ON A NONLINEAR MODEL OF A DYNAMIC SHOCK ABSORBER IN BUILDING STRUCTURES

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Summary: *This paper considers the application of a nonlinear model of a dynamic shock absorber in building structures. The considered dynamic shock absorber forms a strained elastic rope with a concentrated mass on it. This dynamic shock absorber is structurally very simple and can be easily installed on various types of structures such as e.g. platforms, metal structures and other structures. The main advantage of such a dynamic shock absorber is the possibility to achieve its essential dynamic characteristics with the prestressing of the elastic rope (cable). The paper analytically and numerically considers the behavior of a model of a given structure that is exposed to different types of dynamic loading.*

Keywords: *dynamic shock absorber, structures.*

1. INTRODUCTION

Dynamic shock absorber or Tuned mass damper (TMD) is one of the ways to reduce unwanted oscillatory movements in engineering systems. A well-developed theory is an advantage of application. It should be emphasized that in terms of a well-known theory, we mean the theory of linear oscillatory systems. Linear theory implies, above all, small displacements, as well as a linear connection in the constitutive relation of the materials from which the structures are made.

In structures that are important for a civil engineer, these conditions are met in a large number of practical cases. The dynamic shock absorber is a special oscillatory subsystem with its dynamic characteristics. As such, in order for the preliminary dynamic analysis of the structure on which the TMD dynamic damper is mounted to be in the linear domain, it is necessary for the TMD itself to be a linear oscillator. This imposes certain restrictions on the designer.

Namely, the goal is that under a given dynamic load to which the basic structure is exposed, the movement of its elements will be as small as possible. This is possible if, on the other hand, the movement of the TMD is also small. As is known, the oscillatory motion of TMD can significantly reduce the amplitude of the oscillatory motion of the elements of the basic structure. In this regard, it is desirable that the movement of the TMD

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be as much as necessary to achieve the desired effect on the basic structure. However, as previously mentioned, the amplitudes of motion of the TMD elements need to be within certain limits in order for the linear theory to be applicable. In linear theory, the mass and stiffness of TMD completely define its dynamic characteristics, i.e. the frequency of external coercion to be amortized. Here are some well-known examples of the application of TMD in building structures, such as Taipei 101 or John Hancock tower.
The application of nonlinear TMD in the basic structure, which is still in the linear field, is increasingly attracting the attention of researchers and engineers. With such application, in addition to mass and stiffness, TMD amplitudes are also important. In this way, greater possibilities and coverage of a larger frequency spectrum are provided.
On the other hand, the area of nonlinear oscillations is still often a great unknown because, unlike the linear one, it is far less developed. Also, what is valid in linear theory will not be valid in nonlinear oscillatory systems. For example, a typical problem is the method of superposition, which is not valid in nonlinear theory.
This paper will present a part of the research that represents a further continuation of the work [1]. The aim is to show the possibility of applying a single TMD, which is a simple construction, to reduce unwanted vibrations in an engineering structure. Using the analytical analysis introduced in [1], a part of extensive numerical experiments is presented here. It will be shown that the nonlinearity that naturally exists in the introduced TMD can have a beneficial effect on the functioning of the TMD.

2. MECHANICAL MODEL

The structure, Figure 1, is considered to be a platform set on four vertical colums. It will be assumed that the total mass of the structure M is concentrated in the horizontal platform itself, while the masses of the columns will be neglected. It is also assumed that the platform is rigid and that the columns are slender and elastic with pre-known characteristics. The vertical elements are clamped to the base. As for the connection of the columns with the platform, two types of connection will be considered: 1. clamping, 2. cylindrical joint.

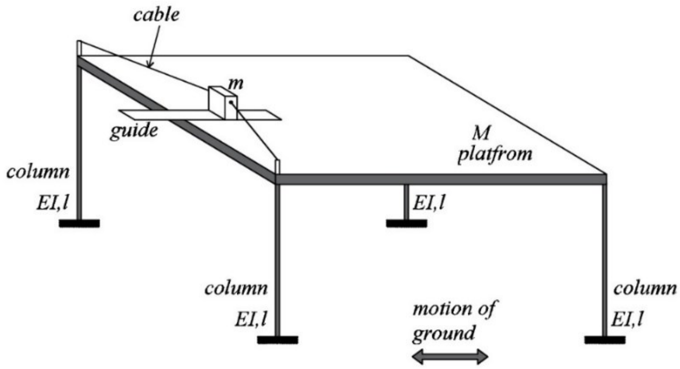

Figure 1.

A TMD is placed on the platform, which consists of a tightened wire (cable) whose ends are attached to the platform itself. The wire is passed through a body of mass m, which

can move along a horizontal guide that is placed on the platform itself. It will be assumed that the dimensions of the body are much smaller than the total length of the cable.
In order to consider the oscillatory motion of the structure from Figure 1, an appropriate mechanical model is introduced, which is shown in Figure 2.

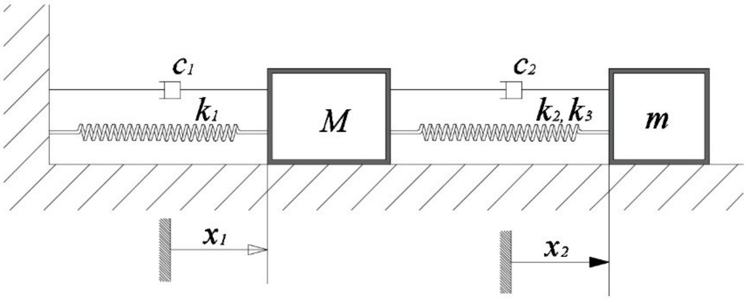

Figure 2.

The presented mechanical model (Figure 2) consists of a discrete system of two concentrated masses so that it is a system of two degrees of freedom of movement (2DOF). The generalized coordinates that define the motion of the masses M and m are denoted by x_1 and x_2 respectively, and they are measured from the stable equilibrium positions of the observed masses. The constant k_1 reflects the rigidity of the elastic vertical elements on which the platform stands. The constants k_2 and k_3 describe the stiffness of a taut cable, where k_2 represents the stiffness of the linear part, while k_3 will appear in the nonlinear part of the assumed model of the restitution force of a taut cable in the form of

$$F_{r2} = k_2 \Delta l_2 + k_3 \Delta l_2^{\alpha} \tag{1}$$

where Δl_2 is the deformation of the second spring in Figure 2, while for degree α holds $\alpha \in \mathbb{R}^+$.
The TMD restitution force model represented by expression (1) was adopted on the basis of experimental research presented in [2]. By adopting suitable values of degree α, it is possible to approximate with the desired accuracy the relationship between force and displacement of a real system. It should be emphasized that model (1) is an extension of the so-called purely nonlinear restitution forces (pure nonlinear), which has occupied the attention of researchers in the previous two decades [3]. For the restitution force of the basic system (platform with elastic columns) the linear form is adopted in accordance with Hooke's law:

$$F_{r1} = k_1 \Delta l_1 \tag{2}$$

where the coefficient k_1 is represented by the expression (3):

$$a)\ k_1 = \frac{48EI}{l^3},\ b)\ k_1 = \frac{12EI}{l^3}. \tag{3}$$

Expression (3a) represents the flexural stiffness of four elastic vertical elements clamped at both ends using an analogy with series-connected springs. In case the vertical elements are clamped at one end and with cylindrical joint at the other end, the corresponding stiffness is given by expression (3b). The reason for taking into account these two methods of bonding is in considering their influence on the dynamic behavior of the structure under a given time-varying load. It is of interest to establish the interrelationship between the stiffness of the basic structure and the stiffness of the TMD and the influence of this connection on the reduction of unwanted vibrations of the basic system.

3. MATHEMATICAL MODEL

The mathematical model of the oscillatory system in Figure 2 consists of a system of two coupled differential equations that can be represented in a concise form with the following matrix equation:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} + [K^*]\{\xi\}^\alpha = \{F(t)\} \tag{4}$$

The following notations are introduced in expression (4): $[M] = \begin{bmatrix} M & 0 \\ 0 & m \end{bmatrix}$ - mass matrix; $[C] = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}$ - damping matrix; $[K] = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$ - stiffness matrix of the linear part of the restitution force; $[K^*] = \begin{bmatrix} k_3 & 0 \\ 0 & k_3 \end{bmatrix}$ - stiffness matrix of the nonlinear part of the restitution force; $\{\ddot{x}\} = \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_1 \end{Bmatrix}$ - acceleration vector; $\{\dot{x}\} = \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}$ - velocity vector, $\{x\} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$ - displacement vector; $\{F(t)\} = \begin{Bmatrix} F_1(t) \\ 0 \end{Bmatrix}$ - vector of external time-varying actions. Value ξ is denoted by $x_1 - x_2$. The damping in the observed system is modeled with viscous friction forces. The time-varying load is adopted to act on the mass M.
The external time-varying load can be, as suggested in [1], modeled as follows:

$$F_1(t) = F_{10} cn^\alpha(\Omega t, \bar{m}) \tag{5}$$

where cn denotes the Jacobi elliptic function whose moduo is $\bar{m}$. Namely, Jacobi elliptic functions are two-parameter functions, which as such enable modeling of different types of loads by choosing suitable values for Ω and parameter $\bar{m}$. It should also be emphasized that, for example, the harmonic functions sin and cos are only special cases of Jacobi elliptic functions, and as such are contained in the mathematical model defined by the expression (5)
As it is known, in case that $[K^*] = 0$, it is a linear system. The basic system (platform with vertical elements) is linear, and its own (natural) circular frequency of undamped oscillations is easily determined as $\omega_n = \sqrt{k_1/M}$. By adding TMD, the system has 2SSK (2DOF), two own circular frequencies, as well as two basic oscillation modes. In the case of linear TMD, a well-known theory allows the determination of the stated quantities.

However, in the case of nonlinear TMD, the analysis is much more complex. Analytical consideration of this problem goes beyond the scope of this paper, so that only the part that arose as a result of extensive numerical analysis will be presented here.

4. NUMERICAL ANALYSIS

A system with the following parameters is observed: $M = 1000kg, m = 90kg, k_1 = 2000\ kN/m,\ k_2 = 50\ kN/m,\ k_3 = 100\ kN/m, \alpha = 2, c_1 = 4\ kNs/m$ and $c_1 = 2\ kNs/m$. In order to consider only forced oscillations, zero initial conditions are adopted. The parameters introduced in this way correspond to a typical construction whose model is shown in Figure 1 for the standard characteristics of vertical elements as well as the two mentioned methods of bonding. In addition, the TMD parameters are consistent with the experimental studies presented in [2].
A harmonic excitation whose amplitude is $F_0 = 10kN$ will be considered. Figure 3 shows the case of resonance of a basic system without TMD, when the excitation frequency Ω is equal to its own circular frequency.

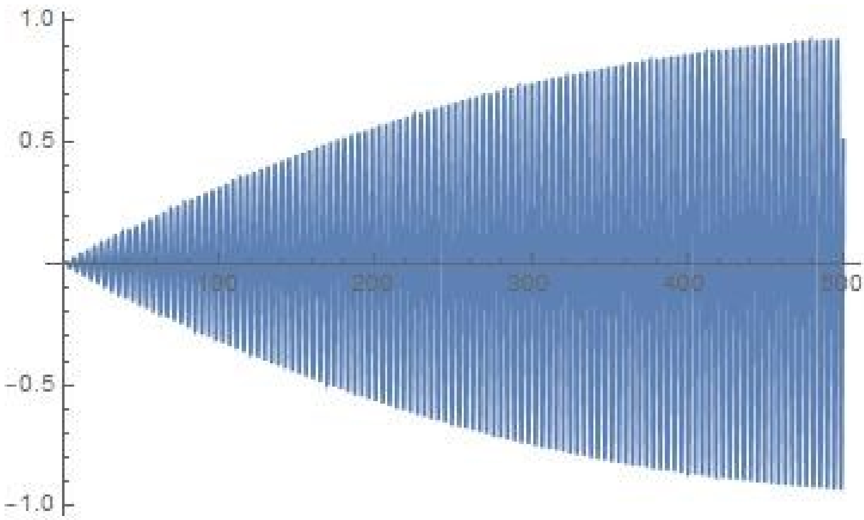

Figure 3. Basic system resonance without TMD

Figure 4 shows the movement of the basic system (defined by the coordinate x_1) when the TMD is placed on it. In this case, only the linear part of the restitution force TMD was taken, i.e., it was assumed to be $k_3 = 0$. Numerical integration was performed for the case of a lower natural circular frequency now of a system with 2SSK. It can be noticed (above enlarged image) that the amplitudes of the basic system are now 60% lower.

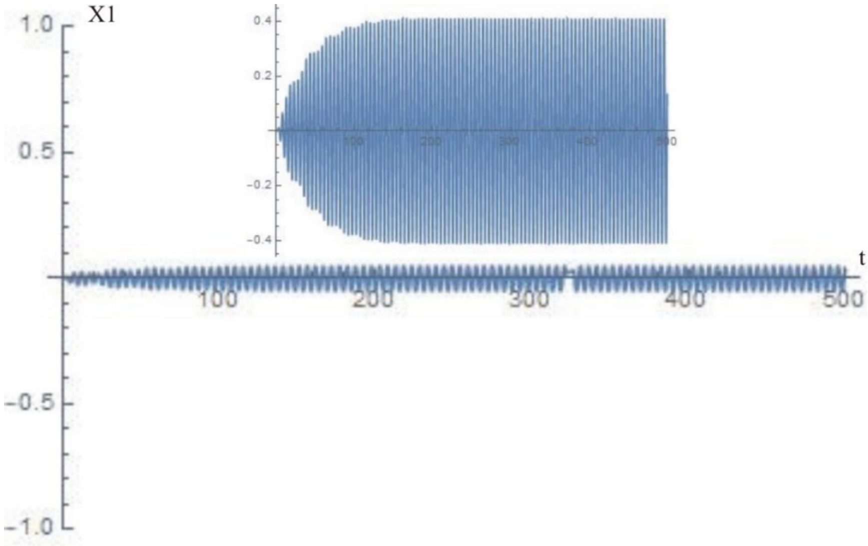

Figure 4. Time history $x_1(t)$ for the case of linear TMD

Figure 5 shows the movement of the basic structure when the nonlinearity of TMD is taken into account and where the nonlinear degree of restitution force is assumed to be 2. It can be noticed that in this case the maximum amplitudes are 10 times smaller.

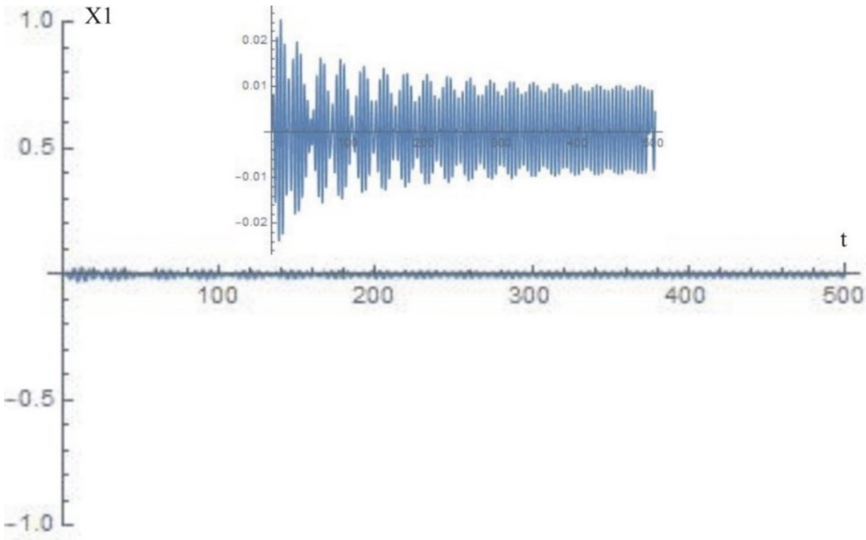

Figure 5. Time history $x_1(t)$ for the case of nonlinear TMD

5. CONCLUSION

The paper presents a case of applying a TMD to reduce unwanted vibrations of a model of a real engineering structure. The introduced TMD is simple construction, but whose dynamic behavior is characterized by pronounced nonlinearity. Numerical experiments indicate that taking into account these nonlinearities can significantly improve the performance of TMD. A significant advantage of this TMD is that with the desired prestressing of the cable, the characteristics of the TMD itself are affected.
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О ЈЕДНОМ НЕЛИНЕАРНОМ МОДЕЛУ ДИНАМИЧКОГ АМОРТИЗЕРА У КОНСТРУКЦИЈАМА

Резиме: *У овом раду се разматра примена нелинеарног модела динамичког амортизера у грађевинским конструкцијама. Разматрани динамички амортизер чини затегнуто еластично уже са концентрисаном масом на њему. Овакав динамички амортизер је конструктивно врло једноставан и може бити лако постављен на разне типове конструкција као што су нп. платформе,металне конструкције и друге структуре. Основна предност оваквог динамичког амортизера је могућност да се његове битне динамичке карактеристике постигну са преднапрезањем еластичне жице (сајле).*
У раду се аналитички и нумерички разматра понашање модела дате структуре која је изложена различитим типовима динамичког оптерећање.

Кључне речи: *динамички амортизер, конструкције*