DYNAMICS OF A STRUCTURE WITH VISCOELASTIC AND FRICTION DAMPERS

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Summary: *Dynamics of a structure consisting of two rigid blocks positioned one on another with a viscoelastic and a friction damper is analyzed. Due to a horizontal ground excitation the viscoelastic plate is exposed to a shear loading. The fractional Zener model is used to describe properties of the viscoelastic body while the friction damper is modelled by the Coulomb friction law in a set-valued form. The horizontal ground motion is described by a simplified earthquake model by means of Ricker's waves.*

Keywords: *viscoelasticity, fractional derivative, earthquake, dry friction*

1. INTRODUCTION

Due to destroying effects of earthquakes, energy dissipating systems are used in numerous engineering constructions to prevent or minimize their damage and to avoid building collapse and losses of human lives. There are several types of seismic isolation systems, for example see [1]. Their installation either in the building or between the foundation and the building is an effective way to reduce structural damage. Active, semi-active and hybrid systems of seismic protection are analyzed in [2], [3] and [4]. Passive systems, very effective in practical usage, are not expensive as previous ones and they does not need any additional energy source for their work, [5], [6], [7]. In this paper we analyze earthquake response of a structure with passive dampers, which dissipate seismic energy by both deformation of a viscoelastic material and dry friction mechanism. Similar problem is treated in [8] where viscoelastic rod was exposed to uniaxial deformation. Here, viscoelastic plate is connected to the structure elements

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which undergo longitudinal displacements during an earthquake, which produce shear deformation in the plate.

2. FORMULATION OF THE PROBLEM

An earthquake response of a structure which consists of two rigid blocks positioned one on another is considered. A viscoelastic plate is positioned between the blocks and it is attached to them with no sliding possibility. Also, it is exposed to a shear loading during the horizontal motion of the upper block relative to the lower one. Another passive damper is presented by a friction element which dissipates seismic energy brought to the structure, see Figure 1a.

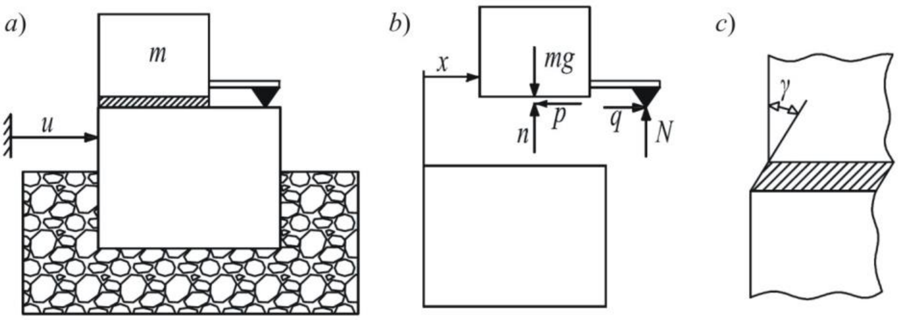

Figure 1. System under consideration

Mass of the upper block is denoted by *m*, while *u* stands for the absolute position of the lower block, which moves horizontally together with the ground. A free body diagram is presented in Figure 1b where *x* is the relative position of the upper block, *p* and *n* are the shear force and the normal reaction force between the block and the plate, while *q* and *N* stand for the friction force and the normal force within the friction damper, respectively, which are explained in [7]. We assume that the upper block moves translatory and that the deformations of the viscoelastic plate are small, so the shear angle reads $\gamma \cong \mathrm{tg}\gamma = x/h$, where *h* denotes the thickness of the plate.

Differential equation of motion and constitutive equation of the viscoelastic plate in the form of the fractional Zener model, see [6], read

$$m \cdot (u^{(2)} + x^{(2)}) = -p + q, \tag{1}$$

$$x(0) = 0, \quad x^{(1)}(0) = 0, \quad p(0) = 0, \tag{2}$$

$$p + \tau_{f\alpha} p^{(\alpha)} = \frac{G_\alpha A}{h}(x + \tau_{x\alpha} x^{(\alpha)}), \tag{3}$$

where $(*)^{(\beta)}=d^{\beta}(*)/dt^{\beta}$, G_{α} and A represent the shear modulus and cross sectional area of the plate. According to the second law of thermodynamics, constants $\tau_{p\alpha}$ and $\tau_{x\alpha}$ with dimension $[\text{time}]^{\alpha}$ and G_{α} must satisfy conditions and $\tau_{x\alpha} > \tau_{p\alpha} > 0$, $G_{\alpha} > 0$, where $0 < \alpha < 1$. Riemann-Liouville fractional derivative of the shear force p and the displacement x is used in constitutive equation (3) and it is defined as

$$\left[z(t)\right]^{(\alpha)} \equiv \frac{d^{\alpha}z(t)}{dt^{\alpha}} = \frac{d}{dt}\left[\frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{z(\tau)}{(t-\tau)^{\alpha}}d\tau\right] \tag{4}$$

see [9]. The friction force q is given by the set-valued Coulomb friction law

$$q \in \begin{cases} -\mu N, & x^{(1)} > 0 \\ \mu N \;\; [-1\;,\;1], & x^{(1)} = 0 \\ \mu N\;, & x^{(1)} < 0 \end{cases} \tag{5}$$

see [10]. The horizontal ground motion is presented by the simplified earthquake model based on Ricker's wavelets, [11]

$$u(t) = u_0 \cdot \sum_{i=1}^{l}(2u_i - 1)\;\; e^{-u_i}, \qquad u_i = \left[\frac{\pi\;\;(t-t_{si})}{t_{pi}}\right]^2, \tag{6}$$

where t_{pi} and $\mathrm{t_{si}}$ are time instants which determine the shape of the wave function $u(t)$.

Introducing nondimensional quantities as in [8]

$$\bar{t} = t\sqrt{\frac{G_{\alpha}A}{mh}},\;\; \bar{x} = \frac{xG_{\alpha}A}{mgh},\;\; \bar{u} = \frac{uG_{\alpha}A}{mgh},\;\; \bar{p} = \frac{p}{mg},\;\; \bar{q} = \frac{q}{mg},$$
$$\bar{\tau}_{x\alpha} = \tau_{x\alpha}\left(\frac{G_{\alpha}A}{mh}\right)^{\frac{\alpha}{2}},\quad \bar{\tau}_{p\alpha} = \tau_{p\alpha}\left(\frac{G_{\alpha}A}{mh}\right)^{\frac{\alpha}{2}},\;\; \bar{\mu} = \frac{\mu N}{mg}, \tag{7}$$

and omitting the bar, the problem is presented in the form of dimensionless equations

$$u^{(2)} + x^{(2)} = -p + q, \quad p + \tau_{f\alpha}p^{(\alpha)} = x + \tau_{x\alpha}\,x^{(\alpha)},$$
$$q \in \begin{cases} -\mu, & x^{(1)} > 0, \\ \mu\;\;[-1\;,\;1], & x^{(1)} = 0, \\ \mu\;, & x^{(1)} < 0, \end{cases} \tag{8}$$

with dimensionless initial conditions which read the same as (2) and with dimensionless ground motion *u* which reads the same as (6). The task is to determine the motion of upper block for a given both ground motion function and parameters of the models of passive frictional and viscoelastic dampers, where the conditions $\tau_{x\alpha} > \tau_{p\alpha} > 0$ have to be satisfied.

3. THE SOLUTION AND NUMERICAL EXAMPLES

In order to solve the posed problem we need to deal with a fractional order differential equation, taking into account non-smooth nature of the friction force, which is not an easy task. Non-smooth dynamical systems are characterized by different motion phases where different sets of equations must be used. To find the solution of the posed problem, it will be treated numerically, like in [12] and [13].

Introducing the time step κ, time is discretized as $t_r = r\kappa$, (r=0,1,2,...). Approximations of the first and the second derivative of a function read

$$z_m^{(1)} = \frac{z_{m+1} - z_m}{\kappa}, \;\; z_m^{(2)} = \frac{z_{m+1} - 2z_m + z_{m-1}}{\kappa^2}, \tag{9}$$

while the Riemann-Liouville fractional derivative can be approximate by the Grünwald-Letnikov definition

$$z^{(\gamma)} = \kappa^{-\gamma} \sum_{j=0}^{m} \omega_j z_{m-j}, \tag{10}$$

where coefficients ω_j are calculated by

$$\omega_0 = 1, \;\; \omega_j = \left(1 - \frac{\gamma+1}{j}\right)\omega_{j-1}, \;\; (j = 1,2,3,...), \tag{11}$$

see [14]. Applying (9)-(11) to the system (2), (6) and (8) we obtain the numerical algorithm for calculation of the shear force

$$p_n = \frac{1}{1+\tau_{fa}\kappa^{-\alpha}} \left\{ x_r \left(1 + \tau_{x\alpha}\kappa^{-\alpha}\right) + \kappa^{-\alpha} \sum_{j=0}^{r} \omega_j \left(\tau_{x\alpha} x_{r-j} - \tau_{p\alpha} p_{r-j}\right) \right\}, \tag{12}$$

and for the relative position x in the case $x^{(1)}>0$

$$x_{r+1} = \kappa^2(-p_n - \mu) - u_{r+1} + 2(x_r + u_r) - x_{r-1} - u_{r-1}, \tag{13}$$

while for the relative position x in the case $x^{(1)}<0$ it reads

$$x_{r+1} = \kappa^2(-p_n + \mu) - u_{r+1} + 2(x_r + u_r) - x_{r-1} - u_{r-1}\,. \tag{14}$$

During the stick phase, where $x^{(1)}=0$, x=const., the friction force q_r can be any value from the interval $[-\mu, \mu]$, and using $(8)_a$, $(9)_b$ it is calculated by

$$q_r = p_r + \frac{u_{r+1} - 2u_r + u_{r-1}}{r^2},\ r > 0\,, \tag{15}$$

where discretized $u_r=u(t_r)$ is obtained by replacing t with $r\kappa$ in the system (6). Discretized initial conditions read $x_0=x_1=p_0=0$, which means the system is at rest at the beginning. As it was mentioned, there are three different phases of motion: sliding to the right, $x^{(1)}>0$, sliding to the left $x^{(1)}<0$ and the sick phase $x^{(1)}=0$, where different numerical algorithms should be used during each of them. Moreover, by the proposed numerical scheme we use the constant time step r, but the time instants when motion phases alternate are usually not integer multiples of r, which makes the problem more difficult. By using the suggested numerical algorithm together with the procedure given in [8], some numerical results are obtained and they are presented in the sequel.

Some simulations of motion are performed for the following values of excitation parameters: u_0=0.1, t_{p1}=1.25, t_{s1}=1.5, t_{p2}=1, t_{s2}=2, t_{p3}=2, t_{s3}=3, t_{p4}=3.5, t_{s4}=4, t_{p5}=3.8, t_{s5}=5, viscoelastic parameters are: α=0.23, τ_{xa}=1.183, τ_{pa} =0.004, time step r=0.01. Ground acceleration $u^{(2)}$ corresponding to the chosen Ricker`s wavelets is shown in Figure 2.

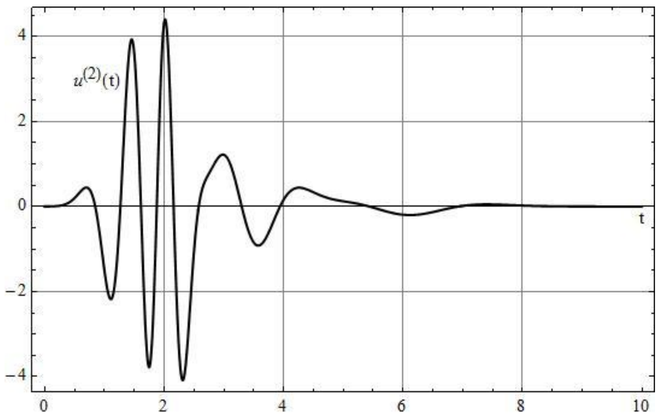

Figure 2. Simplified earthquake excitation - Ricker's waves

In the first simulation the value of the friction coefficient is taken to be μ=0.3 and the relative position x and the shear force p are presented in Figure 3, while the relative velocity of the upper block and the friction force are shown in Figure 4.

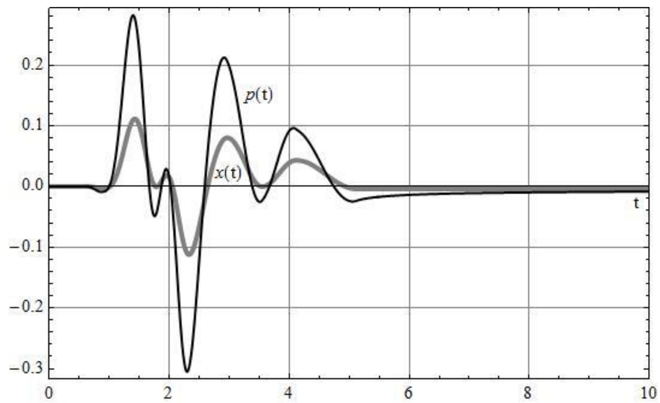

Figure 3. Relative position x(t) and shear force p(t) in the case where μ=0.3

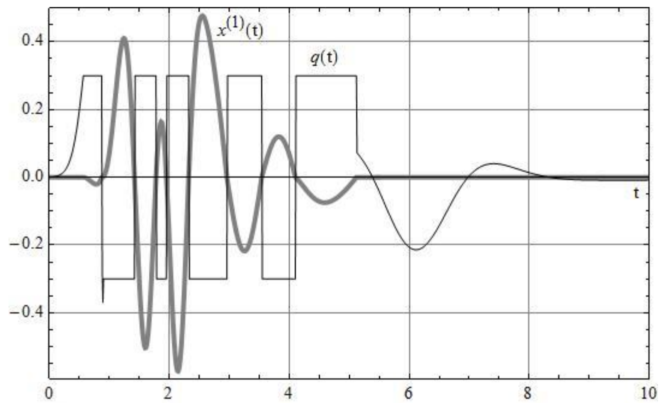

Figure 4. Relative velocity $x^{(1)}(t)$ and friction force q(t) in the case where μ=0.3

The system starts its motion with the stick phase till approximately t=0.7, then sliding phases alternates while the friction force riches its limiting values. After about t=5 the system enters and remain in the stick phase, i.e. the relative motion stops, Figures 3 and 4.

In the second simulation the value of the friction coefficient is greater than in the first case and it is μ=0.9. The relative position x and the shear force p are presented in Figure 5, while the relative velocity of the upper block and the friction force are shown in Figure 6.

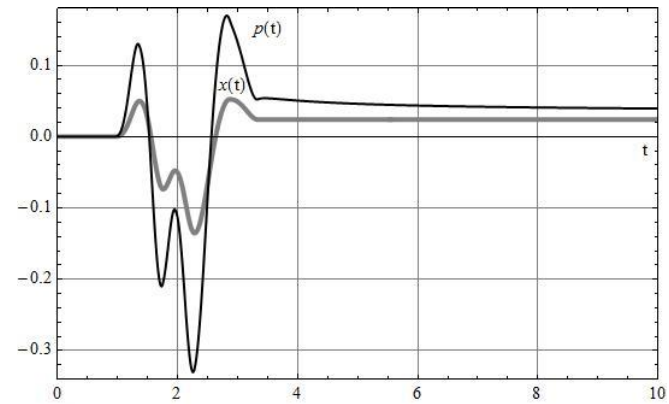

Figure 5. Relative position x(t) and shear force p(t) in the case where μ=0.9

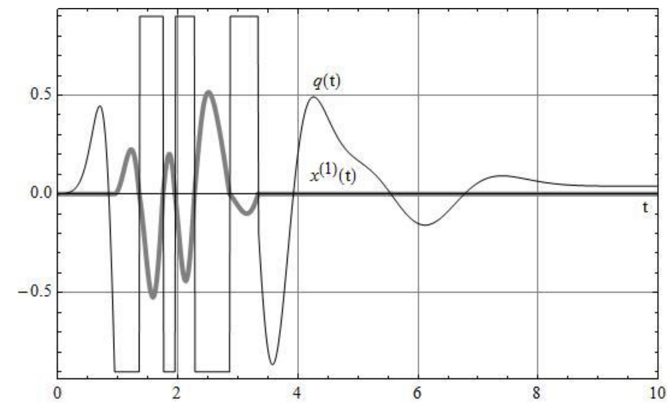

Figure 6. Relative velocity $x^{(1)}(t)$ and friction force q(t) in the case where μ=0.9

Similar to the first case, the system is in the stick phase at the beginning, then it passes through sliding phases reaching the stick phase again in the end. Due to the greater friction coefficient and energy dissipation in this case, Figures 5 and 6, comparing to the first simulation, Figures 3 and 4, time instants of switching from stick to slip phases and vice versa in the second case are smaller, amplitudes of $x(t)$ and $p(t)$ are smaller, while the friction force $q(t)$ is greater.

4. CONCLUSIONS

Dynamics of the non-smooth mechanical system with the fractional derivative constitutive model of a viscoelastic body, under the horizontal seismic excitation in the form of Ricker's waves, Figure (2), is considered. The system is described by the set of equations (1), (3), (5), (6), with initial conditions (2) and restrictions which follow from the second law of thermodynamics. Dimensionless form of equations is given by (8) and (6). Solution is obtained for different value of the system parameters by use of the Grünwald-Letnikov numerical scheme. Non-smooth character of the system is caused by the presence of a dry friction, which is taken by the set-valued Coulomb friction law, and it is considered together with fractional derivatives of the relative position of the upper block and of the shear force as non-local operators. Following the procedure described in [8] for dealing with such type of problems, and for different values of the friction coefficient, numerical results are shown in Figures (3)-(6). The presence of the friction force in the system ensured cessation of the relative motion of the upper block in a finite time. Instead of the simplified horizontal ground excitation in the form of Ricker's waves, given by (6), it would be interesting to analyze the response of the system on some excitations from an earthquake database, but it exceeds the framework of this paper.

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REFERENCES

[1] Torunbalci, N.: Seismic isolation and energy dissipating systems in earthquake resistant design. Proceedings in 13th World Conference on Earthquake Engineering, Vancuver, Canada, **2004**, Paper № 3273.
[2] Datta, T.K.: A state-of-the-art review on active control of structures. *ISET Journal of Earthquake Technology*, **2003.**, vol. 40, № 1, p.p. 1-17.
[3] Soong, T.T., Spencer, B.F.: Active, semi-active and hybrid control of structures. Proceedings in 12th World Conference on Earthquake Engineering, Auckland, New Zealand, **2000.**, Paper № 2834.
[4] Soong, T., Manolis, G.: Active structures. *Journal of Structural Engineering*, **1987**., vol. 113, № 11, p.p. 2290-2302.
[5] Shen, K.L., Soong, T.T.: Modeling of viscoelastic dampers for structural applications. Journal of Engineering Mechanics, **1995.**, vol. 121, № 6, p.p. 694-701.
[6] Zigic, M.M., Grahovac, M.N., Spasic, T.D.: A simplified earthquake dynamics of a column like structure with fractional type of dissipation, Proceedings in 1st Serbian Congress on Theoretical and Applied Mechanics, Kopaonik, Serbia, **2007.**, p.p. 173-180.
[7] Zigic, M., Grahovac, N.: Earthquake response of adjacent structures with viscoelastic and friction dampers. *Theoretical and Applied Mechanics*, **2015.**, vol. 42, № 4, p.p. 277-289.
[8] Zigic M.: *Seismic response of a column like structure with both fractional and dry friction type of dissipation* (in Serbian). Ph.D. thesis, Faculty of Technical Sciences, Novi Sad, Serbia, **2012.**

[9] Samko S.G., Kilbas A.A., Marichev O.I.: *Fractional integrals and derivatives*, Gordon and Breach Sci. Publishers, Yverdon, **1993.**
[10] Glocker Ch. :*Set-valued force laws*, Springer, Berlin, **2001.**
[11] Ricker, N.H.: Transient waves in visco-elastic media. Elsevier, Amsterdam, **1977.**
[12] Grahovac N., Zigic M., Spasic D.:On Impact Scripts with Both Fractional and Dry Friction Type of Dissipation, *Int. J. Bifurcation Chaos Appl. Sci. Eng.*, **2012.**, vol. 22, № 4., Paper № 1250076 (10 pages).
[13] Grahovac N., Spasic D.: Multivalued fractional differential equations as a model for an impact of two bodies, *J. Vib. Control*, **2014.**, vol. 20, 1017–1032.
[14] Podlubny I.: *Fractional differential equations*, Academic Press, London, **1999.**

ДИНАМИКА КОНСТРУКЦИЈЕ СА ПАСИВНИМ ПРИГУШИВАЧИМА

Резиме: *У овом раду је проучена динамика конструкције која се састоји од два крута блока постављена један на други, и која садржи вискоеластични и фрикциони пргушивач. Услед хоризонталне сеизмичке побуде вискоеластична плоча је оптерећена на смицање. Фракционим Зенеровим моделом описана су вискоеластична својства плоче, док је за описивање трења у фрикционом пригушивачу коришћен вишевредносни Кулонов закон силе трења. Хоризонтално кретање подлоге описано је поједностављеним моделом који укључује Рикерове сеизмичке таласе.*

Кључне речи: *Вискоеластичност, фракциони извод, земљотрес, суво трење*