PROBABILISTIC DETERIORATION MODEL FOR TIMBER-CONCRETE COMPOSITE BEAM

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Summary: The paper presents new stochastic process model that will account variability of deterioration of timber-concrete composite beam. We focus on modelling the deterioration of deflection in the mid-span of the beam under a service load. The progress of deterioration and estimate its service life will be presented.

Keywords: Deterioration model, Stochastic proces, Gamma proces

1. INTRODUCTION

The timber-concrete composite (TCC) structure is a structural system in which a timber beam is connected to an upper concrete slab using different types of connectors. The best properties of both materials can be exploited because bending and tensile forces induced by gravity loads are resisted primarily by the timber beam and compression by the concrete slab, while the connection system transmits the shear forces between the two components. TCC structural systems are successfully used in bridges, piers, platforms and in upgrading and strengthening existing timber floors in residential and office buildings. By connecting a concrete slab to a timber beam it is possible to increase significantly (even up to four times) the stiffness of the section than when a timber section is used alone. Because of its lower weight, compared to a reinforced concrete section, the imposed load on the foundationas as well as the seismic effects are reduced. The critical part of any composite structure is the connection between the constituent materials. The shear connector, apart from being stiff, needs to have a certain strength or shear capacity. In order to properly consider the long term behaviour of TCC, time dependent behaviour of constituent materials needs to be considered. Concrete displays creep and shrinkage effects. However timber behaviour is more complex, because its sensitivity to the environmental effects such as the moisture content. Published records of long-term loading experiments have demonstrated that connections can creep, even

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more than timber. Because of the long term processes that occur in the component materials the stress and strain distribution within the TCC section changes in time. For example a decrease in temperature produces a larger shrinkage of concrete slab compared to timber beam, with an overall increase in deflection. In the same manner, an increase in relative humidity leads to timber moistening with swelling and an overall increase in deflection of the beam. A simplified deterministic approach to evaluate the long-term response of TCC beams had been proposed by Fragiacomo and Ceccotti in [1].

2. PROBABILISTIC MODELLING OF DETERIORATION

The probabilistic deterioration models can be generally classified into two categories: random variable models and stochastic process models. The main idea of the random variable models is that one or more of the variables in deterioration model is random variable with certain probaility distribution. Since components in a population are experiencing different rates of deterioration, the rate can be treated as a random variable with an appropriate probability density function, but deterioration of the component will be considered with a fixed deterioration rate throughout its service life. However, it has emerged that for decision making purposes, random variable models have limitations because it is unable to capture temporal effects that could be relevant for long lifecycles such as those in construction. These issues acquire an even greater importance when a composite section such as the one considered here is concerned. Since the late 1990s, stochastic processes have emerged as an alternative to random variable models in providing predicted profiles for componental deterioration.

2.1 Stochastic Gamma Process model

Gamma process is a stochastic process with independent non-negative increments that are gamma distributed with an identical scale parameter. Abdel-Hameed in [2] was the first to propose the gamma process as a proper model for deterioration occurring randomly in time. The gamma process has demonstrated its suitability in modelling gradual damage monotonically accumulating over time, such as wear, fatigue, corrosion, crack growth, erosion, consumption, creep and swell [3]. Firstly, the gamma process is defined in mathematical terms as follows. A random quantity X has a gamma distribution with shape parameter $k > 0$ and scale parameter $\theta > 0$ if its probability density function is given by:

$$Ga(x|k,\theta) = \frac{1}{\Gamma(k)\cdot\theta^k} x^{k-1} \exp\left\{\frac{-x}{\theta}\right\} \tag{1}$$

where

$$\Gamma(a) = \int_{z=0}^{\infty} z^{a-1} e^{-z} dz \tag{2}$$

is the gamma function for $a > 0$. Furthermore, we assume $k(t)$ to be a non-decreasing, right-continuous, real-valued function for $t \geq 0$, with $k(0) \equiv 0$. The gamma process with shape function $k(t)>0$ and scale parameter $\theta>0$ is considered a continuous-time stochastic process $\{X(t);\ t \geq 0\}$ with the following properties:

a) $P(X(0)) = 0$;

b) $\Delta X(t) = X(t + \Delta t) - X(t) \sim Ga(\Delta k(t), \theta)$, $\Delta k(t) = k(t + \Delta t) - \Delta k(t)$

c) $\Delta X(t)$ are independent.

Let $X(t)$ denote the deterioration at time t, $t \geq 0$, and let the probability density function of $X(t)$, in conformity with the definition of the gamma process, be given by

$$f_{X(t)}(x) = Ga(x|k(t), \theta) \quad (3)$$

with expectation and variance

$$E(X(t)) = k(t) \cdot \theta, \qquad Var(X(t)) = k(t) \cdot \theta^2 \quad (4)$$

3. APPLICATION OF GAMMA PROCESS MODEL ON TIMBER-CONCRETE COMPOSITE BEAM

In order to enable efficient management of structure in terms of required maintenance or replacement, it is essential to have enough information about its state. Therefore, the progress of deterioration of the observed structure and estimate its service life, have to be quantified. In particular for TCC beams, component materials will deteriorate at a different pace over the life-cycle. Any analytical model designed to estimate the long term behaviour of TCC sections has to include such diverse effects that develop at different times in the lifecycle. However, closed form solutions will have limited validity in real conditions. We implement the stochastic process approach for TCC sections because temporal variability is very significant and available data is extremely limited.

In this paper we consider a simply supported TCC beam of span 4.2 m, where glued steel bars are used as shear connectors. For medium and long-span composite beams as well as composite beams exposed to external influences (for examle bridges or roof structures), the most serious criterion during the design process is the limit states of maximum deflection [1]. This is so that the focus lies on the modelling deterioration of deflection in the middle range of the TCC beam over time under a service load. Our initial approach is to generate the deterioration model at time t from outcomes that can be based on the available deterministic models or as a result of expert judgement. This serves to simulate the condition of the beam that is monitored through periodic inspections and in that way inspections reveal the progress of the deterioration. Current design code, Eurocode 5, suggests limit value of $l/250$ for long-term deflections of simply supported beams where l is the span length, u_L. Thus, considered TCC beam will reach serviceability limit state when deflection reaches that assumed value. As an initial deflection, deflection at time t_0, is assumed to be elastic deflection u_{el} measured immediately after applying the service load. We observe the relative mid-span deflection of the beam over time, that is define as follows:

$$X(t) = \frac{u(t) - u_{el}}{u_{el}} \quad (5)$$

As we define relative mid-span deflection over time, we can identify its serviceability limit value as ρ:

$$\rho = \frac{u_L - u_{el}}{u_{el}} \tag{6}$$

Due to the nature of this composite beam, relative mid-span deflection is generally uncertain and non-decreasing over time, so it could be regarded as a gamma process.

3.1 Parameter estimation for the gamma process

For modeling of the temporal variability in the deterioration, key input is the trend of the expected deterioration increasing over time.

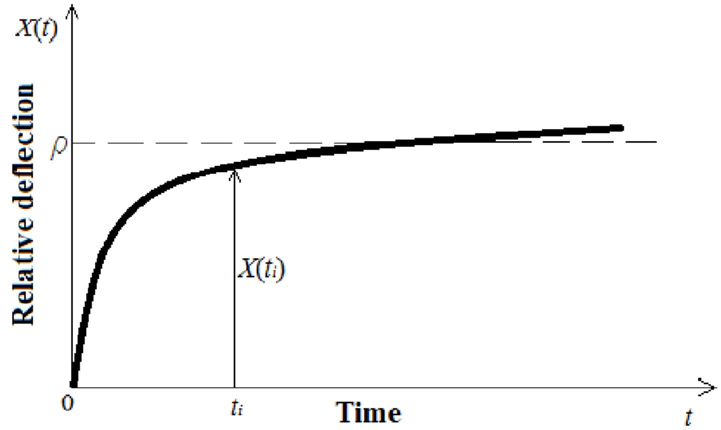

Figure 1. Trend ot the expected relative mid-span deflection increasing over time under service load

Empirical studies show that the expected deterioration at time t can often be represented as a power law:

$$E(X(t)) = k(t) \cdot \theta = ct^b \cdot \theta = at^b \propto t^b \tag{7}$$

for some physical constants $c > 0$, $b > 0$ and $\theta > 0$. The gamma process is called stationary if the expected deterioration is linear in time, i.e., when $b = 1$ and non-stationary when $b \neq 1$. [3] In order to apply the gamma process model to practical examples, statistical methods for the deterioration parameter estimation are required. We can assume that a typical data set for the structure consists of inspection times t_i, $i = 1, . . . ,n$, where $0 = t_0 < t_1 < \cdot \cdot \cdot < t_n$, and corresponding observations of the cumulative amounts of deterioration x_i, $i = 1, . . . , n$, where $0 = x_0 \leq x_1 \leq \cdot \cdot \cdot \leq x_n$. There is often engineering knowledge available about the shape of the expected deterioration in terms of the parameter b. In case that there is no available data of the expected deterioration, as in our case, estimation of the parameter b can be performed based on the least square method. The parameter estimate of the power b used here was 1/6, that suggests that the deterioration is non-linear in time and we can assume that the expected value of the stochastic gamma process follows a non-linear power function. After determining the parameter b, it is necessary to estimate the other two parameters of the gamma process, the shape parameter and scale parameter.

The two most common methods of parameter estimation are Maximum Likelihood Method and Method of Moments. The latter is considered more appropriate and applied here. Estimation of parameters *c* and *θ*, using the Maximum Likelihood Method, can be realized by maximizing the logarithm of the likelihood function of the observed deterioration increments, $\delta_i = x_i - x_{i-1}$, $i = 1, ..., n$, where likelihood function is represented as a product of independent probability density functions:

$$l(c,\theta) = \log L(c,\theta) = \sum_{i=1}^{n}(c[t_i^b - t_{i-1}^b] - 1)\log\delta_i - c\sum_{i=1}^{n}[t_i^b - t_{i-1}^b] - \sum_{i=1}^{n}\log\Gamma(c[t_i^b - t_{i-1}^b]) - \frac{1}{\theta}\sum_{i=1}^{n}\delta_i \quad (8)$$

The maximum-likelihood estimates *c* and *θ* are 371.118 and 0.0049, respectively. Figure 2 demonstrates the use of Gamma process representation for probability density functions of cumulative deterioration of relative mid-span deflection at different time.

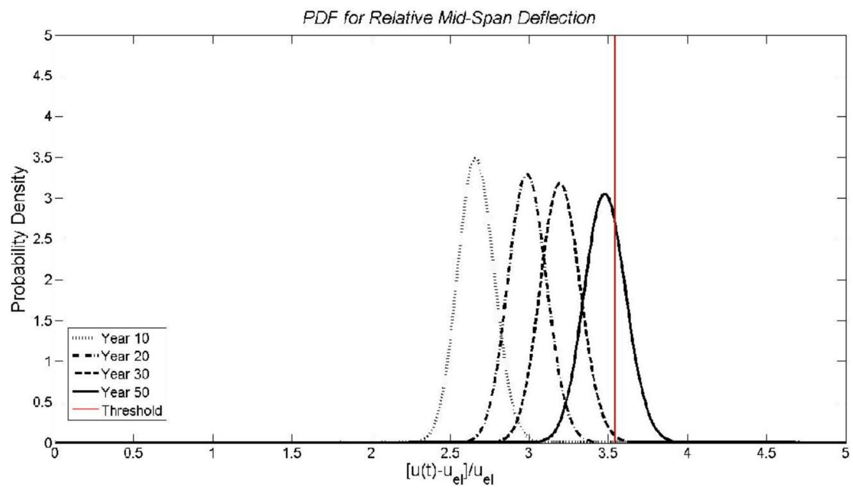

Figure 2. Probability density functions of deterioration X(t) for certain years

3.2 Distribution of serviceability lifetime

For practice the estimate of the time when the structure would reach serviceability limit of some form would be of interest to owners and stake holders. The serviceability lifetime *T* can be defined as the first time when the sample path of $X(t)$ exceeds the limit *ρ*. Due to the gamma distributed deterioration, the serviceability lifetime can then be written as:

$$F_T(t) = \Pr(T \le t | X(t) \ge \rho) = \int_{x=\rho}^{\infty} f_{X(t)}(x)dx = \frac{\Gamma(k(t), \rho/\theta)}{\Gamma(k(t))} \quad (9)$$

where

$$\Gamma(a,x) = \int_{t=x}^{\infty} t^{a-1}e^{-t}dt \quad (10)$$

is the incomplete gamma function for $x \ge 0$ and $a > 0$.

The cumulative probability function of serviceability lifetime is shown in Figure 3.

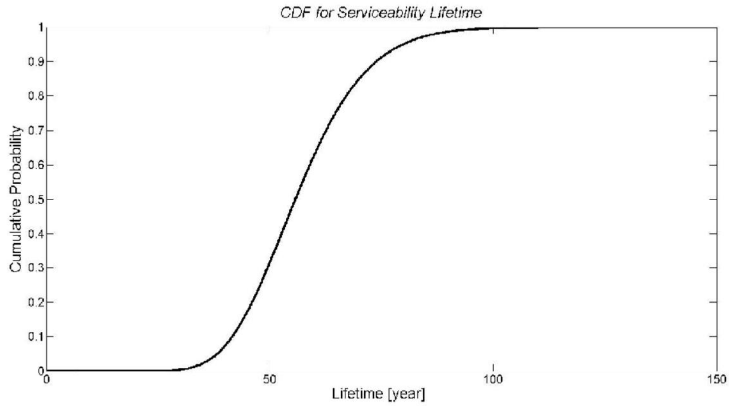

Figure 3. Cumulative distribution function for serviceability lifetime

4. CONCLUSIONS

In order to enable efficient management of structures in terms of required maintenance,and replacement, it is essential to be able to capture the uncertain nature of the deterioration process. In this paper we have considered a stochastic gamma process model that can capture the true nature of deterioration of timber-concrete composite beams. As the increasing deflection can lead to violation of serviceability limit state the serviceability lifetime is of interest and therefore considered in this paper.

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ПРОБАБИЛИСТИЧКИ МОДЕЛ ДЕТЕРИОРАЦИЈЕ СПРЕГНУТЕ ГРЕДЕ ТИПА ДРВО-БЕТОН

Резиме: *Рад представља нови стохастички модел који ће узети у обзир варијабилност детериорације спрегнуте греде типа дрво-бетон. Наш фокус ће бити на моделирању детериорације угиба у средини распона посматране греде под експлоатационим оптерећењем. Биће представљени прогрес детериорације и процена експлоатационог века.*

Кључне речи: *Модел детериорације, Стохастички процес, Гама процес*