# SIMPLE WALL-RIGID FLOOR-SOIL MODEL FOR PREDICTION OF TRAFFIC INDUCED VIBRATION IN BUILDINGS

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***Summary:*** *Simple numerical model for prediction of traffic induced vibration in buildings using wall-rigid floor-soil model is presented in this paper. Only vertical vibrations are considered. The wall is modeled using the axial spectral element, the rigid floor is modeled using effective inertial forces and the effect of soil is taken into account by the dynamic stiffness of rigid foundation on viscoelastic half space. The developed numerical model is used to predict vertical vibrations of 2-, 6- and 13-storey buildings. The calculated building vibrations are compared with the corresponding measured vibrations. The predicted responses closely mimicked the measured ones.*

***Keywords:*** *Traffic induced vibration, numerical prediction, wall-rigid floor-soil model, spectral elements, dynamic stiffness of soil*

## 1. INTRODUCTION

Needs for easily accessible, available and mobile public transport have produced higher level of traffic induced vibrations. These vibrations, produced by interaction between rolling wheels and the road surface, belong to a low frequency disturbance, which lay in the range between 5-30 Hz. Inside the buildings traffic induced vibrations can cause nuisance among residents, malfunctioning to sensitive equipment and in the extreme cases some minor damages to historical buildings. Hence, the prediction of magnitude of the expected vibrations in building caused by traffic is of great interest. Many unknowns and uncertainties, like vibration inputs, structural characteristics, interaction with the ground and damping in the building and soil, make the reliable predictions of traffic-relate vibration in a building practically impossible. The prediction can be reasonable only for low frequency range.

Models for prediction of vibrataions in buildings caused by traffic can be generally devide into two main groups: empirical (semi-empirical) models and numerical models. Numerical models are based on the numerical simulation of moving vehicle

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(emmission), the transmmision of vibrations trough the ground and the analysis of building vibrations due to ground motions (immission). Numerical models can be simple and detailed ones. Detailed models are based on complex numerical analysis, that includes all three above mentioned components of vibrations caused by traffic. Simple numerical models serve to determine whether the prescribed vibration limit is exceeded in specific parts of the building.

In this paper a simple mathematical model for predicting the propagation of traffic-induced vibrations in buildings is presented. Model is based on the wave propagation in the vertical direction through the walls and columns of the building. The computer program in Matlab is developed and the model is verified using the results of vibration measurements performed on the two-, six- and thertheen-story buildings during the passage of a truck crossing a rubber unevenness.

## 2. NUMERICAL MODEL

The simple numerical model for predicting building vibration is based on the fact that traffic produces much larger vertical vibrations than horizontal ones. These vertical vibrations affect building's walls and columns. The disturbances travel through each element independently as longitudinal waves. It is assumed that only one column, or wall, may be considered for wave propagation analysis. Due to that, walls are modeled as one-dimensional spectral elements. Floor slabs are treated as rigid and modeled as lumped mass. Influence of the soil, presented with spring and dashpot in the model at Fig. 1, is taken into account using the dynamic stiffness of the foundation.
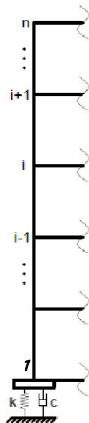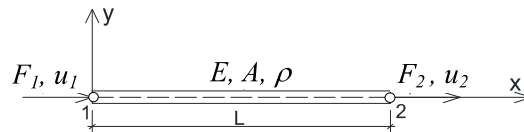


Figure 1. Numerical model                Figure 2. One-dimensional spectral element

*2.1 Wall and column modeling*

. Columns and walls are modeled as two-node axial spectral elements subjected to longitudinal waves, Fig. 2. The dynamic stiffness matrix $\mathbf{K_D}$ relates the vector of nodal forces $\hat{\mathbf{F}} = \{F_1 \ F_2\}^t$ and the vector of nodal displacements $\hat{\mathbf{u}} = \{u_1 \ u_2\}^t$ :

$$\hat{\mathbf{F}} = \mathbf{K_D}\hat{\mathbf{u}} \tag{45}$$

The dynamic stiffness matrix $\mathbf{K}_D$ for an axial element is obtained from the equation of motion in the following form [3]:

$$\left[\mathbf{K}_D\right] = \frac{EA}{e^{ikL} - e^{-ikL}} \begin{bmatrix} ik(e^{ikL} + e^{-ikL}) & -2ik \\ -2ik & ik(e^{ikL} + e^{-ikL}) \end{bmatrix}, \tag{46}$$

where $k = \omega / c_p$ is the wave number, $E$ is modulus of elasticity, $A$ and $L$ are area of cross section and length of element, respectively. In equation (46) the complex modulus of elasticity $E^* = E(1 + 2\xi i)$ can be used to represent damping within element, where $\xi$ is damping ratio and $i = \sqrt{-1}$.

## 2.2 Floor modeling

The dynamic behavior of the floor slabs is modeled as a rigid. Dynamic stiffness of the floor slab is given as:

$$K_f = -m_f \omega^2, \tag{47}$$

where $m_f$ is mass of the floor slab and $\omega$ is circular frequency.

## 2.3 Foundation modeling

The effect of soil-structure interaction (SSI) is taken into account applying the spring and dashpot, which represent the stiffness ($k$) and damping ($c$) of the soil at the base of the model. The dynamic stiffness of the soil in vertical direction is complex number:

$$K_s = k + i\omega c. \tag{48}$$

The foundation parameters, stiffness $k$ and damping $c$, are set according to [1]:

$$k = 3.4 G_S \sqrt{A_S} = 3.4 \rho_S v_S^2 \sqrt{A_S} \qquad c = 1.6\sqrt{G_S \rho_S} A_S = 1.6 \rho_S v_S A_S \tag{49}$$

where $v_s = (G_s/\rho_s)^{0.5}$ is shear wave velocity, $\rho_s$ is soil shear modulus and $A_s$ is area of the foundation.

## 2.4 System of equations

The dynamic equation for the soil-structure interaction analysis in the frequency domain is written as a function of displacements $\hat{u}_{r,i}$, $i=1,...,n$ which are relative to the displacement $\hat{u}_0$ of the base (free field displacement). For the model in Fig. 1, that has $n$ kinematics DOF, the frequency-dependent system of equations in the matrix form is:

$$\begin{bmatrix} K_{n,n} + K_f & K_{n,n-1} & & & 0 \\ K_{n-1,n} & 2K_{n-1,n-1} + K_f & K_{n-1,n+1} & & \\ & & \ddots & & \\ & & K_{23} & 2K_{22} + K_f & K_{21} \\ 0 & & & K_{12} & K_{11} + K_f + K_s \end{bmatrix} \begin{Bmatrix} \hat{u}_{r,n} \\ \hat{u}_{r,n-1} \\ \vdots \\ \hat{u}_{r,2} \\ \hat{u}_{r,1} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -K_s \hat{u}_0 \end{Bmatrix} \tag{50}$$

Based on the presented methodology, the computer program in Matlab is developed and implemented for the numerical analysis presented in the following section.

## 3. NUMERICAL ANALYSIS

### 3.1 Wall model

First, a single wall model is analyzed due to unit harmonic displacements at the base in order to verify the results from the literature [1]. Properties of the wall are the following: height $H=6\cdot3=18m$, length $L=20m$, thickness $d=20cm$, $E_w=3\cdot10^{10}N/m$, $\rho_w=2500kNm^{-1}s^2$, $\xi_w=0.05$. Relative vertical displacements of the wall are calculated for the frequency range between 0 and 100 Hz for two cases: a) without SSI and b) with SSI. Adopted shear wave velocity in the soil is $v_s=200\,m/s$. In the third case the influence of rigid floor slabs 18 cm thick and 10 m wide are taken into account. Vertical displacement amplitudes at each of the six levels are plotted against the frequency for all three cases in Figs. 2$a$, 2$b$ and 2$c$.



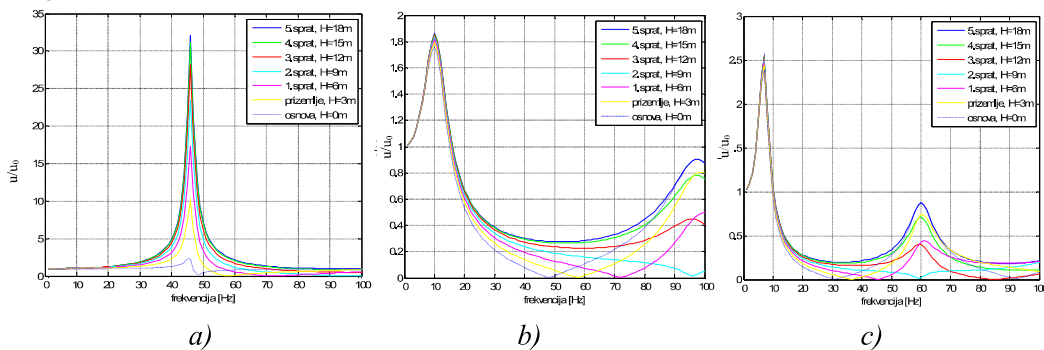a)                               b)                               c)

*Figure 2. a)Wall without SSI,  b)Wall with SSI , c) Wall-rigid floor-SSI*

For the fixed base the resonance frequency is 48 Hz, the amplification factor for the top displacement is 32. For the case with SSI, the first resonance occurs at the frequency of 10 Hz, the second at 96 Hz. Amplification factor is less than previous, 1.8, due to damping in the soil (radiation and material). When rigid floors are taken into account, the resonant frequency decreases to 7.5 Hz and the amplification factor becomes 2.5. The second resonance frequency occurs at 60 Hz and is more pronounced than in the previous case.

### 3.2. Two-, six- and therteen-storey buildings

The proposed numerical models were used to calculate the dynamic response of three typical low-, mid- and high-rise buildings subjected to ground vibrations induced by a 14t truck crossing a 3 cm thick rubber unevenness, at a speed of 50 km/h. The vibrations of the buildings were measured in New Belgrade [2]. The measurements were carried out by the Geophysical Institute, using I/O System One that consists of 5 three-component geophones. The velocities were measured simultaneously at five different points at the buildings site in three orthogonal directions. Mesured velocities in vertical direction at the base of the structures were used as input in the numerical models, while the measured velocities at the top of the structure were used for validation of the numerical results. For the pupose of the analysis measured velocities are transformed to the frequency domain using the Fourier transform and after that vertical base displacements are obtained diveding the velocities by $i\omega$.

Main structural elements for all building are reinforced concrete (RC) walls. In all cases the same properties of the wall elements and the soil top layer are used:

wall: $\rho_w = 2500$ kg/m$^3$, $E_w = 3 \cdot 10^{10}$ N/m$^2$ soil: $\rho_s = 1800$ kg/m$^3$, $v_s = 250$ m/s$^2$ .

*Low-rise building*

The low-rise building has ground floor and two floors above. The height of each floor is 3 m. Cross section of the RC wall parallel to the road is 20x0.25m. The wall is founded on a strip foundation 20x1.5m. All floor slabs are 4.9m wide and 0.12cm tick. The measured velocity time history of vertical vibrations at the base of the building is presented in Fig. 3a. Vertical displacements at the top of the wall are calculated using the wall-rigid floor-soil model. Fig. 3b shows the amplitudes of predicted and measured vertical displacements plotted against the frequency.
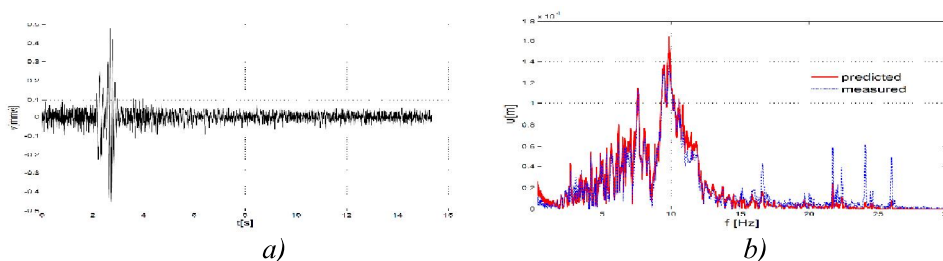


a)                                             b)

*Fig. 3. Low-rise building: a) Ground velocity time history, b) Predicted and measured displacements*

*Mid-rise building*

The mid-rise building consists of basement, ground floor and six floors. The height of each floor is 3 m. The cross section of the RC wall is 20x0.25m while the dimensions of strip foundation are 20x3m. The floor slabs are 5.5m wide and 0.16cm tick. The measured ground velocity and vertical displacements time history (measured and predicted) at the top of the wall are presented in Fig. 4a and 4b.
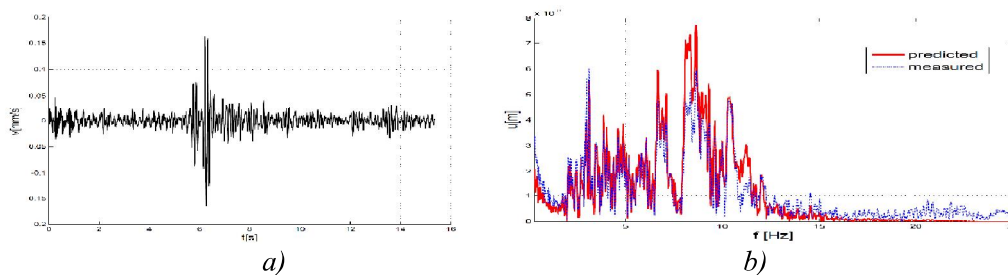


a)                                             b)

*Fig. 4. Mid-rise building: a) Ground velocity time history, b) Predicted and measured displacements*

*High-rise building*

The high-rise building consists of ground floor and 13 floors. The height of each floor is 2.9 m. The reinforced concrete (RC) walls are founded on the reinforced grid foundation. The cross section of the RC wall parallel to the road is 20x0.20m. The dimensions of the foundation grid which belongs to the wall are 20x1m. The floor slabs are 5m wide and 0.12cm tick. The velocity time history at the base of the building and the measured and predicted displacement time histories at the top floor, are presented in Figs. 5a and 5b.
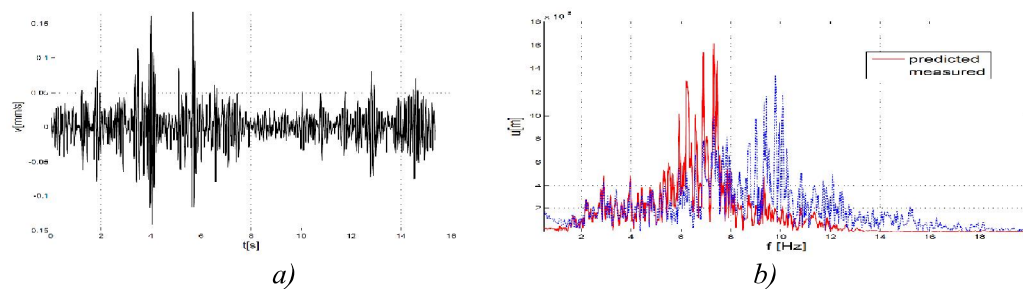
*a)*                    *b)*

*Fig. 5.  High-rise building: a) Ground velocity time history,  b) Predicted and measured displacements*

## 4.  CONCLUSIONS

In this paper the simple numerical model for predicting of building vibrations is presented. The numerical simulations show the dependence of the predicted dynamic response on the dimensions of floor slabs, foundation properties and height of the buildings. The numerical predictions closely matched with the measured responses for low- and mid-rise buildings. In the case of high-rise buildings the discrepancy between obtained results and measured values is obvious, due to many uncertainties of the analysis, e.g.: arrangement of the walls in the buildings, the dimensions of the floor slabs, the details concerning grid foundation and subsoil as well as some disturbances and other side effects.  In addition, in the case of high-rise buildings the influence of bending on the dynamic response should be analysed further.

## ACKNOWLEDGMENT

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# ЈЕДНОСТАВАН МОДЕЛ ЗИД-КРУТА ТАВАНИЦА-ТЛО ЗА ПРЕДВИЂАЊЕ ВИБРАЦИЈА ОД САОБРАЋАЈА

**Резиме:** *У овом раду приказан је једноставан модел зид-крута таваница-тло за прорачун вертикалних вибрација зграда од саобраћаја. Зид је моделиран спектралним елементима, утицај круте таванице узет у обзир преко инерцијалних сила, док је за динамичку крутост темеља на полупростору усвојен израз предложен у литератури. На основу тога је направљен је програм у Matlab-у. Резултати нумеричке симулације су упоређени са резултатима који су добијени мерењем вибрација од саобраћаја на три зграде различите спратности. На основу тога су изведени одређени закључци.*

**Кључне речи:** *Вертикалне вибрације зграда од саобраћаја, спектрални елементи, нумерички модел, верификација модела*