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ELASTO-PLASTIC STABILITY CALCULATION OF THE FRAME STRUCTURES USING THE CODE ALIN

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Summary: *This paper presents the procedure for stability analysis of frame structures in elasto-plastic domain. This analysis is performed using the code ALIN, developed in the C++ programming language. In this code the stiffness matrix is derived using interpolation functions related to the exact solution of the differential equation of bending of a beam according to the second-order theory. Also, the concept of tangent modulus is applied for the stability calculation in the inelastic domain.*

Keywords: *Stability of structures, elasto-plastic analysis, tangent modulus*

1. INTRODUCTION

The phenomenon of instability of frames in elasto-plastic domain is analyzed in this paper. The problems of instability of reinforced concrete structures and, even more, of steel structures, are very contemporary, particularly having in mind desires of engineers to build attractive tall structures with high slenderness. Design of these structures, especially from the viewpoint of their stability, requires an application of complex numerical models. Although there are a significant number of books and papers in the literature devoted to the various problems of structural stability, for example [1-3], there are still a lot of unsolved or inadequately solved problems, especially in the case of the real behavior of structures in elasto-plastic domain.

One of the main goals of this analysis is to develop the corresponding C++ computer program that can be used for the nonlinear, i.e. elasto-plastic stability analysis of frame structures. Because of its complexity, this kind of nonlinear analysis is not present in the standard engineering procedures for design of structures, and therefore it is not present in the standard commercial software for the calculation of the frame structures.

By this procedure it is possible to follow the behavior of the plane frames in plastic domain and to calculate the real critical load in that domain. For the purposes of numerical investigation in this analysis, part of the computer program ALIN was expanded in such a way that this program can be used for the geometric and material nonlinear analysis.

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2. STABILITY ANALYSIS OF PLANE FRAME STRUCTURES IN INELASTIC DOMAIN

The finite element method, as the most effective method for numerical analysis of stability of frame structures is applied in this analysis. As it is well known, using the finite element method, the critical load can be obtained from the homogeneous matrix equation:

$$\mathbf{K} \cdot \boldsymbol{q} = 0 \tag{1}$$

In Eq. (1) **K** is the global stiffness matrix for the whole frame, including the corresponding boundary conditions, while **q** represents the vector of generalized coordinates. This matrix equation can be solved by an incremental process, by increasing the load at specified increments until it reaches the critical value, i.e. until det**K** = 0. In the case of elastic stability problem, the modulus of elasticity E has a constant value. But, elasto-plastic analysis is more complicated. For the structural member where the proportionality limit is exceeded, for each new load increment the member stiffness has to be changed and the corresponding tangent modulus E_t should be used for that member.
The paper is analyzing frames that are made of steel. So, it is necessary to know the physical and mechanical properties of such material. Effective stress-strain diagram of structural steel is given in Figure 1.

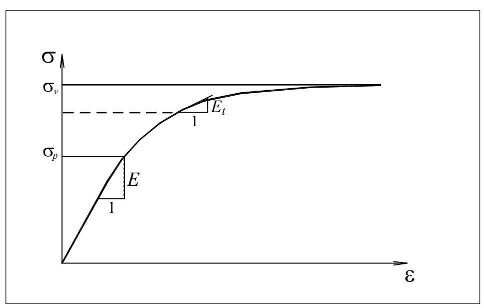

Figure 1. Stress-strain diagram of structural steel

Below a proportionality limit σ_p, the modulus E is constant. Above this point, inelastic behavior occurs with a gradually decreasing resistance of material, measured by the tangent modulus E_t. Whereas E is only a function of the type of material, E_t is stress dependent function. In this analysis relationship between these two modules is used in the form:

$$E_t = 4E \cdot \left[\frac{\sigma}{\sigma_y} \left(1 - \frac{\sigma}{\sigma_y} \right) \right] \tag{2}$$

This is an empirical expression designed to represent the behavior of structural steel columns in the inelastic range.

In order to formulate the exact matrix stability analysis, it is necessary to obtain the corresponding stiffness matrix. So, the interpolation functions should be derived from the solution of the differential equation of bending according to the second order theory. Such interpolation polynomials are obtained as trigonometric or hyperbolic functions of the axially loaded element. The advantage of such approach is in the fact that only one finite element is needed for each beam or column, so the total number of finite elements is 5-10 times less than in the usual approach based on the geometric stiffness matrix. Stiffness matrix for the member of the so-called type “k“ (i.e. clamped at both ends), subjected to compressive force is [4]:

$$K=\frac{E_t I}{l^3\Delta_t}\begin{bmatrix}\omega_t^3\sin\omega_t & \omega_t^2 l(1-\cos\omega_t) & -\omega_t^3\sin\omega_t & \omega_t^2 l(1-\cos\omega_t)\\ & \omega_t l^2(\sin\omega_t-\omega_t\cos\omega_t) & -\omega_t^2 l(1-\cos\omega_t) & \omega_t l^2(\omega_t-\sin\omega_t)\\ & & \omega_t^3\sin\omega_t & -\omega_t^2 l(1-\cos\omega_t)\\ & symm. & & \omega_t l^2(\sin\omega_t-\omega_t\cos\omega_t)\end{bmatrix} \tag{3}$$

where:

$$E_t=4E\cdot\left[\frac{P_{cr,i}}{A\cdot\sigma_v}\left(1-\frac{P_{cr,i}}{A\cdot\sigma_v}\right)\right] \tag{4}$$

$$\omega_t=\frac{1}{2}A\sigma_v\,l\cdot\sqrt{\frac{1}{EI\left(A\sigma_v-P_{cr,i}\right)}} \tag{5}$$

$$\Delta_t=2\cdot\left(1-\cos\omega_t\right)-\omega_t\cdot\sin\omega_t \tag{6}$$

It is obvious that stiffness matrix for nonlinear material behavior has the same form as for the linear behavior of the material, but they are essentially very different. Namely, the difference is primarily in the fact that constant modulus E is replaced by stress dependent tangent modulus E_t.

3. PROGRAM ALIN FOR STABILITY ANALYSIS OF FRAME STRUCTURES

This code is developed using C++ programming language. The program is named ALIN and it enables the complex plane and space analysis of linear frames. The basic possibilities of this program are analysis according to the first and the second order theory, dynamic analysis and stability analysis, i.e. calculation of the critical load in the elastic and inelastic domains. In this paper only the algorithm for the calculation of the critical load in the elastic and inelastic domain is presented. The detailed description of the program can be found in [4], [5].

In the program ALIN, loads may be defined as fixed value or as the variable loads. Variable loads are defined by the value and the load factor. So, the load factors ax, ay and az (depending on the direction of the load) and loading values Px, Py or Pz should

be entered separately. As the final result, program gives „load factor“ which enables to calculate the critical load by multiplying the load factor by the load value.
The procedure for the determination of the critical load in the elastic range is as follows. Firstly, the calculation is performed according to the first order theory.
Obtained axial forces are used in the first iteration to determine the stiffness matrix according to the second order theory. This calculation is performed iteratively using the stiffness matrix according to the second order theory and obtained axial forces at each iteration are used to calculate the stiffness matrix **K*** for the next iteration. Such iterative procedure is carried out until the displacement difference in two consecutive iterations becomes smaller than some pre-set small value. In this calculation the system stiffness matrix is used in the full form, which means that relatively large constants are added at the position and in the direction of restrained generalized displacements. Matrix **K***, which is calculated at the end of the iterative procedure, i.e. after obtaining a converged solution for the second order theory, has to be transformed.
Thus, the rows and columns of the stiffness matrix for the corresponding restrained degrees of freedom are eliminated. So, at the end of this procedure the reduced stiffness matrix of the whole system **K*** only for the active degrees of freedom is considered.
When the reduced stiffness matrix is obtained, it is must satisfy the condition for the existence of the nontrivial solution, i.e. that the determinant of this matrix is equal to zero. This calculation is also performed iteratively. Initial values of the load factor is assumed to be 1.0 and increment delta = 0.01.
The determinant of the system is calculated in each step, with a constant increase of the load factor for delta = 0.01. Calculation is carried out until a negative value of the determinant of the matrix **K*** is reached. When the det|**K***| < 0 is obtained, load factor is reduced for one increment and new "smaller" increment newdelta = 0.001 is defined. Then, a new iterative cycle should be performed. Finally, the value of load factor is calculated with accuracy to three decimal places. This procedure gives the value of the critical force in the elastic domain, i.e. modulus of elasticity E has a constant value which is given in the input XML file.
As it is already mentioned, besides the calculation of critical load in the elastic domain, the program ALIN has also the ability for elastic-plastic stability analysis of frame structures.
Method for calculation of "inelastic" critical load is as follows. Determination of the critical load in "elastic" domain should be performed first. The critical stress is obtained as the ratio of the critical normal force and the cross-sectional area of the analyzed element. When the obtained critical stress is greater than the proportionality limit, calculation continues as it is presented in pervious section.
It means that it is necessary to change stiffness for such columns. So, a new tangent modulus Et is taken in the form (2). Thus, new stiffness matrix in the local coordinate system has to be formed for those columns. Columns with critical stress which did not reach proportionality limit keep "old characteristics".
So, the stiffness matrix for this element is the same as in the first part of this calculation. Then, all matrices should be transformed from the local to the global coordinate system and the stiffness matrix of elements **K*** is formed.
Then an iterative calculation should be performed in the same way as when determining the critical load in the elastic range. It should be emphasized that in the output text file,

besides the critical load factor, the value of tangent tangent modulus (Et) for all elements buckling in the inelastic domain, is also printed.

4. NUMERICAL EXAMPLE

In the engineering practice, the calculation of the stability of frames in the elasto-plastic domain, because of its complexity, is performed by approximate procedure. Namely, the determination of the critical load and the critical stress in the plastic field, for the analyzed members, usually is performed on the basis of the calculation in elastic domain and that applying the empirical expressions and curves that can be found in the corresponding codes. In this paper it is shown that calculation of the stability of frames in the elasto-plastic domain can be performed more successfully using the program ALIN.

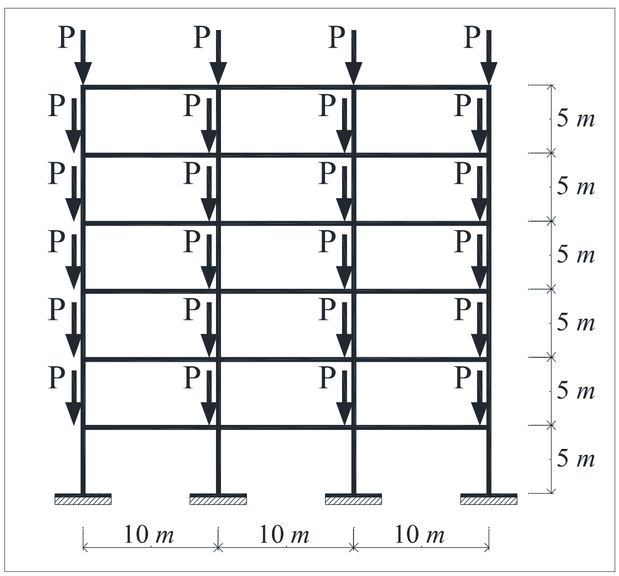

Figure 2. Six-story three-bay sway frame

Considered six-story three-bay sway frame is given in Figure 2.
The frame is clamped at the base, with the load on each column at each story. The numerical analysis is performed for five different cross-sections.
So, it is assumed that all columns and girders in the frame have cross-sections 2⊏12, 2⊏16, 2⊏20, 2⊏26 and 2⊏30. First, applying the elastic analysis, the critical load is obtained. In that case the modulus of elasticity has a constant value E = 210,000,000 kN/m^2.
But, when the stresses in the columns are higher than the proportionality limit, the code ALIN performs inelastic stability analysis. Then the modulus of elasticity (that is now tangent modulus) becomes stress dependent. The yield stress of the steel is σ_v = 240,000 kN/m^2.
The obtained results of the critical load for all five considered cross sections are presented in Table 1.

Table 1. Values of critical load and tangent modulus for the frame given in Figure 2

	Critical load	Tangent modulus		
		3. floor	2. floor	1. floor
2[12	$P_{cr,el}$ = 38.92 kN	E=210,000,000 kN/m^2	E=210,000,000 kN/m^2	E=210,000,000 kN/m^2
2[16	$P_{cr,el}$ = 78.23 kN	E=210,000,000 kN/m^2	E=210,000,000 kN/m^2	E=210,000,000 kN/m^2
2[20	$P_{cr,inel}$= 143.81 kN	E=210,000,000 kN/m^2	E=210,000,000 kN/m^2	E_t=207,146,069 kN/m^2
2[26	$P_{cr,inel}$= 289.63 kN	E=209,999,927 kN/m^2	E_t=196,952,146 kN/m^2	E_t=157,685,260 kN/m^2
2[30	$P_{cr,inel}$= 387.91 kN	E=208,072,513 kN/m^2	E_t=181,288,957 kN/m^2	E_t=122,984,728 kN/m^2

The results of the elastic modulus and tangent modulus at the moment of buckling are also presented in the Table 1. Since the axial force in columns is not constant, the elastic-plastic stability analysis leads to different behavior of the columns in the different floors. Therefore, the results for the columns for the three most loaded stories of the analyzed frame are given separately in this paper.

On the basis of these results it is possible to accurately calculate the buckling length and the slenderness of the columns of the analysed frame. These results are necessary in order to define the real load bearing capacity of the analysed centrically loaded members.

5. CONCLUSIONS

The paper is presenting the procedure for stability analysis of the frame structures in elastic-plastic domain. Stiffness matrices are derived using the tangent modulus which is stress dependant and follows changes of the member stiffness in the inelastic field. These matrices have been implemented in the computer code ALIN. The presented algorithm introduces more accurate calculation of buckling in plastic domain and it shows the advantages of this procedure compared to the approximate solutions used in the standard engineering practice.

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ЕЛАСТО-ПЛАСТИЧНА АНАЛИЗА СТАБИЛНОСТИ ОКВИРНИХ НОСАЧА ПОМОЋУ ПРОГРАМА ALIN

Резиме: *У овом раду је приказан поступак анализе стабилности оквирних носача у еласто-пластичној области. Прорачун је обављен применом програма ALIN који је написан у C++ програмском језику. При томе су матрице крутости изведене коришћењем интерполационих функција које се односе на тачно решење диференцијалне једначине савијања штапа према теорији другог реда. Такође, прорачун стабилности у нееластичној области је обављен применом концепта тангентног модула.*

Кључне речи: *Стабилност конструкција, еласто-пластична анализа, тангентни модул*