CRITICAL CRACK LENGTH IN SMOOTH REINFORCEMENT BARS

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Summary: *The cases that occur in the implementation of the smooth reinforcement steel bars are analyzed. Geometry of the initial cracks on axially tensioned cylindrical rods are taken from calculations based on the rheological-dynamical analogy (RDA). Plastic zone size is determined by Dugdale model. From the fracture mechanics standpoint, model is reduced to a plane with symmetrically placed edge cracks.*

Keywords: *Dugdale model, RDA, plastic zone, critical crack*

1. INTRODUCTION

Linear-elastic fracture mechanics (LEFM) assumes that the plasticity in the zone ahead of the crack front can be neglected because the nonlinear deformation of materials is limited to a very small area around the crack tip. However, in a highly ductile material that is not happening. In the soft steel, a crack expansion is preceded by large plastic deformation. Although the element load is below the yield point, just before the expansion of the crack, the stress field in the zone around the crack tip reaches the yield point or close to its value. In the elastic-plastic fracture mechanics (EPFM) various models and approaches for determining the length of plastic zone ahead of the crack tip are proposed. Dugdale model [1] attributes the reduction of the stress at the top of crack to macroscopic plasticity while by Barenblatt model [2] this effect is caused by the large cohesive forces in the vicinity of the crack tip, and its distribution is a constant of a material. Inclusion of RDA to the analysis of problems related to fracture mechanics starts with a paper published by Milašinović [3][4]. This paper shows how the RDA can be used in the study of stress concentration at the crack tip, calculate: fatigue fracture frequency, crack depth, fatigue strength, cyclic ductility, crack width, crack opening and stress intensity factor (SIF).

2. DUGDALE MODEL

Elastic-plastic crack expansion by Dugdale is the classical theoretical basis of EPFM.

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The plastic zone around the crack tip is included in the linear-elastic approach by having the real crack length *2a* expanded by the value of *2c,* Figure 1.

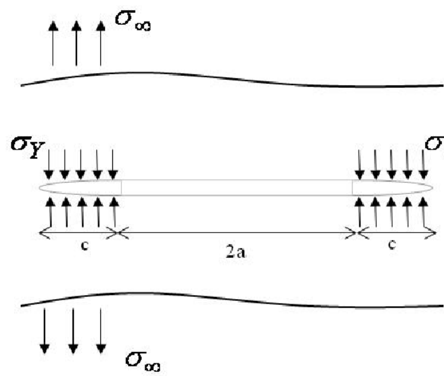

Figure 1. Dugdale model

Length *2 (a + c)* gives the equivalent elastic crack in Dugdale model. External forces are present at length *c* with intensity σ_Y, which prevent crack opening on the part of plastic deformation. Length of plastic zone *c* is determined so that the stress on the tip of an imaginary crack is not singular (K_I =0) and can be calculated using Equation (1)

$$\frac{a}{a+c} = cos\frac{\pi}{2}\frac{\sigma_\infty}{\sigma_Y} \tag{1}$$

With symmetrically positioned edge cracks, Figure 2,

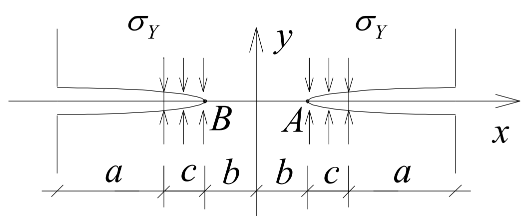

Figure 2. Geometry for edge cracks

the length of the plastic zone by Dugdale is also determined from the condition that the SIF on the edge of an imaginary crack equals zero:

$$K_I\left(\sigma_\infty, a+c\right) + K_I\left(\sigma_Y, a+c\right) = 0 \tag{2}$$

Stress intensity factor (SIF), at the top of an imaginary crack, due to an external load σ_∞ is as follows [5]

$$K_I\left(\sigma_\infty, a+c\right) = C\ \sigma_\infty\sqrt{\pi\left(a+c\right)} \tag{3}$$

where *C*=1.12.

At the top of an imaginary crack due to stress intensity σ_Y, which prevents crack opening, SIF can be determined based of its expression due to the concentrated force at arbitrary point:

$$K_{IA}\left(\sigma_Y, a+c\right)=\frac{Q}{\sqrt{\pi b}}\sqrt{\frac{x+b}{x-b}} \tag{4}$$

$$K_{IB}\left(\sigma_Y, a+c\right)=\frac{Q}{\sqrt{\pi b}}\sqrt{\frac{x-b}{x+b}} \tag{5}$$

where

$Q=-\sigma_Y\,dx$ for $b\le x\le b+c$

Displayed equation for SIF is actually Greene's function for its determination.
SIF at point A due to the stress σ_Y is given by the following equation

$$K_{IA}\left(\sigma_Y, a+c\right)=\int_{-(b+c)}^{-b}\frac{-\sigma_Y}{\sqrt{\pi b}}\sqrt{\frac{x+b}{x-b}}dx+\int_{b}^{b+c}\frac{-\sigma_Y}{\sqrt{\pi b}}\sqrt{\frac{x+b}{x-b}}dx \tag{6}$$

With substitution of variable x=-u, first integral becomes

$$\int_{-(b+c)}^{-b}\frac{-\sigma_Y}{\sqrt{\pi b}}\sqrt{\frac{x+b}{x-b}}dx=\left|\begin{matrix} x=-u & -b=-u \\ & b=u \\ dx=\left(-1\right)du & -\left(b+c\right)=-u \\ & b+c=u \end{matrix}\right|=\int_{b+c}^{b}\frac{-\sigma_Y}{\sqrt{\pi b}}\sqrt{\frac{-u+b}{-u-b}}(-1)du=$$

$$=\int_{b}^{b+c}\frac{-\sigma_Y}{\sqrt{\pi b}}\sqrt{\frac{u-b}{u+b}}du$$

Thus, SIF can be expressed as

$$K_{IA}\left(\sigma_Y, a+c\right)=\frac{-\sigma_Y}{\sqrt{\pi b}}\int_{b}^{b+c}\left[\sqrt{\frac{x-b}{x+b}}+\sqrt{\frac{x+b}{x-b}}\right]dx=-\frac{2\sigma_Y}{\sqrt{\pi b}}\int_{b}^{b+c}\frac{x}{\sqrt{x^2-b^2}}dx \tag{7}$$

By solving the integral

$$\int_{b}^{b+c}\frac{x}{\sqrt{x^2-b^2}}dx=\sqrt{c(2b+c)}$$

on top of an imaginary crack due to stress σ_Y is obtained

$$K_{IA}\left(\sigma_Y, a+c\right)=-\frac{2\sigma_Y}{\sqrt{\pi b}}\sqrt{c(2b+c)} \tag{8}$$

By substituting (3) and (8) into (2) yields

$$1.12\;\sigma_\infty\sqrt{\pi\left(a+c\right)}-\frac{2\sigma_Y}{\sqrt{\pi b}}\sqrt{c(2b+c)}=0$$

therefore, the Dugdale crack tip plasticity model for the symmetrically placed edge cracks in an infinite strip is

$$1.12\frac{\pi}{2}\frac{\sigma_\infty}{\sigma_Y}=\sqrt{\frac{c(2b+c)}{b(a+c)}} \tag{9}$$

in which

$$b=\frac{D_0}{2}-\left(a+c\right) \tag{10}$$

Also the SIF at point B due to the stress σ_Y is given by the following equation

$$K_{IB}\left(\sigma_Y,a+c\right)=\int_{-(b+c)}^{-b}\frac{-\sigma_Y}{\sqrt{\pi b}}\sqrt{\frac{x-b}{x+b}}dx+\int_{b}^{b+c}\frac{-\sigma_Y}{\sqrt{\pi b}}\sqrt{\frac{x-b}{x+b}}dx$$

With substitution of variable x=-u, first integral becomes

$$\int_{-(b+c)}^{-b}\frac{-\sigma_Y}{\sqrt{\pi b}}\sqrt{\frac{x-b}{x+b}}dx=\left|\begin{array}{ll} x=-u & -b=-u \\ & b=u \\ dx=\left(-1\right)du & -\left(b+c\right)=-u \\ & b+c=u \end{array}\right|=\int_{b+c}^{b}\frac{-\sigma_Y}{\sqrt{\pi b}}\sqrt{\frac{-u-b}{-u+b}}(-1)du=$$

$$=\int_{b}^{b+c}\frac{-\sigma_Y}{\sqrt{\pi b}}\sqrt{\frac{u+b}{u-b}}du$$

Thus, SIF can be expressed as follows

$$K_{IB}\left(\sigma_Y,a+c\right)=\frac{-\sigma_Y}{\sqrt{\pi b}}\int_{b}^{b+c}\left[\sqrt{\frac{x+b}{x-b}}+\sqrt{\frac{x-b}{x+b}}\right]dx=-\frac{2\sigma_Y}{\sqrt{\pi b}}\int_{b}^{b+c}\frac{x}{\sqrt{x^2-b^2}}dx$$

and

$$K_{IB}\left(\sigma_Y,a+c\right)=K_{IA}\left(\sigma_Y,a+c\right)$$

This means that the lengths of plastic zones at both end cracks are equal.

3. NUMERICAL RESULTS

For the analysis of critical crack length, the axial stretched cylindrical rod was selected: elastic modulus $E=210\,GPa$, toughness $G_C=107\,KJm^{-2}$.

Critical crack lenght is calculated for smooth reinforcement bars with diameters $D_0=19,\ 16,\ 14,\ 12,\ 10$ and $8\,mm$.

Rod model is analyzed as a three-dimensional rotationally symmetric and reduced to the case of plane strain in classical fracture mechanics model with symmetrically placed edge cracks. Milašinović [4] proposed that initial cracks geometry a_f of the rod are obtained as a result of calculation based on the RDA.

Dugdale considered the plastic regions to take the form of narrow strips extending a distance c from each crack tip. For purpose of the analysis in this paper, the elastic edge crack of length $a=a_f$ is allowed to extend elastically to a critical crack length $a_{cr}=a_f+c=a+c$; however, an internal stress is applied in the plastic regions c to close extended part of crack.

Initial crack depth $a_f=a$=5.26485 mm was obtained as a result of calculation based on the RDA at a load of σ_∞=142 Mpa [4], which corresponds to the stress proportionality σ_P for a given material. According to load, appropriate yield stress is σ_Y=258.17 MPa.

We can define the length of plastic zone c using Equation (9) as follows

$$1.12\frac{\pi}{2}\frac{142}{258.17}=\sqrt{\frac{c(2(\frac{0.019}{2}-0.00526485-c)+c)}{(\frac{0.019}{2}-0.00526485-c)(0.00526485+c)}}$$

Solving this equation we get

$$c=0.00224705\,m$$

Critical crack depth is the sum of the initial crack depth $a_f=a$ and the length of a plastic zone c

$$a+c=5.26485\,mm+2.24705\,mm=7.5119\,mm$$

Result obtained with RDA concept improved by Dugdale crack tip plasticity model for the symmetrically placed edge cracks in an infinite strip corresponds to the experimental results for cylindrical rods which can be calculated using the following equation [6]

$$K_I=\left(1.72\frac{D_0}{d}-1.27\right)\frac{P}{D_0^{\frac{3}{2}}}$$

where $d=D_0-2(a+c)$, while SIF in the case of plane strain is $K_I=\sqrt{G_C E(1-\nu^2)}$
By solving

$$\left(1.72\frac{0.019}{d}-1.27\right)\frac{\frac{0.019^2\pi}{4}142\times10^6}{\sqrt{0.019^3}}=1.41327\times10^8$$

$$d=0.0031233\,m$$

we get critical crack depth

$$a+c=\frac{D_0-d}{2}=\frac{19\,mm-3.1233\,mm}{2}=7.93835\,mm$$

Results for other diameters are shown in Table 1.

Table 1.

D_0 [mm]	σ_P [MPa]	σ_Y [MPa]	a_f=a [mm]	c [mm]	$a+c$ [mm]	experimental results [mm]
8	142	216,99	2,4167	1,0959	3,5126	3.55434
10	142	224,48	3,27138	1,23676	4,50814	4.38316
12	142	231,96	3,92944	1,44556	5,37500	5.19613
14	142	239,45	4,43302	1,69330	6,12632	5.99498
16	142	246,93	4,82427	1,93967	6,76394	6.78102
19	142	258,17	5,26485	2,24705	7,51190	7.93835

4. CONCLUSION

The present study has analyzed the physical mechanism and experimental proof for the transition from the complex three-dimensinal RDA crack geometry into a simple plane one using the Dugdale model.
A new analytical RDA model for the predictions related to fatigue crack growth and failure of rods made of ductile materials has already been presented [4]. Considering of the initial crack geometry at a load which corresponds to the stress proportionality for a given material and using the Dugdale model [1], the experimental results [6] show that the model predictions are reasonable.

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КРИТИЧНА ДУЖИНА ПРСЛИНЕ ГЛАТКОГ АРМАТУРНОГ ЧЕЛИКА

Резиме: *Анализирани су случајеви који се јављају у примени глатког арматурног челика. Геометрија почетне прслине на аксијално затегнутом цилиндричном штапу добија као резултат прорачуна базираног на реолошко-динамичкој аналогији (РДА). Величина пластичне зоне одређена је према Дагејловом моделу. Са аспекта механике лома модел је сведен на равански са симетрично постављеним рубним прслинама.*

Кључне речи: *Дагдејлов модел, РДА, пластична зона, критична прслина*