= 1.4m



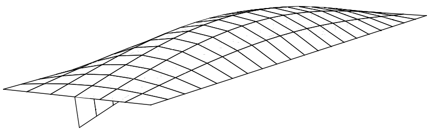
Steel failure buckling



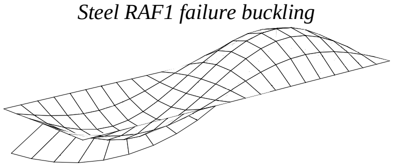
Steel elastic and RAF10 failure, l= 2m



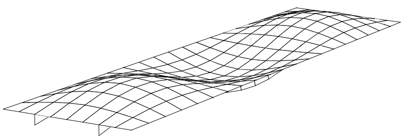
Steel RAF1 failure buckling



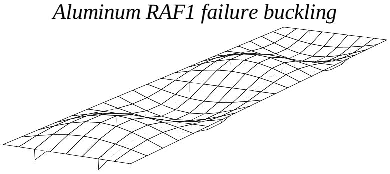
Aluminum elastic and RAF10 failure, l= 0.3m



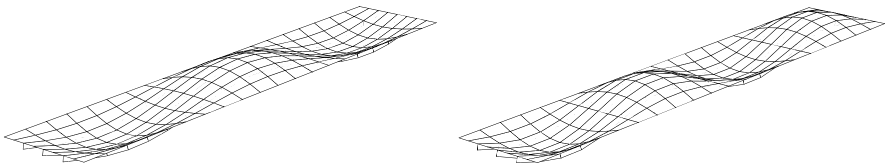
Aluminum RAF1 failure buckling



Steel elastic and RAF10 failure, l= 5.5m



Steel RAF1 failure buckling



Steel elastic and RAF10 failure, l= 5m *Steel RAF1 failure buckling, l= 5m*

Figure 10. Typical buckling modes of stiffened panel

3. CONCLUSION

Derived theory is composed into algorithm and implemented into software package. Intensive numerical study is performed and the infulence of various parameters is examined. The most important results obtained in this paper are the failure stresses by which the failure buckling curves are determined. The failure stress present the ultimate strength of longitudinally stiffened plate girder under compression.

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ГРАНИЧНА ЧВРСТОЋА ПРИТИСНУТИХ ПОДУЖНО УКРУЋЕНИХ ПЛОЧАСТИХ НОСАЧА

Резиме: *У раду је дат заједнички оквир за квазистатичко и динамичко нееластично извијање и граничну чврстоћу подужно укрућених плочастих носача при једнако расподијељеном притиску на крајевима носача. Конструкције су моделиране примјеном метода коначних трака. Материјална нелинеарност је укључена реолошко-динамичком теоријом. Према овој теорији, компликован нелинеаран проблем у подручју нееластичних деформација је рјешен као једноставан линеаран динамички проблем. У претходним радовима су изведене ортотропне конститутивне релације за нееластично извијање као и поступак за итеративно рјешавање нелинеарних једначина, док је овдје приказана исцрпна нумеричка анализа.*

Кључне речи: *Метод коначних трака, реолошко-динамичка теорија, гранична чврстоћа*