GRAPHICAL SIMULATION OF MINIMUM THICKNESS OF CLAMPED POINTED ARCH

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Summary: *Nearly three centuries ago, Couplet proposed the problem of finding the minimum thickness of a uniform semicircular arch subjected to its own weight, and it was Serbian scholar Milutin Milankovitch who first gave the complete and correct solution almost two centuries later. After remarkable mathematical elaboration concerning thrust line theory, searching for the expression more appropriate for iterative procedure, Milankovitch finds the solution numerically. Since iterations may be easily done by using computer programs nowadays, the mathematical calculation which concerns the finding of the minimum value of equation by the mean of differentiating can be omitted, and the iterative procedures can be done at the earlier stage of the computation, i.e. with the more complex expressions. Recently, the minimum thickness of elliptical arches has been computed, as well. However, there are very few contemporary researches dealing with the mechanical behaviour of pointed arches.*

In this paper, on the basis of the appropriate correlation between the shape of an arch and corresponding collapse mode, particular iterative procedures have been derived. In order to determine the minimum thickness of the chosen arch, the appropriate collapse mode and corresponding iterative procedure have been adopted, and the thickness, as well as the position of the application point of the horizontal thrust force at the crown or of the reaction force at the springing, have been modified, regarding the distance of thrust line from the intrados or extrados at the critical sections. The analysis has been conducted on pointed arches having various eccentricities and embrace angles, including both incomplete and overcomplete arches, and the numerical values for the minimum thickness of more than hundred arches have been provided for the first time. Developed procedures may be applied in the analysis regarding maximum and minimum thrust, span to thickness ratio or weight to span ratio, as well as in the analyses concerning the different types of employed stereotomy.

Keywords: *pointed arches, minimum thickness, iterative procedure, thrust line analysis*

1. INTRODUCTION

Nearly three centuries ago, Couplet [1] proposed the solution to the problem of the determining the minimum thickness of a uniform semicircular arch subjected to its own weight. He defined some assumptions about material behaviour and introduced the

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mechanism of collapse caused by the formation of rupture joints, or so-called hinges (see [2]). However, it was Serbian scholar Milutin Milankovitch who, almost two centuries later, in his paper on the thrust line theory [3] not recognized until nowadays (see [4] as well), first gave the complete and correct solution. After remarkable mathematical elaboration, searching for the expression more appropriate for the iterative procedure, Milankovitch finds the solution numerically. Since iterations may be easily done by using computer programs nowadays, the mathematical calculation, which concerns the finding of the minimum value of equation by the mean of differentiating, can be omitted, and the iterative procedures can be done at the earlier stage of the computation, i.e. with the more complex expressions.

Over the last few years, in the frame of limit equilibrium analysis, Alexakis and Makris computed the minimum thickness of elliptical arches [5]. Although pointed masonry arches are very common in historic structures, particularly in Gothic architecture, the researches considering their minimum thickness are barely present (in [6] or [7] one can find the numerical values for only a few different shapes of pointed arches). In this paper, on the basis of the research on thrust line theory applied to pointed arches conducted in [8] and [9], and their admissible collapse modes provided in [10], appropriate iterative procedures concerning the derivation of numerical solutions for the minimum thicknesses have been developed. The analysis has been conducted on pointed arches having various angle of embrace, considering both incomplete i.e. segmental and overcomplete i.e. horseshoe arches, with varying eccentricity as the measure of pointedness. The usual eccentricities of pointed arches present in historic structures as well as in medieval manuscripts (see for example [11]), namely 1/5, 1/4, 1/3, 1/2, 3/5, 4/5, 1, 3/2, 2, 5/2 and 3 are particularly considered. Furthermore, for the sake of desirable accuracy and the complete insight in the effect of the eccentricity on the collapse mode and the corresponding minimum thickness, where it was necessary, the unconventional eccentricities were included into the analysis. This particularly refers to the eccentricities resulting in the limit collapse modes having five and seven hinges. The comprehensive review of the conducted analysis is given in table that provides the numerical values for the theoretical minimum thicknesses of all the analysed arches.

2. COMPUTATIONAL PARAMETERS IDENTIFICATION

In [10] it is shown that the admissible collapse mode of a pointed arch depends on its eccentricity and the angle of embrace, and, for a fixed angle, the correlation between eccentricity and the order of the occurrence of collapse modes is provided. Furthermore, when limit state is assumed, five different collapse modes (including the circular arch) are obtained (Fig. 3), and for the determination of the corresponding minimum thickness of each collapse mode, the particular computational procedure is required.

In this paper, the characteristic elements necessary for the computation, such as eccentricity, the position of the application points of relevant forces, as well as the critical sections regarding intrados or extrados, that diverse the pointed from semicircular arches, have been particularly investigated. Due to the symmetry of the arch, only half-arches are considered. In accordance with Fig. 1 (a), R and t denote the mean radius and the thickness of the arch ring, respectively. The minimum value of thickness to radius ratio, t/R, i.e. the minimum possible thickness of the pointed arches normalized by the radius, has been set as the main aim of this paper. Further, the value *e*, which measures the deviation from the circular shape, is the horizontal distance between the centreline's centre *O* and the centre *C* of the pointed arch. The angle *α* represents the angle of embrace, which is the complement of springing angle.

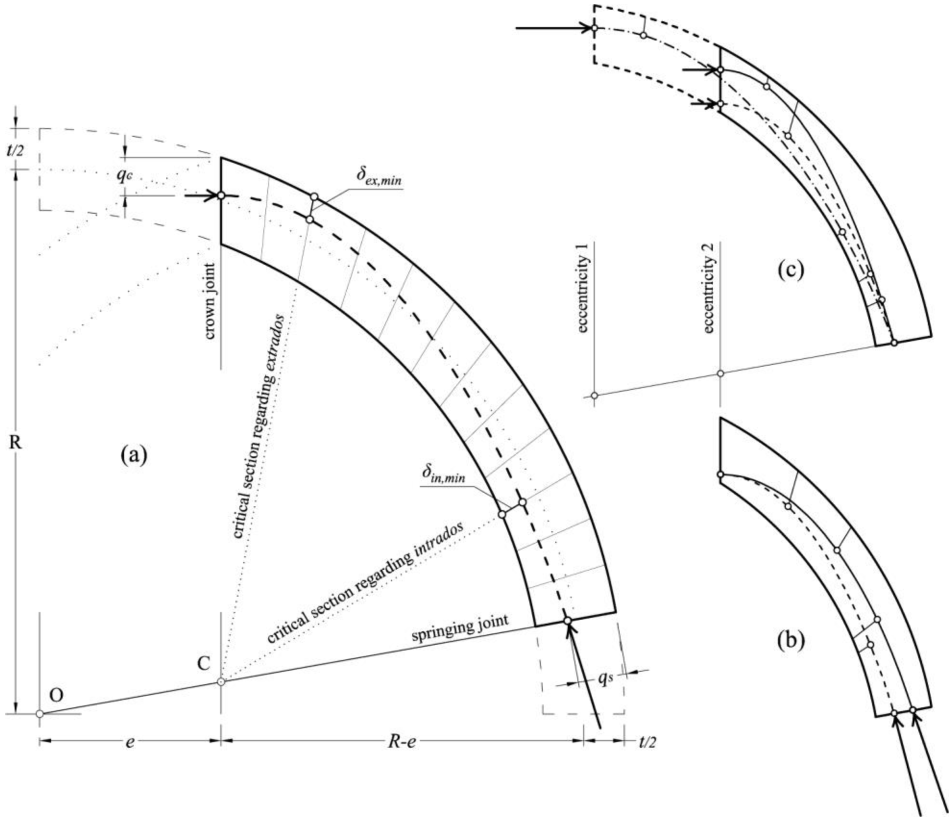

Figure 1. Pointed arch with indicated characteristic parameters (a), influence on the thrust line of the position of the application point of the reaction force acting at the springing (b) and of horizontal thrust acting at the crown as well as eccentricity (c)

The substantial parameter of pointed arch is its eccentricity, denoted by ξ, being the measure of pointedness, and following [7], represents the ratio between *e* and the difference between *R* and *e* (i.e. the mean radius or semispan of the corresponding circular arch). The relation between these three parameters is given by the following expression:

$$\xi = \frac{e}{R - e} \tag{1}$$

Thrust line, representing the load path, is the locus of the application points of the resultant thrust forces, which develop at the joints (beds) between the voussoirs of the arch [5], and according to the analytical expression provided in [9] is traced with the dashed line in Fig. 1 (a). Hence, for each generic section, the distance of the thrust line, i.e. of the application point of the resultant thrust force, from the extrados and intrados, δ_{ex} and δ_{in} respectively, is computed and temporarily stored in the appropriate list. The critical section regarding extrados and intrados refers to the joint containing the minimal value of δ_{ex} and δ_{in}, respectively. Moreover, the minimal value of δ_{ex}, denoted by $\delta_{ex,min}$, is determined as the last value in the list δ_{ex}, which is less than the next one in the list (this form of criteria is valid for positive as well as negative values which develop during the computation). Minimal value of δ_{in}, denoted by $\delta_{in,min}$, is determined as the minimal value in the list δ_{in} between the critical section regarding extrados (joint containing $\delta_{ex,min}$) and the springing (see Fig. 1 (a)).
The parameters of particular importance are the application points of the horizontal thrust force acting at the crown joint, and the reaction force acting at the springings, since, among the eccentricity by default (Fig. 1 (c)), their position affects the position of thrust line through the arch, and therewith the value of horizontal thrust, changing the location of critical sections as well, as one can see in Fig. 1 (b) and (c). The distance of the application point of the reaction force is defined by its radial distance from extrados, and is denoted by q_s. Since it is measured along the direction of springing joint, its maximum value equals t. Further, the positions of the application point of horizontal thrust force, is defined by its vertical distance from extrados, having positive value downward, and is denoted by q_c. Since thrust line cannot pass below intrados of the crown joint, its maximal practical value $q_{c,max}$ is:

$$q_{c,max} = \sqrt{(R+t/2)^2 - e^2} - \sqrt{(R-t/2)^2 - e^2} \qquad (2)$$

In Fig. 1 (a) the pointed arch of thickness greater than the minimum is shown; when the thickness of the arch is sufficiently reduced and thrust line touches the intrados and extrados in more than four points (rupture joints, hinges), the arch reaches a limit equilibrium state by developing a hinged mechanism, i.e. it reaches the point of collapse [12]. Thus, the thickness of the arch and the position of relevant application points have to be modified according to the values $\delta_{ex,min}$ and $\delta_{in,min}$, which simultaneously have to reach zero. In order to determine such state, the sections neighbouring to the critical sections, referred in this paper as the critical area, have to be thoroughly analysed.

3. CRITICAL AREA SELECTION

In pre-processing step, the arch is initially divided into five portions having equal angle of embrace, as shown in Fig. 2 (a) by thick dot-dashed lines. Furthermore, each arch portion is subdivided into appropriate number of segments regarding generic sections, where the greater number of segments should correspond to the portion that will contain critical section; for example, in Fig. 2 (a) the subdivision 3–7–3–7–3 is shown. For the calculations conducted in this research, the subdivision 10–30–10–30–10 turned out to be satisfying. Regarding the critical section, the second preceding and the second succeeding section (thin dot-dashed lines) are adopted as the bordering sections which determine the critical area for the next iteration, along with thickness modification, and so on, until the thrust line reaches extrados and intrados, up to a satisfactory precision.

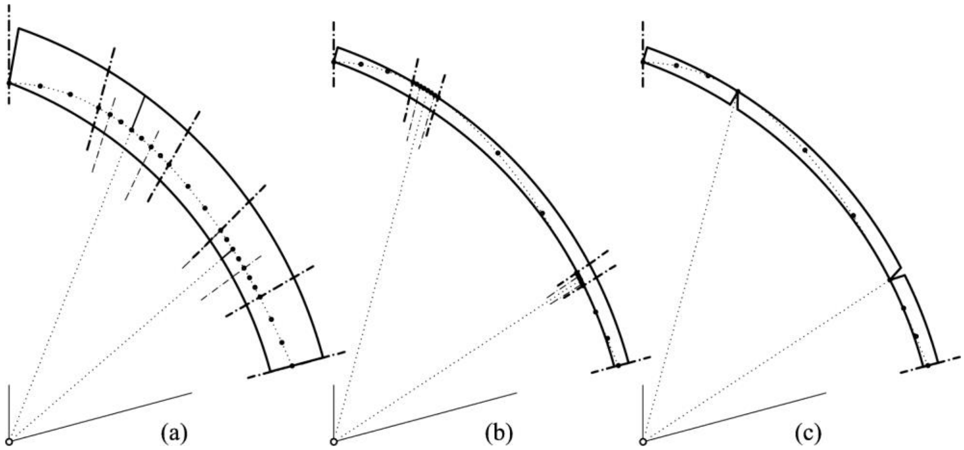

Figure 2. Appropriate subdivision of the arch according to the critical sections and the gradual narrowing of the critical area selection

In order to achieve appropriate gradual successive narrowing of the critical area regarding its embrace angle, being the difference between the angles corresponding to the bordering joints of the critical area, the precision of the subdivision of that embrace angle is adjusted according to the magnitude of the values $\delta_{ex,min}$ and $\delta_{in,min}$. For example, in the conducted calculations, all the values have been computed with the minimum precision of 10^{-12}, and the precision regarding critical area selection (*cap*) is adjusted according to the following relations:

(a) for | $\delta_{ex,min}$ or $\delta_{in,min}$ | $< 5\cdot 10^{-4} \rightarrow cap = 5\cdot 10^{-5}$
(b) for | $\delta_{ex,min}$ or $\delta_{in,min}$ | $< 5\cdot 10^{-10} \rightarrow cap = 5\cdot 10^{-7}$
(c) for | $\delta_{ex,min}$ or $\delta_{in,min}$ | $> 5\cdot 10^{-4} \rightarrow cap = 5\cdot 10^{-2}$.

4. GENERAL GUIDELINES FOR THE DETECTION OF MINIMUM THICKNESS OF POINTED ARCHES

When minimum thickness is assumed, thrust line passes through one extremity (i.e. endpoint on the extrados or intrados) of the springing or crown joint [9], as one can see in Fig. 3. Hence, one of two application points, regarding horizontal thrust at the crown and reaction force at springing, is known, and the other one can be set as the variable, the change of which affects the position of thrust line along the arch. Furthermore, the thickness of the arch is modified with respect to the other critical joint. Accordingly, if the position of the one of the application points is varied with respect to the critical joint regarding extrados, thickness is varied with respect to the critical joint regarding intrados, and vice versa, if the first is varied with respect to the critical joint regarding intrados, the latter is varied with respect to critical joint regarding extrados.

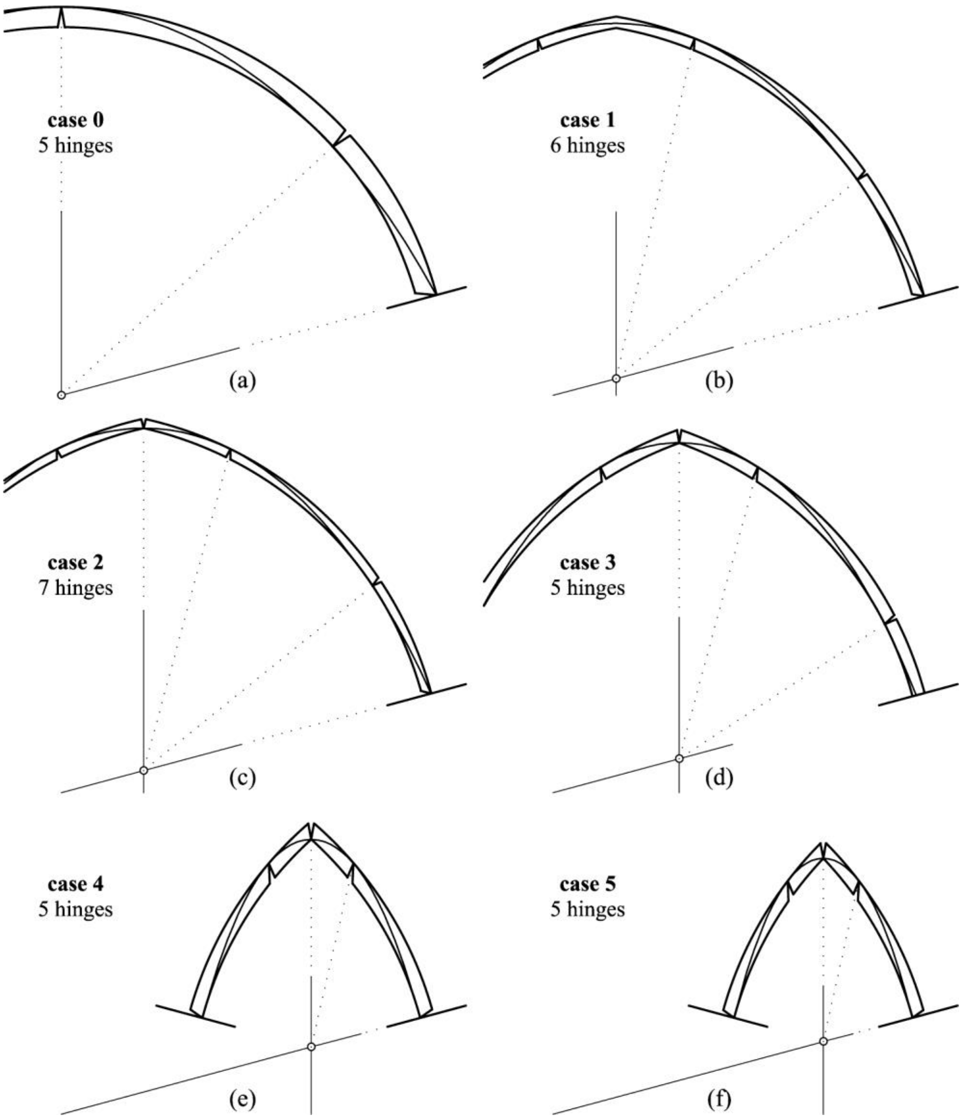

Figure 3. Admissible collapse modes of pointed arches having minimum thickness and corresponding thrust lines: (a) circular arch, (b) extrados hinge at springings, (c) limit eccentricity having seven hinges, (d) intrados hinge at crown, (e) limit eccentricity having five hinges, (f) intrados hinge at springings and crown (after [10])

In other words, if thrust line passes through the endpoint of the springing joint, thickness is varied with respect to the critical joint regarding intrados, and the position of the application point of the horizontal thrust at the crown is varied with respect to the critical joint regarding extrados, according to Eq. (3) and Eq. (4), respectively:

$$t_{\text{int}} = t - \delta_{in,\min} \tag{3}$$

$$q_{c,\min} = q_c - \delta_{\text{ex},\min} \tag{4}$$

For the absence of concentrated loads, thrust cannot pass through the apex of the crown, and therefore the thrust line touches extrados on the both sides near the crown [7]. In order to keep the rise of the thrust line as high as possible, striving for the minimum thrust [13], although the minimum and the maximum thrust will coincide when the limit state is attained [7], the application point of the horizontal thrust must be as close as possible to the extrados crown, i.e. the value q_c should be as small as possible.

On the contrary, if the thrust line passes through the endpoint of the crown, thickness is varied with respect to the critical joint regarding extrados, and the position of the application point of the reaction force at the springing is varied with respect to the critical joint regarding intrados, according to Eq. (5) and Eq. (6), respectively:

$$t_{ext} = t - \delta_{ex,\min} \tag{5}$$

$$q_{s,\max} = q_s - \delta_{\text{in},\min} \tag{6}$$

In order to keep the span of the thrust line as narrow as possible, striving for the minimum thrust [13], the application point of the reaction force must be as close as possible to the intrados at the springing, i.e. the value q_s should be as large as possible.

Although the changing of the position of these application points affects the thrust line as a whole, as can be seen in Fig. 1 (b) and (c), it is adopted that they change regarding the closer critical section, since the effect is usually more immediate in that way. Therefore, the application point of horizontal thrust changes with respect to the critical section regarding extrados, and the application point of reaction force changes with respect to the critical section regarding intrados. However, when both the application points are known, as is the collapse modes shown in Fig. 3 (a) and (f), there exists only one critical section, and then the thickness is varied according to it.

5. GENERAL GUIDELINES FOR THE DETECTION OF THE ECCENTRICITIES OF LIMIT COLLAPSE MODES

When the limit collapse mode with seven hinges is considered (Fig. 3 (c)), since the change of eccentricity immediately affects the application point of horizontal thrust at crown, and, as stated, the crown is closer rather to the critical joint regarding extrados than intrados, it is adopted that eccentricity modifies with respect to the critical section regarding extrados, according to the following expression:

$$\xi_{\lim,ex} = \xi + f \cdot \delta_{ex,\min} \tag{7}$$

On the other hand, thickness is varied with respect to the critical section regarding intrados (Eq. (3)). Regarding the limit collapse mode comprising five hinges (Fig. 3 (e)), there exist only one critical section, and that regards extrados. Thus, the thickness is varied according to Eq. (5). On the other hand, eccentricity is varied with respect to the critical section regarding intrados, according to the following expression:

$$\xi_{\lim,in} = \xi - f \cdot \delta_{in,\min} \tag{8}$$

Namely, since thrust line passes through intrados at both crown and springing joint, when the eccentricity is less than the limit one, thrust line also passes outside the arch ring, toward the arch centre. Thus, by gradually increasing eccentricity smaller than the limit one, according to Eq. (8), along with the adjusting of thickness, critical section regarding intrados approaches the springing, and when it reaches it, limit eccentricity is obtained. Thus, to determine the eccentricity of limit collapse mode, its value needs to be known only approximately. It should be noted that in Eq. (7) and Eq. (8), value f is appropriately chosen factor which represents the measure in which the actual value $\delta_{ex,min}$ or $\delta_{in,min}$, respectively, affects the next value ξ; it has to be chosen in such a way that in every iteration the magnitudes of $\delta_{ex,min}$ and $\delta_{in,min}$ change in the approximately same scale.

6. RESULTS

Six procedures for six collapse modes (two of them have limit eccentricity) have been derived. Characteristic positions of the application points of horizontal thrust acting at the crown and reaction force acting at the springings, regarding the collapse mode and corresponding iterative procedure, are given in Table 1. Therewith, the guidelines for their modification (defined by Eqs. (2), (4) and (6)) and the variation of the thickness (defined by Eqs. (3) and (5)) regarding critical sections, as well as for the determination of the particular values of eccentricity (given by Eqs. (7) and (8)) corresponding to the limit collapse modes with seven and five hinges are provided as well.

Table 1. Parameters used in iterative procedures for the detection of minimum thickness of pointed arches as well as the values of limit eccentricities (bold numbers indicate the collapse mode having limit eccentricity)

Collapse mode	0		1		**2**		3		**4**		5	
Application point	ext.	int.	ext.	int.	ext.	int.	ext.	int.	ext.	int.	ext.	int.
horizontal thrust at crown $\boldsymbol{q_c}$	*0*		$q_{c,min}$			$q_{c,max}$		$q_{c,max}$		$q_{c,max}$		$q_{c,max}$
reaction force at springings $\boldsymbol{q_s}$	*0*		*0*		*0*			$q_{s,max}$		t		t
thickness at critical section $\boldsymbol{t}$		t_{int}		t_{int}	$\xi_{lim,ex}$	t_{int}	t_{ext}		t_{ext}	$\xi_{lim,in}$	t_{ext}	

The iterative processes have been performed by the procedures developed by Visual Basic for Application, i.e. compiling macros in Excel Visual Basic Editor, linked to Excel worksheet. In this research, more than hundred pointed arches of different shape have been analysed, using the iterative procedures based on the presented guidelines; their theoretical minimum thicknesses have been computed and are presented in Table 2.

Table 2. Minimum thickness t/R of pointed arches having various eccentricities and embrace angles with indicated collapse mode and corresponding iterative procedure

ξ \ α [°]	15	30	45	60	75	90	105	120	135
0,00	0,000097	0,00153	0,00750	0,02284	0,05369	0,10748	0,19394	0,32761	0,53654
0,0005	0,000075								
0,00141	0,000049								
0,002	0,000078				*Case 0*: circular arch *(5)*				
0,00301	0,000133				*Case 1*: extrados springing hinge *(6)*				
0,005	0,000252	0,00112	0,00681		*Case 2*: limit eccentricity comprising 7 hinges				
0,01	0,000548	0,00084	0,00623	0,02112	*Case 3*: intrados crown hinge *(5)*				
0,01162		0,00077			*Case 4*: limit eccentricity comprising 5 hinges				
0,02		0,00152			*Case 5*: intrados springing and crown hinge *(5)*				
0,03007		0,00251							
0,04177			0,00373						
0,05	0,002447	0,00441	0,00460	0,01600					
0,10	0,003754	0,00830	0,00983	0,01181	0,03823	0,08856	0,17271	0,30650	0,52722
0,10848			0,01068						
0,11199				0,01103					
0,20	0,003797	0,01296	0,01817	0,01880	0,02917	0,07587	0,15729	0,29058	0,52204
0,25	0,002912	0,01411	0,02106	0,02241	0,02587	0,07090	0,15101	0,28398	0,52058
0,27860					0,02423				
0,33	/	0,01483	0,02443	0,027271	0,02727	0,06399	0,14205	0,27450	0,51944
0,37938				0,02943					
0,50	/	0,01330	0,02743	0,03353	0,03371	0,05370	0,12815	0,25971	0,52087
0,60	/	0,01126	0,02770	0,03557	0,03608	0,04902	0,12161	0,25278	0,52354
0,80	/	0,00596	0,02632	0,03727	0,03872	0,04180	0,11120	0,24188	/
0,88735						0,03928			
1,00	/	/	0,02356	0,03709	0,03962	0,03964	0,10324	0,23381	/
1,50	/	/	0,01485	0,03329	0,03850	0,03855	0,08957	0,22105	/
1,94827					0,03610				
2,00	/	/	0,00629	0,02822	0,03580	0,03588	0,08081	/	/
2,50	/	/	/	0,02332	0,03285	0,03303	0,07468	/	/
3,00	/	/	/	0,01893	0,03002	0,03038	0,07014	/	/

Case 0 regards the arch having zero eccentricity, and represents, indeed, the circular i.e. segmental arch. This type of arch has been discussed in detail in [14], but it is presented here for the completeness of the analysis. The cases 4 and 5 represent the same collapse mode, but the case 4 is the limit one. One can notice that for each analysed embrace angle, the limit eccentricity regarding the collapse mode with seven hinges, corresponds to the theoretically thinnest possible arch (in the range of common shapes of pointed arches, excluding very large eccentricities for the greater embrace angle), having maximum use of its thickness, and therefore the optimal one.

7. FINAL REMARKS AND CONCLUSION

Computational methods present nowadays provide the revision of old theories through the extensive elaboration of a problem, providing the complete insight into the matter. The main challenge is to precisely set the problem, regarding the adequate framework for the analysis within which the appropriate procedures have to be developed. In this paper, the characteristic elements of the computation in the frame of thrust line analysis, particularly applied to pointed arches, have been noticed. Critical sections regarding the joints where the thrust line approaches closest to intrados and extrados, along with the neighbouring critical area, have been particularly treated. The results obtained in recent study, concerning appropriate correlation between the shape of an arch and corresponding collapse mode, have been the basis for the general guidelines statements, on which the particular iterative procedures regarding each collapse mode have been derived. It has been shown that, for any of admissible collapse modes of pointed arch having minimum thickness, at least one of two application points, regarding horizontal thrust at crown or reaction force at springing, is known, and the other one can be set as the variable whose modification regarding one critical section affects the position of thrust line along the arch. On the other hand, the thickness of the arch is modified with respect to the other critical joint. The iterative procedures for the detection of the limit collapse modes having seven and five hinges, and corresponding limit eccentricity, are developed as well. Hence, knowing the appropriate collapse mode, the appropriate procedure can be applied.

In this research, the analysis has been conducted on various pointed arches having angle of embrace from 15 to 135 stepwise 15 degrees, and varying eccentricity from 0 to 3, as the measure of pointedness. Accordingly, the numerical values for the minimum thickness of more than hundred pointed arches of different shape, comprising overcomplete arches as well, have been provided for the first time, and the optimal arch has been identified.

Further research, albeit rather theoretical importance than practical necessity, may consider the analysis of the convergence of iterative procedures proposed in this paper, and eventual improvements regarding the number of iterations necessary for obtaining of the solution. However, developed procedures may be applied in the analysis regarding maximum and minimum thrust, span to thickness ratio or weight to span ratio, in order to identify the optimal shape of the arch regarding the quantity of the used masonry. Moreover, the presented procedures are not strictly connected to the radial stereotomy employed in this paper, but apply to the other types of stereotomy, as well.

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ГРАФИЧКА СИМУЛАЦИЈА МИНИМАЛНЕ ДЕБЉИНЕ ПРЕЛОМЉЕНОГ УКЉЕШТЕНОГ ЛУКА

Резиме: *Купле је, пре скоро три века, поставио проблем одређивања минималне дебљине полукружног лука константне дебљине оптерећеног сопственом тежином, а српски научник Милутин Миланковић је први дао целовито и тачно решење, скоро два века касније. Изванредном математичком разрадом теорије потпорне линије, трагајући за изразом погоднијим за итеративни поступак,*

Миланковић нумерички добија решење. С обзиром на то да данас итерације могу бити лако изведене уз помоћ рачунарских програма, математички прорачуни, који обухватају налажење минималне вредности функције у смислу диференцирања, могу бити изостављени, а итеративни поступци могу бити спроведени у ранијој фази прорачуна, односно користећи сложеније изразе. Недавно је израчуната и минимална дебљина елиптичких лукова. Међутим, врло мало савремених истраживања је посвећено механичком понашању преломљених лукова по теорији потпорне линије.
У овом раду су, на основу успостављене одговарајуће повезаности облика лука и придруженог облика слома, тј. граничног стања равнотеже, изведени посебни итеративни поступци. Како би се одредила минимална дебљина одабраног лука, усвојени су одговарајући облик слома и придружени итеративни поступак, те су, с обзиром на удаљеност потпорне линије од интрадоса и екстрадоса на месту критичног пресека, мењани дебљина лука, као и положај нападне тачке хоризонталне силе у темену и реакције на ослоначкој спојници. Анализирани су преломљени лукови различитог ексцентрицитета и централног угла, узимајући у обзир и сегментне и потковичасте лукове, те су први пут добијене нумеричке вредности минималне дебљине, и то за више од сто лукова. Изведени поступци могу бити примењени у анализи максималног и минималног потиска лука, односа распона лука и његове дебљине или тежине, као и у анализама које се тичу других облика стереотомије.

Кључне речи: *готски лукови, минимална дебљина, итеративни поступак, анализа по теорији потпорне линије*