FREE VIBRATION AND BIFURCATION BUCKLING ANALYSIS OF STRUCTURES USING THE HARMONIC–COUPLED FINITE STRIP METHOD

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Summary: *This paper analyzes the problem of solving the free vibration and bifurcation buckling of folded plate structures with different boundary conditions. For the boundary conditions different from those of the strip simply supported on both ends, the orthogonality characteristics for the integral expressions that appear in the stiffness matrix formulation are no valid. To calculate the blocks of the stiffness matrix, it would be necessary to know values of the integrals for all coupled series terms. This kind of the finite strip analysis is named the harmonic-coupled finite strip method. This paper present the solutions of the characteristic equations of the basic functions (or eigenfunctions) derived from the beam vibration equation for the beams with different boundary conditions.*

Keywords: *Harmonic-coupled finite strip method, basic functions, roots of characteristic equations, accuracy of numerical evaluation, numerical analysis*

1. INTRODUCTION

Typical folded-plate structures are supported by edge diaphragms and may have arbitrary longitudinal support conditions. For these structures, taking into consideration different edge boundary conditions, the design process should define the optimal shape of the transversal cross-section, which means its geometry, size, and shape. However, for the boundary conditions different from those of the strip simply supported on both ends, the orthogonality characteristics for the integral expressions that apear in the stiffness matrix formulation are no valid. To calculate the blocks of the stiffness matrix, it would be necessary to know values of the integrals for all coupled series terms. This kind of the finite strip analysis is named the harmonic-coupled finite strip method (HCFSM) [1]. This method takes into account the important influence of the interaction between the buckling modes. In contrast, the conventional FSM, in its usual form, ignores this interaction and therefore cannot be used in analysis of folded-plate structures for the boundary conditions different from those of the strip simply supported on both ends.

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Basic dynamic features of the folded-plate structures are determined by free vibration, which is measured by natural frequencies and mode shapes of vibration. This is of great importance in defining of the response of structures to dynamic loading.
Stability and instability of a statically equilibrium state are conventionally defined in terms of the free motions of the system following an infinitesimal and once-and-for-all disturbance from the equilibrium state. Static bifurcation buckling behavior of the folded-plate structures can be obtained from linear equations, solving the standard characteristic-value problem by a matrix of initial stress instead of the mass matrix in free vibration.
This paper present the solutions of the characteristic equations of the basic functions (or eigenfunctions) derived from the beam vibration equation for the beams with different boundary conditions.

2. BASIC FUNCTIONS SOLUTION

In the case of different edge boundary conditions, the solution of the differential equation of the free vibration of a finite length beam can be selected

$$\frac{d^4 Y_w(y)}{dy^4} - Y_w(y)\left(\frac{\mu^4}{a^4}\right) = 0. \tag{1}$$

In this paper we've chosen to examine only one particular boundary condition. The finite strip where one edge of the finite strip is clamped and the other is free is selected. The m^{th} mode of Y_{wm}, is

$$\begin{aligned} Y_{wm}(y) &= \sin\left(\frac{\mu_m y}{a}\right) - \sinh\left(\frac{\mu_m y}{a}\right) \\ &\quad - \frac{\sin(\mu_m) + \sinh(\mu_m)}{\cos(\mu_m) + \cosh(\mu_m)}\left[\cos\left(\frac{\mu_m y}{a}\right) - \cosh\left(\frac{\mu_m y}{a}\right)\right] \\ &= \frac{\left[\begin{array}{c} \sin\left(\frac{\mu_m y}{a}\right)\cos(\mu_m) + \sin\left(\frac{\mu_m y}{a}\right)\cosh(\mu_m) \\ -\sinh\left(\frac{\mu_m y}{a}\right)\cos(\mu_m) - \sinh\left(\frac{\mu_m y}{a}\right)\cosh(\mu_m) \\ -\cos\left(\frac{\mu_m y}{a}\right)\sin(\mu_m) + \cosh\left(\frac{\mu_m y}{a}\right)\sin(\mu_m) \\ -\cos\left(\frac{\mu_m y}{a}\right)\sinh(\mu_m) + \cosh\left(\frac{\mu_m y}{a}\right)\sinh(\mu_m) \end{array}\right]}{\cos(\mu_m) + \cosh(\mu_m)}. \end{aligned} \tag{2}$$

in which μ_m is obtained from the characteristic equation

$$\cos(\mu_m)\cosh(\mu_m) = -1. \tag{3}$$

The characteristic equation must be accurately solved for μ_m (m=1,...,100). The obtained roots μ_m are then used to define the basic function $Y_{wm}(y)$ for further solving the eigenvalue FSM problems, as shown in [1].

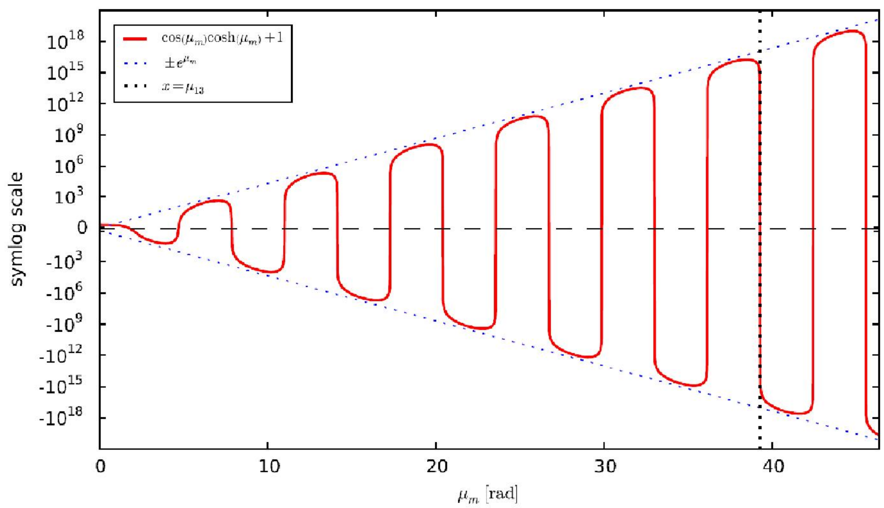

Figure 1. Characteristic function of clamped-free beam (LJ=3) when computed at IEEE 754 binary64 precision. To improve readability only the first 15 modes are shown

As can be seen from Figure 1 when approaching the μ_m roots the characteristic function changes its sign suddenly, mimicking the behavior of the signum function while in the $\mu_m \pm \varepsilon$ area. The local extrema of the characteristic function tend to approach $\pm e^{\mu m}$, making the resulting root-finding errors even larger - the function "falls from huge height" when approaching its root, as observed by the interaction with the $x=\mu_{13}$ vertical. With each increasing mode the root-finding error grows exponentially and once the higher modes (up to 100) are reached (not shown on Fig. 1 to improve its readability) it becomes a very serious issue.

These two properties of the selected characteristic function lead to very large root-finding errors, with the absolute error going up to 10^{121} when using the (semi-)analytic solution from Eq. (4) for solving the selected characteristic equation.

$$\mu_m = (2m - 1)\pi/2 \qquad (4)$$

As shown on Figure 2, the analytic solution fails to yield accurate roots for most modes. Furthermore, the analytic solution fails to produce accurate roots even when computed using the high arbitrary-precision floating point arithmetic [2], as evidenced by Figure 3. This makes the resulting root-finding errors unacceptable, especially for higher modes.

To combat the approximation error caused by the analytic solution we tried to find the exact closed-form solution of the selected characteristic equation, but both SymPy [3], and Mathematica have failed to find the exact closed-form solution, even when given substantial computation time and resources.

Thus we turned to numerical root-finding solvers for better results, while using the analytic solution as the initial guesstimate of the roots location. To avoid guessing which of the existing root-finding solvers would yield the best results we decided to build a tournament based system where all the supported root-finding solvers can compete in solving the characteristic equation, the winner being the solver with the smallest root-finding error.

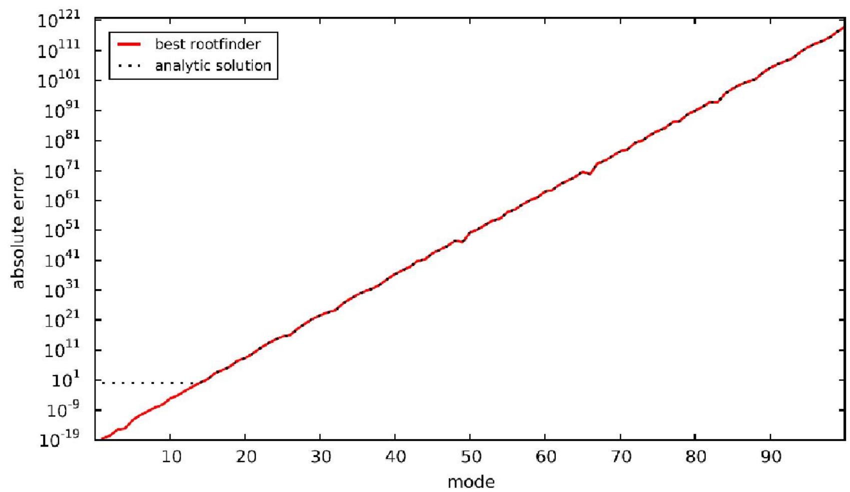

Figure 2. Characteristic function root-finding errors of clamped-free beam when computed at IEEE 754 binary64 precision

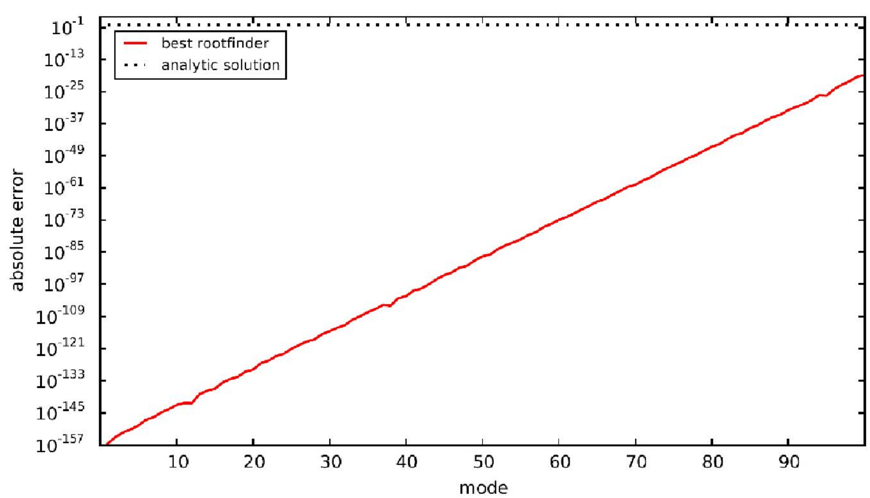

Figure 3. Characteristic function root-finding errors of clamped-free beam when computed using the high arbitrary-precision floating point arithmetic (at 155 decimals)

When analyzing the numerical root-finding tournaments performance shown in Figure 2 (presented as the “best rootfinder” line) it can be seen that our tournament based system outperforms the analytic solution. Unfortunately, it can also be observed that the numerical root-finding tournaments alone are not enough to provide accurate roots for the higher modes.

By coupling our numerical root-finding tournament based system with the high arbitrary-precision floating point arithmetic, we form a hybrid method capable of accurately solving the characteristic equations. We provide a reference Open Source implementation of the hybrid method [4] under an OSI approved BSD Open Source license [5]. We have also provided public access to our testing infrastructure [6], which continuously runs our extensive test suite with over 2300 individual test cases.

The advantages of the hybrid method can be visually confirmed on Figure 3 (presented as the "best rootfinder" line) where it's shown the hybrid method is capable of yielding accurate roots for all modes, unlike the other approaches discussed so far.

The hybrid method also poses substantial benefits when computing the basic function $Y_{wm}(y)$, as demonstrated on Figure 4 – especially when compared to the analytic solution.

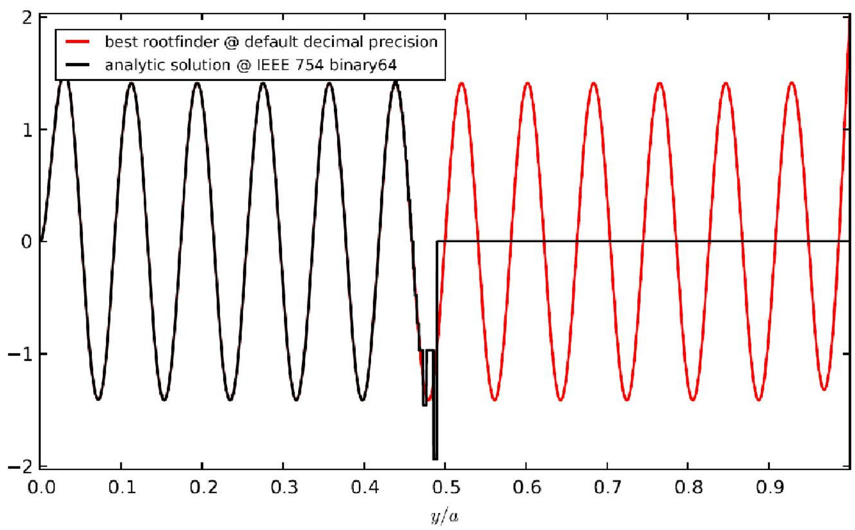

Figure 4. Basic function $Y_{wm}(y)$ of clamped-free beam (LJ=3) for mode 25

Based on this, it can be said the presented hybrid method should always be used when solving the characteristic equations or computing the basic function $Y_{wm}(y)$.

3. CONCLUSIONS

This paper has demonstrated that the analytic solutions of characteristic equations may result in very large root-finding errors, especially for higher modes, and that tournament based numerical optimization should be used instead. Furthermore it has been shown that the computations should be performed with arbitrary-precision floating point arithmetic, as the IEEE 754 standard fails to provide sufficient precision.

A hybrid method, which couples these two approaches, has been proposed along with its reference Open Source implementation, while demonstrating their effects and benefits.

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АНАЛИЗА СЛОБОДНИХ ВИБРАЦИЈА И БИФУРКАЦИОНОГ ПРОБЛЕМА ИЗВИЈАЊА ХАРМОНИЈСКИ-СПОЈЕНИМ МЕТОДОМ КОНАЧНИХ ТРАКА

Резиме: *Рад анализира проблем слободних вибрација и проблем бифуркационог извијања полиедарксих конструкција са различитим условима ослањања на ослоначким дијафрагмама, применом метода кончних трака. За ослоначке услове, различите од слободно ослоњених, услови ортогоналности функција помјерања не вриједе. За срачунавање блокова матрица крутости је неопходно познавати вриједности интеграла за све међусобно различите чланове реда, који се појављују на основу базних функција помјерања. Овај вид метода коначних трака је назван хармонијски-спојен метод коначних трака. Рад представља рјешења карактеристичних једначина базних функција изведених из једначине попречних вибрација греде са различитим условима ослањања на њеним крајевима.*

Кључне речи: *Хармонијски-спојен метод коначних трака, базне функције, коријених карактеристичних једначина, тачност нумеричког срачунавања, нумеричка анализа*