

## EXPERIMENTAL ANALYSIS OF SHEAR STRENGTH OF BEAMS AND APPLICATION OF STM

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**Summary:** Results of experimental testing of shear strength of the beams were used to analyze the carrying capacity of beams using the Strut-and-Tie Model (STM). Tests were performed on beams made of normal and high strength concrete. Strain of concrete in the D-region was measured in the experimental test, and the appropriate stresses were calculated. Relationship between stresses that can be used for calculation of capacity of STM elements and strain is examined. The impact of concrete strength classes on the effective strength of STM elements is analyzed.

**Keywords:** Experiment, Strut-and-Tie Model, shear, strength of the strut

### 1. INTRODUCTION

In some reinforced concrete elements, such as for example deep beams, there are areas where calculation based on hypothesis of linear strain distribution in section cannot be applied. Such areas are the areas of discontinuity or D-regions. For the calculation of shear strength of deep reinforced concrete beams where the fracture occurs in D-regions, strut-and-tie model (STM) is applied.

However, the results of experimental tests are not always consistent with the results obtained using the STM. Experimental researches also show that results obtained using the strut-and-tie models for high-strength concrete are less conservative than for normal strength concrete. Because of this, the effect of concrete strength on the effective capacity of the elements of strut-and-tie model must be taken into account. Although STM has been implemented in various building codes throughout the world, there are many details of the process that have yet to be well-documented, [1].

### 2. EXPERIMENTAL PROGRAM

The experimental research has been done on pairs of identical beams made of high and normal strength concrete. For high strength concrete (HSC), characteristic value of compressive strength is  $f_{ck}=90$  MPa, of tension strength is  $f_{ct}=4.0$ MPa, while for normal strength concrete (NSC) these values are  $f_{ck}=35$  MPa, and  $f_{ct}=2.5$ MPa. The beams have been exposed the action of one concentrated force in the middle of the span brought to

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the failure which can be characterized as shear failure. Strains are measured using strain gauges – rosetes (type TML) on three positions along each strut in D regions of the experimental models, [2].

The results of experimental research are used for analysis of STM.

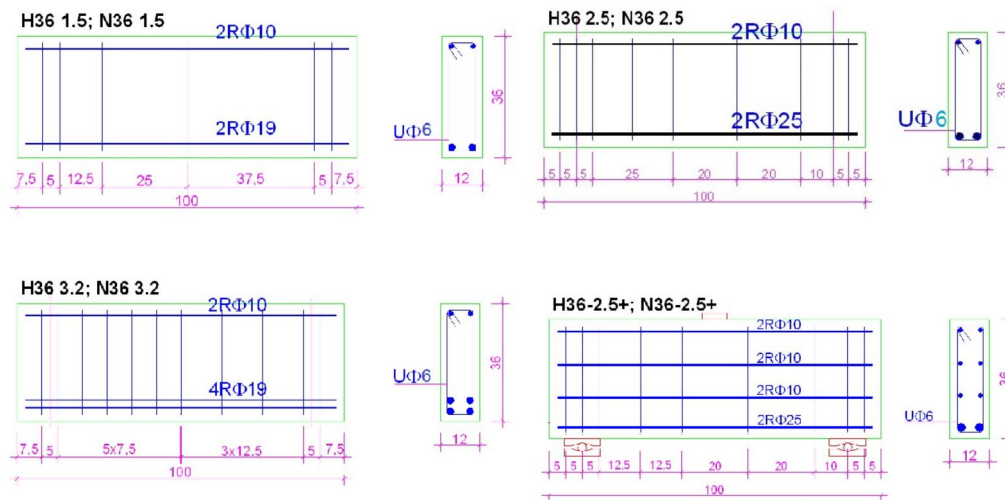


Figure 1. Dimensions and reinforcement of the tested models, [2]

### 3. STRUT-AND-TIE MODEL

STM is a very simple idea with wide-ranging applications. The difficulty lies in translating a behavior model into a code specification. The critical part for determining the strength of a truss model is determining the strength of the individual components (struts, ties, and nodes). STM is governed by the lowerbound theorem of plasticity which requires only static equilibrium and yield conditions to be satisfied. Therefore, the appropriate yield condition for struts and nodes must be identified. The yield conditions for ties are easily quantifiable and readily available to design engineers, but the yield conditions for struts and nodes vary greatly from code to code, [1].

The observed experimental investigation was carried out on reinforced concrete beams with shear span to depth ratio of 1.25. The shape and dimensions of the beams, as well as the plan of reinforcement, are shown in Figure 1. Strut-and-tie model of the tested beams is shown in Figure 2. It is composed of two compressed struts and one tie, which are connected with the nodes. Node B is of type CCC, where the elements in compression come together, while the A and C are nodes of CCT type which connect elements in compression and one tie in tension. Geometric characteristics of the strut-and-tie model are determined from the geometrical characteristics of the beam, the position of the reinforcement geometry and supports, as well as pattern of cracks. The system is always in balance and equilibrium conditions of the individual nodes get the relation:  $V_u = F_{AB} \cdot \sin\theta$ ;  $F_{AC} = F_{AB} \cdot \cos\theta$ ;  $V_u = F_{AC} \cdot \tan\theta$ . From the geometry of the model the following

is obtained:  $\theta=38^\circ$ ;  $w_t=8\text{cm}$ ;  $w_{s1}=13\text{cm}$ ;  $w_{s2}=6,1\text{cm}$ . For beams H36-32 and N36-32  $\theta=36^\circ$ ,  $w_t=12\text{cm}$ ,  $w_{s1}=16\text{cm}$  and  $w_{s2}=6,4\text{ cm}$  is obtained.

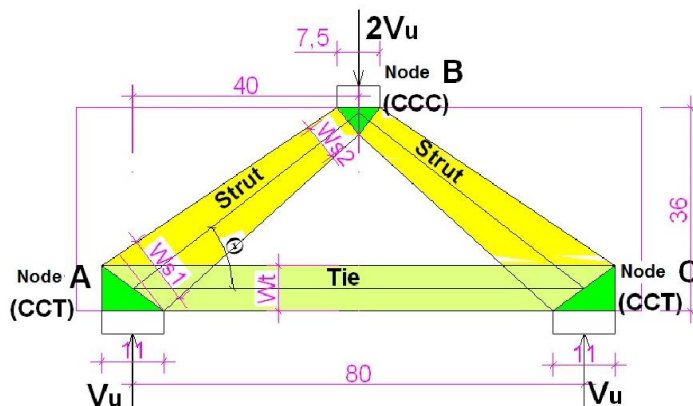


Figure 2. Strut-and-tie model used in stressed analysis

In all tested beams the failure has occurred in the compressed strut B-C in the part directly alongside the node B. Fracture zone in some cases included the area of the node B. Design capacity of node B is significantly higher than the design capacity of strut B-C.

#### 4. STRENGTH OF STRUTS AND NODES

The stress levels in the struts and nodes must be kept below allowable stresses. The allowable stress is determined differently by different codes. Formulas for calculation of bearing of STM elements are a function of concrete strength under uniaxial compression.

Table 1. Ultimate shear load by test and STM according ACI and EC2 [2,3,4]

Sign of beam	$V_u$ - test		$V_u$ - ACI (kN)		$V_u$ - EC2 (kN)		$V_u$ - test/ $V_u$ - ACI		$V_u$ - test/ $V_u$ - EC2	
	Load (kN)	Element of failure	Strut B-C	Node B	Strut B-C	Node B	(B)	(B-C)	(B)	(B-C)
N36-15	135.3	B,B-C	80.4	134.1	81.4	135.6	1.01	1.68	1.00	1.66
N36-25	200	B-C	100.5	134.1	81.4	135.6	1.49	2.0	1.47	2.46
N36-32	225.6	B-C	100.7	134.3	81.5	135.9	1.68	2.24	1.66	2.77
N36-25+	219.8	B-C	100.5	134.1	81.4	135.6	1.64	2.19	1.62	2.70
H36-15	205	B,B-C	195.6	325.4	151.9	252.8	0.63	1.05	0.81	1.35
H36-25	315	B,B-C	244.2	325.4	151.9	252.8	0.97	1.29	1.25	2.07
H36-32	335	B,B-C	244.6	325.9	152.1	253.2	1.03	1.37	1.32	2.20
H36-25+	337.5	B-C	244.2	325.4	151.9	252.8	1.04	1.38	1.34	2.22

Most building codes, as ACI 318-08 provide correction coefficients in the form of constants, independent on the strength of concrete. EC2 takes in to account the characteristics compressive strength of concrete using correction coefficient  $v$ . Design

concrete strength in cracked concrete (with longitudinal cracks) is calculated using the following expression:

$$\sigma_{Rd\ max} = 0.6\nu f_{cd} \tag{1}$$

The coefficient  $\nu$  is calculated by the equation:

$$\nu = 1 - \frac{f_{ck}}{250} \tag{2}$$

where  $f_{ck}$  is characteristic compressive strength of concrete and  $f_{cd}$  is design concrete compressive strength. For compressed nodes (CCC), according EC2, stress on the node face is limited to  $\nu f_{cd}$ , while for nodes with one tie (CCT) the stress is limited to  $0.85\nu f_{cd}$ , and for nodes with more than one tie (CTT) the stress is limited to  $0.75\nu f_{cd}$ . [3].

Results of test failure load and ultimate shear load according STM are shown in Table 1.

### 5. ANALYSIS OF STRESSES

The behavior of struts and nodes are inseparably linked. A strut must abut a node. Therefore, there will be a common plane where the strut and node must have equal stress. The minimum cross-sectional area of a strut, occurs where the strut frames into a node. In this location the stresses in the strut will be the greatest. To maintain the equilibrium across the plane of intersection of a strut and a node the stresses on opposite sides of the plane must be equal. Therefore, the peak stress in a strut will be equal to the stress at the face of the abutting node, [1].

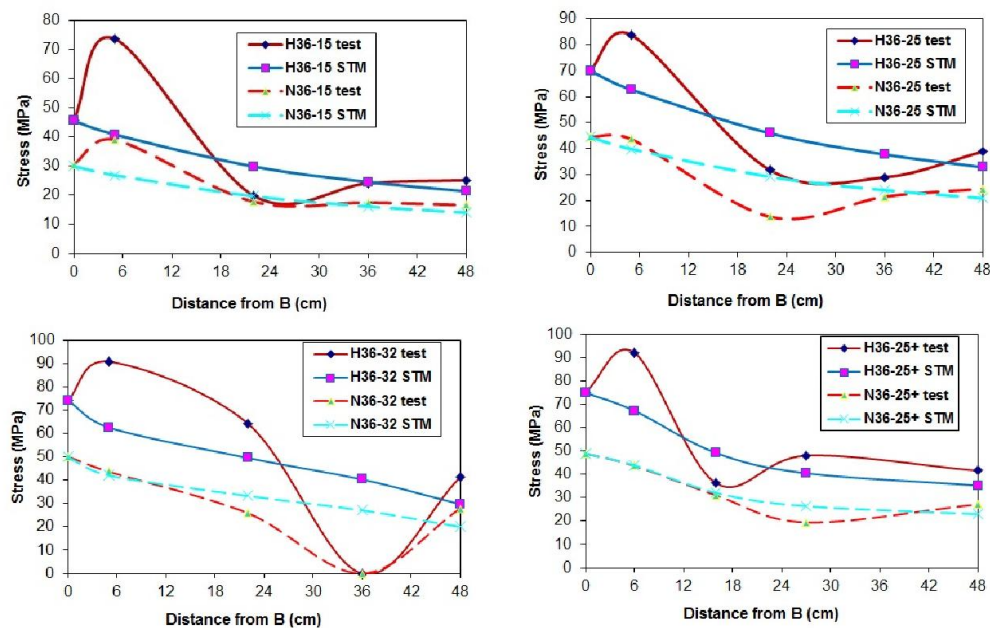


Figure 3. Stresses along the strut BC (BA) according STM and according strain measured in the test

Stresses shown in diagrams on distance 0 (in node B) have values that are calculated according to STM, for both options (“test” and “STM”). Stresses in remaining points along the strut, except last point on distance 48, for option “test” are defined based on experimental stress-strain diagram for uniaxial compression and measured strain. The most critical elements in the strut-and-tie model will be those that are in contact with the external boundary of the member. Boundary elements will be influenced by bearing areas and support reactions. Bearing areas and reactions will limit the size of the adjacent nodes, and therefore limit the allowable force in those elements, [1]. Experimental analysis of beams under failure load shown that the most critical element in the model is the strut BC (or BA) on the place where it frames in node B. Diagrams on Figure 3 show that stresses on connections of node B and struts BC and BA are the most critical.

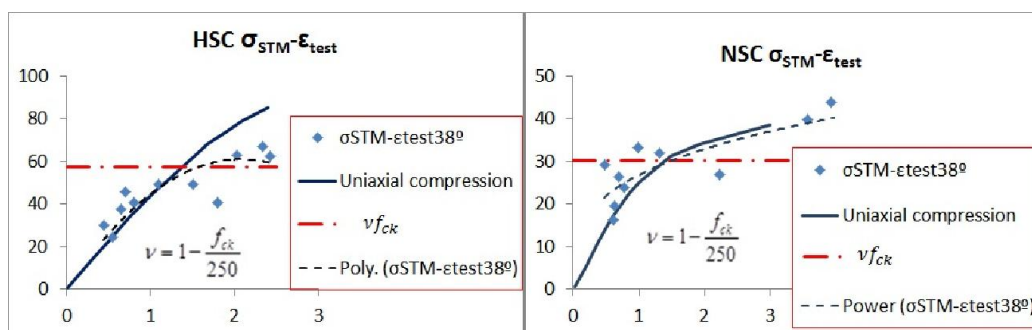


Figure 4. Correlation of stresses according STM and strain measured during the test

Diagrams on Figure 4 show correlation between stresses in the strut calculated according STM for ultimate shear load and the strain measured in the experimental research. Data from all strain gauges and all beams are shown on the diagram. Strains which are higher than 2‰ (HSC), or 3‰ (NSC) are measured on the compressed strut B-C in part directly alongside the node B. Line „long dash dot” shows level of stresses  $v f_{ck}$ , where  $v$  is correction factor according (2). This line on HSC (C90/105) diagram is very close to values of stresses  $\sigma_{STM}$  with corresponding strain  $\epsilon \approx 2-2.5\text{‰}$ . For NSC max measured strain are between 3-4‰. Corresponding stresses according STM have significantly higher values than  $v f_{ck}$  for concrete C35/45.

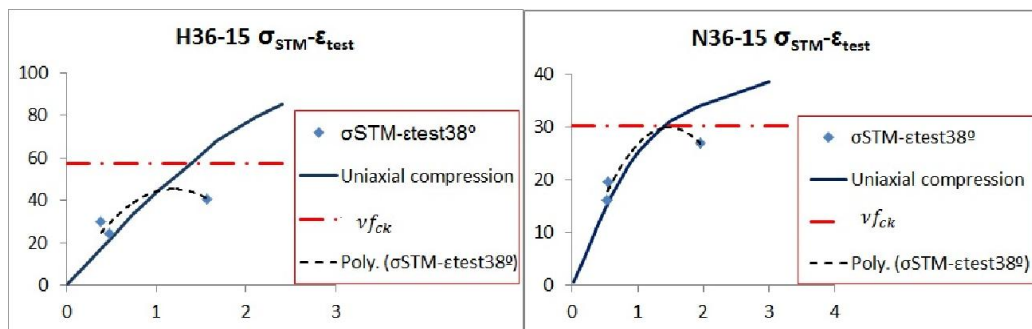


Figure 5. STM stress and test strain in D region of the beams without shear reinforcement

Maximal strain in the beams marked with H36-15 and N36-15, which have no shear reinforcement, reached smaller values than the strain in the other beams with more shear reinforcement. Corresponding stresses which were calculated according STM have significantly smaller values than  $\nu f_{ck}$ , especially for beam made of HSC, as shown on Figure 5.

## 6. CONCLUSION

In the Strut-and-Tie Model which is used for design of shear strength of the beam the concrete strut is critical element. Some design codes propose that design strength for concrete struts should be calculated based on the concrete compressive strength reduced using correction coefficient depending of the type of nodes connected with strut. EC2 proposes that design strength of concrete strut depends on  $f_{ck}$  using corrective factor  $\nu$  which is a function of  $f_{ck}$ . Analyses show that the recommended factor  $\nu$  should be reviewed and the impact of distributed reinforcement around the strut should be considered.

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## ЕКСПЕРИМЕНТАЛНА АНАЛИЗА СМИЧУЋЕ НОСИВОСТИ ГРЕДА И ПРИМЈЕНА STM

**Резиме:** Резултати експерименталних испитивања смичуће носивости греда искоришћени су за анализу носивости греда примјеном Strut-and-Tie Model (STM). Испитивања су обављена на гредама од бетона нормалне и високе чврстоће. У експерименталном испитивању мјерене су деформације бетона у D-региону, и прорачунати одговарајући напони. Проучава се веза између деформација и напона у испрском бетону која може послужити за прорачун носивости елемената STM и анализира утицај класе бетона на ефективну чврстоћу.

**Кључне речи:** Експеримент, Strut-and-Tie Model, смицање, чврстоћа притиснутог штапа