Међународна конференција
Савремена достигнућа у грађевинарству 24.-25. април 2014. Суботица, СРБИЈА

RELIABILITY ANALYSIS OF TIMBER STRUCTURES

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UDK: 624.011.1:624.046
DOI: 10.14415/konferencijaGFS2014.008

Summary: *This paper presents a parametric analysis of the reliability of bent wooden beams. Reliability coefficient is determined for the cases of utilization of section beams due to the normal stress, shear stress and deformation of beams. Out of these three conditions only one is authoritative, and the one that gives the smallest load capacity beams. Parametric analysis was performed according to the requirements of the Eurocodes, both in terms of stress and strain analysis, and in part that requires the reliability of structures. To determine the reliability coefficient, well-known structural reliability of analytical methods is used, as well as application programs. The goal of the analysis is to show how the theoretical results of reliability of bent wooden beams are used in practice according to Eurocodes.*

Keywords: *Reliability, Timber Structures, Eurocodes.*

1. INTRODUCTION

In the modern design of timber structures, in addition to other conditions and requirements that must be met, great attention is paid to the resistance, i.e. durability of the building or designing buildings to life design construction. Basic requirements for capacity, serviceability and durability of structures are given in the Eurocodes [1]. The wooden structure should be designed so that capacity and usability throughout the lifetime of the structure are not affected, and this is achieved by regular maintenance. Also every timber structure must have adequate: structural resistance, serviceability, and durability [1]. General principles of durability and reliability of structures are described in the Eurocodes [1], [2], [3]. The basic reliability elements considered in these papers include probability of failure p (or equivalent reliability index β) corresponding to a certain reference period T used in verification of structural reliability [4]. Reliability index β is determined by the beam loaded by uniform load for three typical cases, bending, shear and deflection of the beam. Service life of the structure is determined by the importance of the building and its importance for the environment. At the beginning

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of design must be clearly and precisely define the life cycle of construction and service life which is defined by Eurocode [1] as the assumed period of construction or part of the structure with regular maintenance that does not require a large recovery and repair.

2. RELIABILITY ANALYSIS METHODS

Of all requirements to be met by a wooden structure, the most important features are: safety and serviceability. Both requirements must be guaranteed for some predefined period of time – **durability**, and achieved by minimum cost [4]. Each of these conditions can be written in the form of **limit state** conditions, which are generally expressed as:

$$g(X_1, X_2, ...X_n) \geq 0 \tag{1}$$

The X_i are random variables that are not necessarily physical quantities such as size, load, etc... They may also be of some other size, such as deflection of the beam etc. [4]. **Limit state** equation clearly separates the two opposite zones as follows: *safe* area and the area of the *fracture*.

$$g(X_1, X_2, ...X_n) = 0 \tag{2}$$

Collapse of structures or failure is defined by the *failure* condition as:

$$g(X_1, X_2, ...X_n) < 0 \tag{3}$$

Based on the above it follows:

$$P_f = P\{g(X_1, X_2, ...X_n) < 0\} \tag{4}$$

P_f represents a probability *of failure.* Method for the determination of this probability depends on the complexity of the function. Some of the basic methods are: Monte-Carlo method, Basler, in the notation of Cornell, method of Hasofer and Lind etc. In further are given basic principles of the methods of Hasofer - Lind. The **limit state** function *g* is given as:

$$g = R - E \tag{5}$$

In these expressions R and E are *stochastic variables* representing, in terms of structural engineering, the resistance of a section and the stress in this section due to applied loads and actions, respectively [4]. Toward a solution Hasofer and Lind, performed the transformation limit state function in the so-called *standard space*. The random variables *R* and *E* are transformed into U_1 and U_2:

$$U_1 = \frac{R - \mu_R}{\sigma_R} \Rightarrow R = U_1 \cdot \sigma_R + \mu_R \tag{6}$$

$$U_2 = \frac{E-\mu_E}{\sigma_E} \Rightarrow E = U_2 \cdot \sigma_E + \mu_E \tag{7}$$

The variables U_1 and U_2 are sometimes called *reduced variables* [5]. The limit state function g can be expressed in terms of the reduce variables by using Eqs.6 and 7. The result is:

$$g = R - E = \left(U_1 \cdot \sigma_R + \mu_R\right) - \left(U_2 \cdot \sigma_E + \mu_E\right) = \left(\mu_R - \mu_E\right) + U_1 \cdot \sigma_R - U_2 \cdot \sigma_E \tag{8}$$

The line of interest to us in reliability analysis is the line corresponding to $g\left(U_1, U_2\right) = 0$ because this line separates the safe and failure domains in the space of reduced variables [5]. From the above it follows that the **reliability index** as the *shortest* distance from origin of reduced variables to the line $g\left(U_1, U_2\right) = 0$. According to Eurocode [1] reliability index is marked with β and in literature, often with HL (*Hasofer-Lind*) safety index (Fig.1).

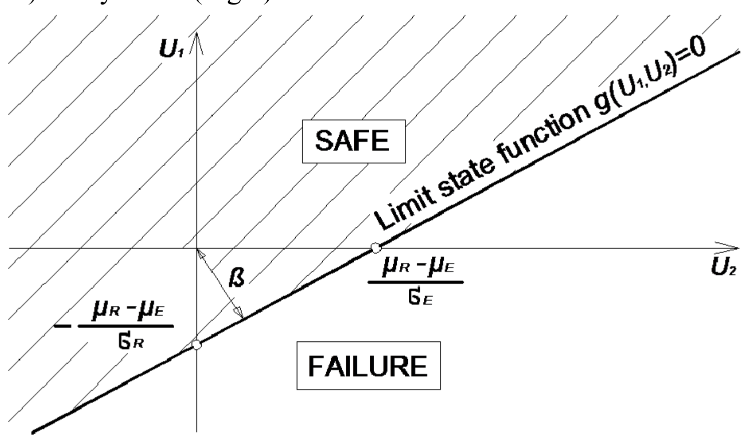

Figure 1. Reliability index

Using geometry easily be reached to **reliability index** (shortest distance):

$$\beta = \frac{\mu_R - \mu_E}{\sqrt{\sigma_R^2 + \sigma_E^2}} \tag{9}$$

From the above follows:

$$\beta = -\Phi^{-1}\left(P_f\right) \qquad P_f = \Phi\left(-\beta\right) \tag{10}$$

3. RELIABILITY ANALYSIS OF FLEXURAL TIMBER MEMBERS

The three types of failure modes (failure limit state function) are represented by following failure criteria:

mode 1 -bending failure: $$g_1\left(q,b,f_{m,g,k,},h,l\right)=\frac{q}{b\cdot f_{m,g,k}}-\frac{4}{3}\cdot\left(\frac{h}{l}\right)^2=0 \qquad (11)$$

mode 2-chear failure: $$g_2\left(q,b,f_{v,g,k},h,l\right)=\frac{q}{b\cdot f_{m,g,k}}-\frac{4}{3}\frac{f_{v,g,k}}{f_{m,g,k}}\left(\frac{h}{l}\right)=0 \qquad (12)$$

mode 3-deflection: $$g_3\left(q,b,E_{0,g,mean},h,l\right)=\frac{q}{bf_{m,g,k}}-\frac{384}{60}\frac{E_{0,g,mean}}{m\cdot f_{m,g,k}}\left(\frac{h}{l}\right)^3=0 \qquad (13)$$

It is requested the minimum β value of the three modes.

4. EXAMPLE

In this example, the β values are determined for three cases of structural failure in terms of the ratio l/h simply supported beam of laminated wood loaded by Fig. 2.

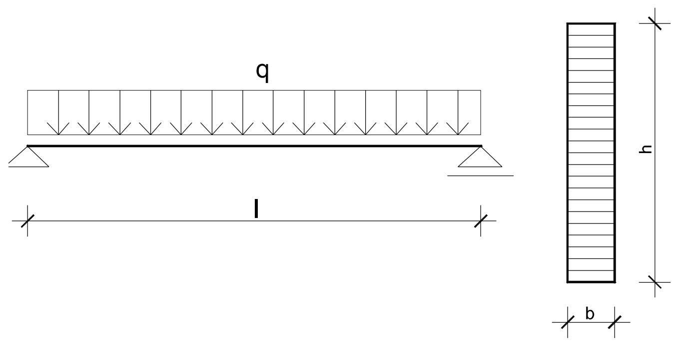

Figure 2. Simply supported beam

MATERIAL CONSTANTS
- characteristic bending strength: $f_{m,g,k}=28N/mm^2$
- characteristic shear strength: $f_{v,g,k}=3,2N/mm^2$

-modulus of elasticity : $E_{0,g,mean}=12600N/mm^2$

The beam corresponds to the GL 28 class, whose characteristics are displayed in the Table 1 in EN 1194, as well as the characteristic strength and stiffness in N/mm^2 and densities in kg/m^3 for homogenous glulam [6].

mode1: Calculation of reliability index β -usage of normal bending stresses

Adopted values [6], For example: $\frac{l}{h}=12$, $l=20m$, $h=1,67m$, $b=0,20m$, $q=20000N/m$

$\beta=6,09$ **- bending stress (the smallest β value)**

$\beta=9,92$ - shear stress

$\beta=6,29$ - maximal deflection

For target value of reliability index β for Class RC2 strucural members, for 50 years, $\beta=3,8$ (ultimate limit states) [1] is obtained the load $q=22850N/m$

mode2: Calculation of reliability index β - usage of shear stress

Adopted values [6], For example: $\frac{l}{h}=8$, $l=20m$, $h=2,50m$, $b=0,20m$, $q=45000N/m$

$\beta=4,67$ - bending stress

$\beta=4,08$ **- shear stress (the smallest β value)**

$\beta=5,42$ - maximal deflection

For target value of reliability index β for Class RC2 strucural members, for 50 years, $\beta=3,8$ (ultimate limit states) [1] is obtained the load is $q=49300N/m$

mode3: Calculation of reliability index β -usage of maximal deflection

Adopted values [6], For example: $\frac{l}{h}=20$, $l=20m$, $h=1,00m$, $b=0,20m$, $q=4000N/m$

$\beta=13,40$ - bending stress

$\beta=31,90$ - shear stress

$\beta=2,94$ **- maximal deflection (the smallest β value)**

For target value of reliability index β for Class RC2 strucural members, for 50 years, $\beta=1,5$ (serviceability limit states) [1] is obtained the load is
is $q=4715N/m$

5. CONCLUSION

The example shows that, depending on the selected parameters of the beam in three modes, a given load q, for each mode corresponds to the value, so that the smallest value is determined by what mode to calculate reliability. Target values of reliability index β are taken [1] for Class RC2 structural members, for 50 years. For these values are calculated corresponding load on the reliability limits.

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Рад је написан у оквиру истраживања на Катедри за материјале и конструкције Грађевинско архитектонског факултета Универзитета у Нишу, под менторством Проф.др Драгослава Стојића

АНАЛИЗА ПОУЗДАНОСТИ ДРВЕНИХ КОНСТРУКЦИЈА

Резиме: *Рад презентује параметарску анализу поузданости савијене греде. Коефицијент поузданости се одређује за случајеве искоришћења пресека греде услед нормалних напона, смичућих напона или услед искоришћења деформација греде. Од ова три услова само је један меродаван, и то онај који даје најмању носивост греде. Параметарска анализа је рађена према захтевима Еврокодова, како у вези анализе напрезања и деформација, тако и у делу захтева који се односе на поузданост конструкција. За одређивање коефицијента поузданости користе се познате методе конструкцијске поузданости аналитички као и применом програма. Циљ анализе је да покаже како се теоријски резултати поузданости савијене греде користе у пракси примене Еврокодова.*

Кључне речи: *Поузданост, дрвене конструкције, Еврокодови*