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DEVELOPMENT OF A 1-D FLOW MODEL FOR A LOOPED RIVER NETWORK

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Summary: This paper presents the development of a one-dimensional, looped river *network model with a dam as an internal boundary condition. The one-dimensional de Saint-Venant equations were discretized using the Preissmann's scheme. The discretized equations are then linearized and solved using the Newton-Raphson's iterative algorithm combined with the Thomas (double-sweep) algorithm. In order to accommodate flow simulation in looped river networks, the regular equations are supplemented with additional equations necessary when simulating flow in complex river systems.*

Keywords: *One dimensional numerical model, looped river network*

1. INTRODUCTION

The objective of this paper is to present the development of a one-dimensional flow model for a looped river network. The significance of 1-D models is reflected through their suitability to help solve various hydraulic problems. For example, Ward et al. developed a numerical model used to estimate the net flow through the Chesapeake and Delaware canal [1]. Islam et al. established a model for steady and unsteady flow simulation in an irrigation canal network [2], capable of handling different hydraulic structures. These open channel flow models also found their relevance in long term simulations, river network modeling, flood predictions, etc.
The developed numerical model was formulated to support water flow modeling in looped river network with a dam as an internal boundary condition.

2. GOVERNING EQUATIONS

The one-dimensional de Saint-Venant equations, used in modeling open channel flow [3, 4, 5], consist of the continuity and momentum equation, Eqs.(1),

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$$\frac{\partial \omega}{\partial t}+\frac{\partial Q}{\partial x}=0, \quad \frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\alpha \cdot \frac{Q^{2}}{\omega}\right)+g \cdot \omega \cdot \frac{\partial Z}{\partial x}+g \cdot \omega \cdot S_{f}=0, \tag{1}$$

where ω denotes the flow cross-section area, t is the time parameter, Q marks the flow discharge, x is the spatial coordinate consistent with the flow direction (Fig. 1), α is the velocity distribution coefficient, g the gravitational acceleration, Z the water free-surface elevation and S_f the friction slope.

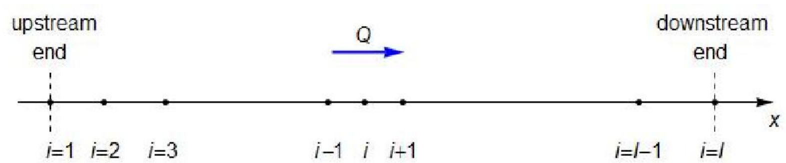

Figure 1. Computational grid

3. DISCRETIZATION AND THE NUMERICAL SOLUTION

The 1-D space is defined by a number of computational points (Fig. 1) along x. The position of these points is determined by the index i, that takes values form $i=1$ at the upstream to $i=I$ at the downstream end. The governing Eqs. (1) can be discretized with various schemes. Here the Preissmann's scheme is applied. The obtained Eqs. (2) expressed as functions of the unknowns and respectively present the continuity and momentum equation,

$$\begin{aligned}
&F\left(\omega_{i}^{n+1}, \omega_{i+1}^{n+1}, Q_{i}^{n+1}, Q_{i+1}^{n+1}\right)=0, \\
&F\left(Q_{i+1}^{n+1}, Q_{i}^{n+1}, \omega_{i}^{n+1}, \omega_{i+1}^{n+1}, \alpha_{i}^{n+1}, \alpha_{i+1}^{n+1}, Z_{i}^{n+1}, Z_{i+1}^{n+1}, K_{i+1}^{n+1}, K_{i}^{n+1}, \varphi_{i+1}^{n+1}, \varphi_{i}^{n+1}\right)=0,
\end{aligned} \tag{2}$$

where index i marks the computational point along x, while K denotes hydraulic conveyance defined through the Strickler's coefficient φ as

$$K=\varphi \cdot \omega \cdot R^{2/3}, \tag{3}$$

where R is the hydraulic radius. The appearance of φ in Eqs. (2) insinuates the possibility to include it in the computation process as a variable, enabling different options for the calibration process. Since Eqs. (2) are nonlinear, they are linearized and solved with the Newton-Raphson iterative algorithm. The linearized equations are

$$\begin{aligned}
&-C \cdot \Delta Z_{i}-D \cdot \Delta Q_{i}+A \cdot \Delta Z_{i+1}+B \cdot \Delta Q_{i+1}=G, \\
&-C' \cdot \Delta Z_{i}-D' \cdot \Delta Q_{i}+A' \cdot \Delta Z_{i+1}+B' \cdot \Delta Q_{i+1}=G',
\end{aligned} \tag{4}$$

where ΔZ is the water level increment computed as $\Delta Z=Z^{n+1}-Z^n$, $\Delta Q=Q^{n+1}-Q^n$ is the discharge increment, and A, B, C, D, G and A', B', C', D', G' are coefficients that consist of variables known from the previous iteration or the previous time step. Employing the Thomas (double-sweep) algorithm, results in equations

$$\Delta Q_{i+1} = E_{i+1} \cdot \Delta Z_{i+1} + F_{i+1}, \quad \Delta Z_i = L_i \cdot \Delta Z_{i+1} + M_i \cdot \Delta Q_{i+1} + N_i, \tag{5}$$

that can be written for each computational point. Coefficients E and F in Eq. (6), determine the influence of the computational point on the solution, while coefficients L, M and N, given by Eq. (11), present the influence of the computational reach between two subsequent computational points on the solution.

$$\begin{gathered} E_{i+1} = \frac{L_i \cdot \left(C_i + D_i \cdot E_i\right) - A_i}{B_i - M_i \cdot \left(C_i + D_i \cdot E_i\right)}, \quad F_{i+1} = \frac{N_i \cdot \left(C_i + D_i \cdot E_i\right) + D_i \cdot F_i + G_i}{B_i - M_i \cdot \left(C_i + D_i \cdot E_i\right)}, \\ L_i = \frac{A_i \cdot D_i' - A_i' \cdot D_i}{C_i \cdot D_i' - C_i' \cdot D_i}, \quad M_i = \frac{B_i \cdot D_i' - B_i' \cdot D_i}{C_i \cdot D_i' - C_i' \cdot D_i}, \quad N_i = \frac{D_i \cdot G_i' - D_i' \cdot G_i}{C_i \cdot D_i' - C_i' \cdot D_i}. \end{gathered} \tag{6}$$

Equations (6) are complemented with the boundary condition equation

$$\xi \cdot \Delta Z + \eta \cdot \Delta Q = \zeta \tag{7}$$

to form a system of equations. Since this work considers only subcritical flow, boundary conditions are needed on both ends of the computational domain. Using the universal form of the boundary condition Eq. (7), both upstream and downstream boundary condition equations can be obtained. Coefficients ξ, η and ζ are known coefficients that depend on the selected boundary condition. The upstream boundary condition is known discharge through time for which the coefficients in Eqs. (7) and (6) are given as

$$\xi_{us} = 0, \quad \eta_{us} = 1, \quad \zeta_{us} = Q_{us}\left(t^{n+1}\right) - {}^{m}Q_1^{n+1}, \quad E_1 = -\frac{\xi_{us}}{\eta_{us}}, \quad F_1 = \frac{\zeta_{us}}{\eta_{us}}, \tag{8}$$

where upper index m marks the value in previous iteration in the Newton-Raphson algorithm. The downstream boundary condition is known water level through time, and the corresponding coefficients are not required since ΔZ_I can be computed straightforward from equation

$$\Delta Z_I = Z_{ds}\left(t^{n+1}\right) - {}^{m}Z_I^{n+1}. \tag{9}$$

Equations (5) and (7) form a system that is solved with the Thomas algorithm. The computation now boils down to computing the unknown values of discharge and water level increments using Eqs. (5). The developed model also incorporates the possibility of dam modeling as an internal boundary condition between computational points *i* and *i+1*, which results in including equations

$$Q_i^{n+1} = Q_{i+1}^{n+1}, \quad Z_i^{n+1} = Z\left(t^{n+1}\right), \tag{10}$$

in the aforementioned system of equations. Equations (10) are the continuity equation, suggesting the equality of discharges upstream and downstream of the dam, and the known water level upstream of the dam. The matching coefficients are

$$\begin{aligned} &A = 0, \quad B = 1, \quad C = 0, \quad D = 0, \quad G = {}^mQ_{i+1}^{n+1} - {}^mQ_i^{n+1}, \\ &A' = 1, \quad B' = 0, \quad C' = 0, \quad D' = 0, \quad G' = Z\left(t^{n+1}\right) - {}^mZ_i^{n+1}. \end{aligned} \tag{11}$$

Since the developed model is capable of modeling a looped river network, instead of Eqs. (5), Eqs. (12) need to be used.

$$\Delta Q_{i+1} = E_{i+1} \cdot \Delta Z_{i+1} + F_{i+1} + H_{i+1} \cdot \Delta Z_1, \quad \Delta Q_1 = E'_{i+1} \cdot \Delta Z_{i+1} + F'_{i+1} + H'_{i+1} \cdot \Delta Z_1. \tag{12}$$

Equations (12) give the ΔQ at any computational point as a function of ΔZ in that point and the first point of the considered reach (Fig. 2), and the ΔQ in the first computational point of the considered reach as a function of the ΔZ in the *i*-th and the first computational point. Coefficients *E* and *F*, for all the sequential computational points are determined using Eqs. (6), while the remaining coefficients are

$$\begin{aligned} &H_{i+1} = \frac{D_i \cdot H_i}{B_i - M_i \cdot \left(D_i \cdot E_i + C_i\right)}, \quad E'_{i+1} = E'_i \cdot \left(M_i \cdot E_{i+1} + L_i\right), \\ &F'_{i+1} = E'_i \cdot \left(N_i + M_i \cdot F_{i+1}\right) + F'_i, \quad H'_{i+1} = H'_i + E'_i \cdot M_i \cdot H_{i+1}. \end{aligned} \tag{13}$$

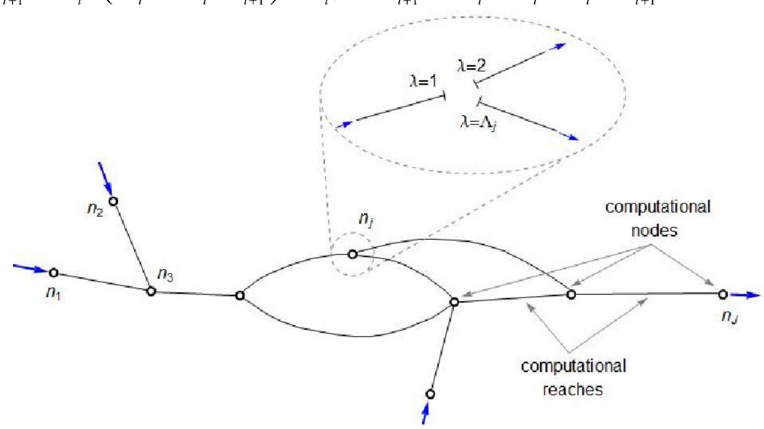

Figure 1. Looped network

Due to their recursive character, coefficients *E* and *F* and *H* must be initialized. These expressions can be obtained from Eqs. (4), when applied to the computational reach between points *i=1* and *i=2* to give

$$E_2 = \frac{A_1' \cdot D_1 - D_1' \cdot A_1}{D_1' \cdot B_1 - B_1' \cdot D_1}, \quad F_2 = \frac{D_1' \cdot G_1 - D_1 \cdot G_1'}{D_1' \cdot B_1 - B_1' \cdot D_1}, \quad H_2 = \frac{D_1' \cdot C_1 - C_1' \cdot D_1}{D_1' \cdot B_1 - B_1' \cdot D_1},$$
$$E_2' = \frac{A_1' \cdot B_1 - A_1 \cdot B_1'}{D_1' \cdot B_1 - D_1 \cdot B_1'}, \quad F_2' = \frac{G_1 \cdot B_1' - G_1' \cdot B_1}{D_1' \cdot B_1 - D_1 \cdot B_1'}, \quad H_2' = \frac{C_1 \cdot B_1' - C_1' \cdot B_1}{D_1' \cdot B_1 - D_1 \cdot B_1'}. \tag{14}$$

A looped river network consists of multiple links connected to computational nodes n_j, $j=1,...,J$ (Fig. 2), hence two additional equations are needed. The equation used at node n_j, where multiple links marked $\lambda=1,2,...\ \Lambda_j$ are connected, is the continuity equation

$$\sum_{\lambda=1}^{\Lambda_j} {}^{m+1}Q_{\lambda}^{n+1} = 0, \tag{15}$$

where Λ_j is the total number of links connected to the node. Equation (16) equalizes the free surface ΔZ for the first or last points of all links connected to the same node.

$$\Delta Z_{\lambda=1} = \Delta Z_{\lambda=2} = ... = \Delta Z_{\lambda=\Lambda_j} = \Delta Z_{n_j} \tag{16}$$

The continuity Eq. (15) is linearized and implemented on all nodes. As a consequence of the linearization, ΔQ appear in these continuity equations, and they are substituted with (12). Finally employing the principle given by (16) gives a system of linear equations, with unknown ΔZ in all nodes, that are computed by inverting the matrix

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1J} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2J} \\ \vdots & & & & & \\ a_{j1} & a_{j2} & \cdots & a_{jj} & \cdots & a_{jJ} \\ \vdots & & & & & \\ a_{J1} & a_{J2} & \cdots & a_{Jj} & \cdots & a_{JJ} \end{bmatrix} \cdot \begin{bmatrix} \Delta Z_{n_1} \\ \Delta Z_{n_2} \\ \vdots \\ \Delta Z_{n_j} \\ \vdots \\ \Delta Z_{n_J} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_j \\ \vdots \\ b_J \end{bmatrix}, \tag{17}$$

where a and b are known coefficients, index n_j is the node for which the equation is written, and $j=1,...,J$ shows the node corresponding to the considered ΔZ_{nj}, where n_J denotes the total number of nodes. After acquiring the ΔZ in all nodes, using Eqs. (12) gives the ΔQ, while Eq. (5) gives the ΔZ in all computational points.

4. CONCLUSION

A 1-D numerical model for simulating flow in a looped river network implementing the de Saint-Venant equations was developed. The governing equations, along with the modeling concept and a solution acquiring procedure for an arbitrary looped river network, are given in Sections 2 and 3. Equations (2) suggest the ability of the developed model to allow the change of the Strickler's coefficient through the computation. For future references, the developed model should undergo a series of schematic and real life situation simulations in order to determine its accuracy and validity.

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КАЛИБРАЦИЈА И ВЕРИФИКАЦИЈА ЛИНИЈСКОГ МОДЕЛА ТЕЧЕЊА У МРЕЖИ ОТВОРЕНИХ ТОКОВА

Резиме: *Овај рад даје опис развоја линијског модела течења у мрежи отворених токова са браном као унутрашњим граничним условом. Полазећи од de Saint-Venant-ових једначина које су дискретизоване применом Preissmann-ове шеме, једначине су линеаризоване и решаване комбиновањем Newton-Raphson-ове методе и Thomas-овог алгоритма. Класичне једначине које се користе при моделисању линијског течења, су допуњене додатним једначинама које су неопходне приликом моделисања течења у сложеним системима отворених токова.*

Кључне речи: *Линијски нумерички модел, мрежа отворених токова*