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THRUST LINE DEPENDENCE ON THE LOAD AND SAG

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Summary: *The arch is one of the basic and frequently used linear structural elements. Compared to beams, the advantages of arches are shown in the dominant influence of axial pressure forces and in slight bending moments and transverse forces. The characteristics of the arch mostly depend on the span, height of the arch, transverse and longitudinal section, as well as on the selected static system. One of the most important parameters for the rational design of arches is the knowledge of the type of its thrust line. In this paper we have illustrated the dependence of the thrust line on the sag of the arch and the character of load. For this, we have used the mathematical models created in the computer program Sofistik. The aim of this work is better understanding of the importance of the thrust line and its application in the design of arches.*

Keywords: *Arch, thrust line, sag*

1. INTRODUCTION

The arch is one of the basic and frequently used linear structural elements for the design of large structures [1]. Based on the geometrical form of the arch, several structural systems have developed, first, linear arch structural system, then dome structural system, and later on, simply and doubly curved thin-wall structural systems.
The application of arches in constructions arose as early as prehistoric times. At first, they were used as the elements for bridging the small spans, without sufficient knowledge of their characteristics and use of their properties. Only much later, when arches started to be studied as independent structural elements, their properties, depending on a large number of various parameters, were fully utilized [2]. The first arches were made of stone and bricks, as the available and most commonly used materials of that time. Later, with the appearance of new kinds of materials, arches were made of reinforced concrete, steel, and glued-laminated timber.

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The main parameters that influence the behaviour of arches are: selected static system, geometry of cross and longitudinal section, monolithic quality, the span and height of the arch**.** Based on the knowledge of longitudinal geometry of arches, i.e. the span and height of the arch, arches can be divided into several types. Depending on the ratio of span and the height of the arch, arches are divided into shallow, if this ratio is greater than 4, medium deep, if the ratio is between 1 and 4, and deep, if the ratio is less than 1 [3]. In the design of arches, two main problems appear: designing of the arch which is capable of self-standing, and designing the supports which are able to accommodate a large horizontal thrust. Stability of arches and their ability to independently stand is closely connected with the knowledge of the form of the thrust line. This paper presents the forms of thrust lines for certain types of arches and types of load.

2. GEOMETRY OF LONGITUDINAL SECTION

Arches represent curved girders of large radius compared to the cross-sectional dimensions, where the knowledge of the longitudinal arch geometry is one of their most important characteristics. It is longitudinal geometry that is responsible for the types of stress which occur in arches. Regardless of the type of load, the axial pressure forces in arches are dominant compared to bending moments and transverse forces [4]. Axial pressure forces occur because of fixed suports of the arches and the curvature of the arch axis, which most commonly has a form of a circular arch, parabola or higher order polynomial.

We have analysed the changes of bending moments and axial force pressure depending on the change of the longitudinal geometry of arches, for the same span and the same intensity of evenly distributed load, Figure 1. The analysis has been conducted for one girder, three circular arches with variable height of the arch**,** and one parabolic arch, applying software package Sofistik. Geometric features of these models are not detailed in this paper.

On the basis of the obtained diagrams of bending moments, we can notice that the beams have the greatest bending moments, and that the shallowest arch has the smallest bending moment of all circular arches. Based on the analysis obtained for circular arches, we can conclude that shallow arches have smaller bending moments than deep arches, so shallow semicircular arches can be applied in practice as more rational. In the example of parabolic arches, we can see that the axial pressure forces are dominant, and there is no bending moment.

Namely, the depth of the arch is not directly correlated with the intensity of the bending moment. Bending moments primarily depend on how the geometry of the longitudinal section differs from the geometry of the thrust line. If the geometry of the longitudinal section follows the thrust line, the moment in the whole arch is equal to zero. If they do not match, then the intensity of the bending moment, depends not only on the differences in geometry, but on the nature and intensity of load, too.

Therefore, a small bending moment and high axial force pressure in shallow arches**,** represent their positive features regarding the capacity utilization of materials. The disadvantage of shallow arches**,** and the reason they are rarely used in practice, are large horizontal reactions that occur in the supports. From the architectural point of view, the height in the corners is not sufficient and therefore this space is often unusable. They also form a space with a small volume of air, which in some cases can be beneficial [5].

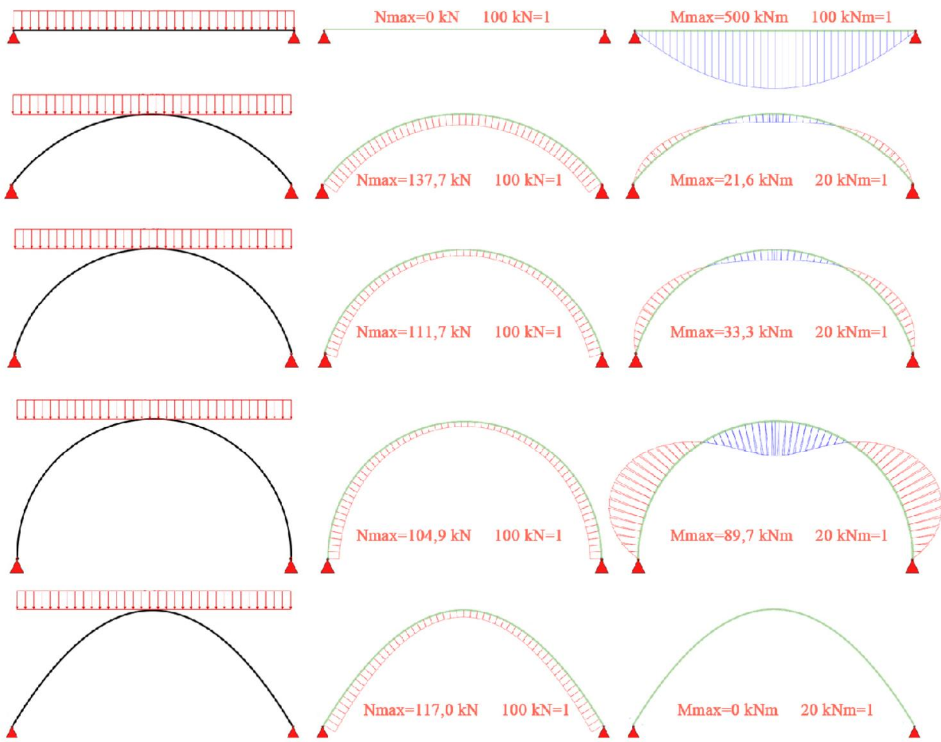

Figure 1. Beam and arches, and corresponding diagrams of bending moments and axial forces

3. THRUST LINE

Hook was among the first who in 1670 dealt with the importance and shape of the thrust line and he tried to solve the problem of the shape and thrust. To illustrate the thrust line, he uses the catenary because he believes that the thrust line is an inverted catenary. He argues that, in the same way that flexible thread hangs, so will, but conversely, stand rigid arch, but he cannot not define the mathematical formulation of catenary [6]. However, this approach should be taken with reserve because it has certain limitations.

La Hire (1695-1712) also dealt with the stability of arches and he considered the balance of the wedge-shaped stone blocks by using appropriate chain polygon and thus unconsciously defined the thrust line**.** Based on previous research, Couplet (1729-1730) in the analysis of arch accurately introduces material properties and states that the friction that occurs between the stone blocks prevents them from moving. Poleni (1748) explicitly states that, for the stability of masonry arches, it is only necessary that the line

of thrust lies anywhere within the thickness of the arch. Mathematical formulation of the stability of masonry brackets was fully set by Coulomb (1773).
By connecting the centres of pressure at any cross section of the arch we get a curved line, called a thrust line. Geometry of the thrust line largely depends on the geometry of the arch, the type of load, and the type of support. It primarily depends on the sag of the arch, i.e. on the depth of the arch, so increasing depth of the arch comes to changing the shape of the thrust line. The thrust line is not directly dependent on the sag of the arch, but on the ratio of sag and range, i.e. depth of the arch. Knowledge of the forms of thrust lines is of great importance especially in the design of masonry arches.
The trust line shape also depends on the character of the load. Thus, for different types of loading of the same intensity, we obtain different thrust lines for the same static system of the arch, Figure 2. It should be noted that the shape of the thrust line does not depend at all on the load intensity.

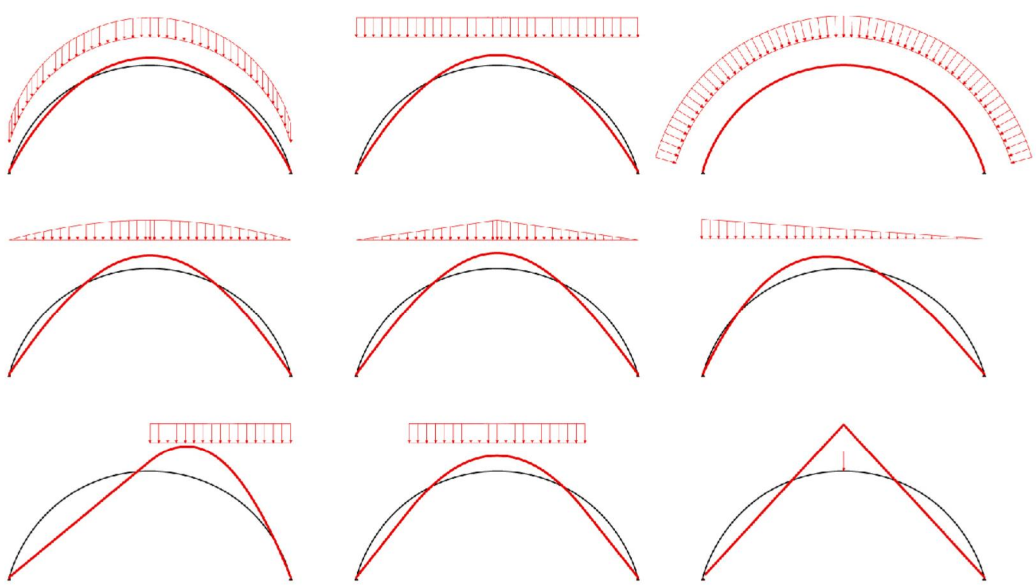

Figure 2. Thrust lines of circular arch for different load types

Based on the analysis, we can see that only in the case when the arch is under the influence of evenly distributed load that is perpendicular to the arch axis, the thrust line has the shape of a circular arc. In all other types of different loading, the thrust line deviates from the shape of a circular arc. When the load is of variable intensity, the thrust line has a greater curvature where the load intensity is higher, while the thrust line is a straight line on the unloaded part of the arch. In the case of action of point loads, the thrust line changes the direction in the place where the force acts, while remaining straight in unloaded parts. In the case of combined action of concentrated forces and evenly distributed load, the thrust line would be curved, with the direction changes in the places of action of point loads.

Figure 3 shows the thrust lines for the arch with different sags which is loaded by its own weight. Thrust lines have the same shape, whereby the increasing of the sag of the arch leads to the increasing of the depth of the thrust line.

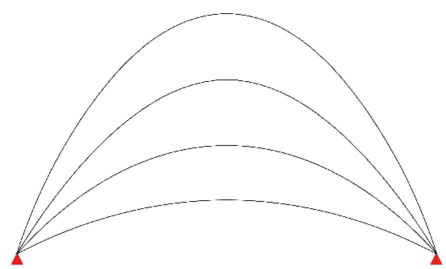

Figure 3. Thrust lines for different sags of the arch for self-weight load

4. CONCLUSION

Although arches have been used for centuries, their use today is still very common and popular. The best proof of it are some of the most spectacular world's buildings where the major structural elements are arches. Besides bridge structures, where the use of arches is common and where they reach the highest spans, they are used in other types of buildings as well. Although the application of the arch structural system has not been considered the most attractive in recent decades, arches, as structural elements, are more often used in hybrid structural systems. Because of this, it is obvious that arches in the future will also play an important role among applied structural systems.

This paper deals with the importance of knowledge of the thrust line shape and its dependence on the load and sag. The thrust line is one of the most important characteristics of the arch but very little is written about its dependence on the load and sag. This paper presents the influence of different types of load on the thrust line as well as the influence of the sag on the thrust line. The obtained results have been analysed and presented for the purpose of better understanding of these dependences and more rational designing of arches in practice.

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ПОТПОРНА ЛИНИЈА ЛУКА – ЗАВИСНОСТ ОД ОПТЕРЕЋЕЊА И СТРЕЛЕ ЛУКА

Резиме: *Лучни носач је један од основних и често коришћених линијских конструктивних елемената. У поређењу са гредним носачем предности лучних носача огледају се у доминантном утицају аксијалних сила притисака, а незнатним моментима савијања и трансверзалним силама. Карактеристике лука у многоме зависе од распона лучног носача, стреле лука, попречног и подужног пресека као и усвојеног статичког система. Један од битнијих параметара за рационално пројектовање лучног носача је и познавање облика његове потпорне линије. У овом раду илустрована је зависност потпорне линије лука од величине стреле лука и карактера оптерећења. За приказ су коришћени математички модели израђени у компјутерском програму Софистик. Циљ овог рада је боље разумевање значаја потпорне линије лука и њене примене у пројектовању лучних носача.*

Кључне речи: *Лучни носач, потпорна линија, стрела лука*