ESTIMATION OF TEMPERATURE TRANSFER FUNCTION IN FACADE WALL HEAT TRANSPORT

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Summary: *This paper is presenting a method for temperature transfer function (TTF) estimation by filtering an experimentally collected data. The experimental data were obtained in simultaneous measurements of inside and outside temperatures variations during the period of 3 months of a building located in Belgrade, Serbia. The TTF estimation is based on Wiener filtering technique for the dynamic systems with finite impulse response (FIR). TTF is derivate in time and complex domain and the correctness of the acquired transfer function is tested on the new input data set. The estimated TTF in complex domain is used to get decrement factor (DF) and time lag (TL) between the temperatures.*

Keywords:*transfer function, heat transport, decrement factor, time lag*

1. INTRODUCTION

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In building design industry, variable weather conditions strongly influence heating and cooling load calculation, which is a starting point for building energy consumption analysis and equipment sizing [1-3]. Building envelopes are exposed to various weather conditions and they are required to provide a comfortable indoor environment and to help to minimize heating and cooling energy demand, or even almost eliminate the energy needs. How the building envelope reacts when it is subjected to the variable conditions is determined by thermal properties of building envelope composition layers.

In engineering practice, a transfer function (TF) is a convenient representation of a linear time invariant (LTI) dynamical systems and compactly describes the input-output relation of the system. By making an analogy between a building envelope, as a thermal system, and a dynamic system, the building wall can be described through a TF. TF characterizes the relationship in frequent domain between input excitation, which a building component is exposed to, and the output response. In time domain TF is called impulse responce (IR). More specifically, TF is the Fourier transform of Green function which is defined as impulse response of a dynamic system to Dirac delta excitation. Observing the building envelopa, as a LTI dynamical system, since both are described by a similar set of differential equations, heat transfer through a planar multi-layer wall could be analised trought the transfer function of the system where the temperatures at the both sides of the wall are considered as input and output quantities of the system.

The main objective of this study is to present a novel approach for estimating the temperature transfer function (TTF) for multi-layer façade wall using only the air temperatures measurements at discrete time instants nearby the wall surfaces and Wiener filtering technique. In general, a Wiener filter (WF) is an optimal filter, in the mean square error (MSE) sense that maps an input signal to an output that is as close to a desired signal as possible. This procedure does not require any prior knowledge of thermal properties, internal structures or dimensions of the considered building structure. Thus obtained Wiener coefficients are related to the TTF. Additionally, using Discrete Fourier Transform (DFT) the Wiener coefficients are transferred into complex domain to obtain the discrite transfer function (DTF), as well dynamical parameters: time lag (TL) and decrement factor (DF).

This dynamical paramitars are related to propagation of the thermal fluctuations through a wall. The relevant International Standard ISO 13786 [4], which describes the thermal exchange in a steady periodic regime between indoor and outdoor environments, defines these dynamical thermal paramitars and gives a procedure for calculating them. Standard defines DF as a ratio of the modulus of the periodic thermal transmittance to the steady-state thermal transmittance U, and TL is defined as a period of time between the maximum amplitude of a cause and the maximum amplitude of its effect. The Standard calculation procedure define DF and TL as a function of angular friquency and requres prior knowledge of the thermal and physical properties of building composition layers and their thicknesses.

However, a commonly used definition for TL is a difference between the temperature peaks at the outer and inner side of the fasade wall, whereas the DF is a ratio of decrease of the amplitude during the propagating of the temperature variations through a wall. In experimental part of the investigation, authors [5-7] used this simplified definitions where TL and DF have been obtained only from the temperature variations experimentally measured. None of the above reasearches considered a building wall under the real environmental conditions, but in laboratory conditions, and the result themselves apply only to the daily thermal variationa. The analysis of the frequency response in [8] has highlighted how, the dynamic properties of a wall depend on the pulsation, on the amplitude and on the argument of the sinusoidal boundary conditions, which vary with the month and the location. All autors agreed that analysing these factors can be useful in the thermal design of the wall in order to reduce power peaks and energy requirements during summer and winter conditioning.

For the purpose of this study the measurements of the inside and outside near wall air temperatures, under real environmental conditions, have been carried out. Using these quantities as inputs and outputs of the system, the influence of the conditions in the boundary layer and thermal properties of the building envelope are included. The thermal model for a façade wall is adapted as one-input, one-output linear time invariant (LTI) dynamic system that is Single Input and Single Output (SISO) system. The outside air temperature variations are considered as input signal, while the inside air temepratures as the output. FIR Wiener filter, has been applied to obtain TTF in time domain. One half of the measurements is use to estimate the TTF and the second half as a control data set. Thus, one derived and validated TTF can be used for inside air temperature prediction in the next longer period and energy load calculation in order to maintain heat balance in the room.

2. EXPERIMENTAL MEASUREMENTS

USB Temperature/Humidity Data Loggers - EL-USB-2PLS have been used for measurements of the air temperature inside and outside the building (Figure 1). Inside and outside air temperatures were measured at the same moments, every 5 minutes. Response time of the logger to step excitation is 20s and the measuring range is from -35°C to 80°C. The logger is protected against water and dust with a plastic cap and seal. Figure 1. represents the logger and the measurement accuracy of the air temperature according to manufacturer specifications. Within a measuring range of 10°C to 40°C the maximum absolute error of measurement is less than ±1°C.

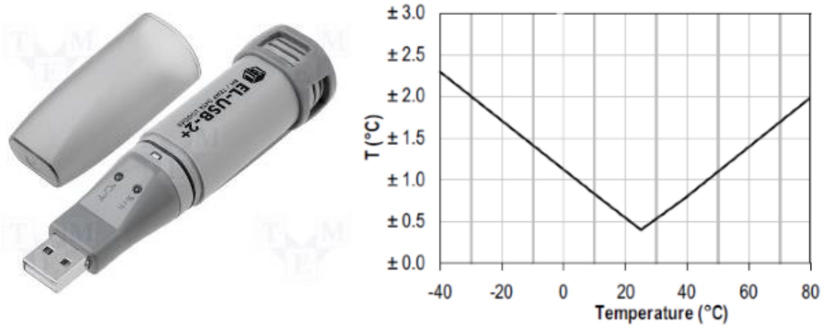


Figure 1. Data logger used for measuring and accurancy according by the manufacturer

Subjected façade wall was multilayered with 5 homogeneous layers. The dimensions and thermal properties of the wall materials are presented in the Table 1. The data from the Table 1 are used only for testing of the acquired results in later section.

Table 1Dimensions and thermal properties of the wall material

Num.	Layer	d[m]	ρ [kg/m3]	c[J/(kg*K)]	λ[W/(m*K)]
1	Interior plaster	0.03	1900	1050	0.99
2	Clay block	0.25	1400	920	0.61
3	Glass wool	0.05	23	840	0.034
4	Solid brick	0.12	1800	920	0.76
5	Exterior plaster	0.02	1900	1050	0.99

Legend: d- Thickness of a layer; ρ - Density of a material; c - Specific heat capacity of a layer; λ - Thermal conductivity

Building wall faces south. Outdoor temperatures are measured in vicinity of the building close to the facade wall, in a place sheltered from direct sunlight. Indoor temperatures were measured by second logger placed near the inside surface of the same facade wall. During the entire course of measurements the heating system has been turned on, while there were no people present and no power consumption in the apartment.

3. PHYSICAL MODEL AND FILTER DESIGN

As it is stated in Table 1, we consider a multi-layer building wall where each layer is homogeneous and has a different thickness and physical properties. The non-stationary heat transfer problem through such structure can be described by Fourier partial differential equation (PDF) with corresponding initial and boundary conditions and matching conditions on interfaces between layers. Our problem is to reconstruct or estimate the TTF of this structure from the dynamical measurements of the external and internal air temperatures on the both sides of the wall: $T_e[n]$, $T_i[n]$, respectively, where $n = 1...L$, and L is a number of measurements ie. signals length. Since our measurements are obtained at discrete time instants all equations below will be showen in discretized notation. The outdoor air temperature is treated as an input of the system, and the indoor air temperature profile as a response. Facilitated by the use of the Furier transform, it is known that for a linear dynamic system, the relation between input and output in the complex variable s domain is given by the multiplication:

$$\hat{T}_{\mathrm{i}}[j\omega] = H[j\omega] \cdot \hat{T}_{\mathrm{e}}[j\omega] \tag{1}$$

where the corresponding complex quantities of temperatures are labeled with "^". From the convolution theorem, in time domain, the output of the system $T_i[k]$, is given by the convolution of the transfer function $h[k]$ and the input signal $T_e[k]$:

$$T_i[n] = h[n] * T_e[n] = \sum_{k=0}^{N} h[k] \cdot T_w[n-k] \tag{2}$$

where the operator $*$, denotes the convolution sum, N is the filter length in terms of time samples of the impulse response. The row vectors $[H(j\omega)]_{1xN+1}$ and $\mathrm{h}[k]_{1xN+1}$ are called the temperature transfer function, TTF, for s and time domain, respecively, and the elements $H_k, k = 0 \dots N$ and h_k, are basically the responces in inside air temperature at the time k, to excitations: $T_e(k), T_e(k-1), T_e(k-2), \dots T_e(k-N)$, at the time $k, k-1, k-2, \dots k-N$. These elements depend on the thermal and physical properties of each layer.

4. ESTIMATION OF THE WIENER FILTER

We propose to estimate the coefficients of the TTF as Wiener filetr coefficients. We choose the Finite Impulse Response (FIR) Wiener filter, which use a finite amount of past data to estimate the coeffifients. The Wiener coefficients are obatined minimizing the mean square error (MSE) between the estimated random process and the desired process. Finaly, the goal is to compute a statistical estimate of an unknown signal by filtering a related known signal and obtaned Wiener coefficients.

Block diagram view of the FIR Wiener filter for discrete series are showen in Figure 2. At the diagram an input signal *x[n]* is convolved with the Wiener coefficients *h[k]* and the result is compared to a reference signal *s[n]* to obtain the filtering error *e[n]*.

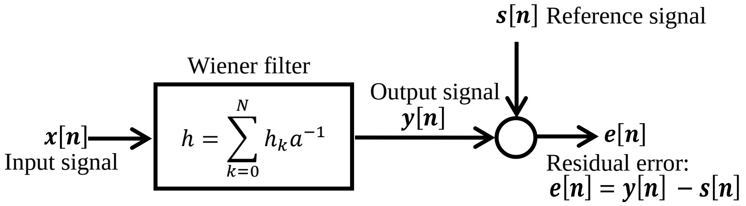


Figure 2. Block diagram view of the FIR Wiener filter for discrete series

The residual error is denoted *e[n]* and is defined as *e[n] = y[n]−s[n]*. The output signal is given by the expression:

$$y[n] = \sum_{k=0}^{N} h_k x[n-k], \qquad k = 0, \dots, N \tag{3}$$

where k is a linear predictor of order N and the coefficients denoted as h_k are Wiener coefficients. The calculation of the Wiener filter coefficients by [9], are given by the expression:

$$h_k = argmin\, E\{e^2[n]\} \tag{4}$$

where *E { }* is an expectation operator. Letting the derivative to be equal to zero:

$$\frac{\partial}{\partial h_k} E\{e^2[n]\} = 0 \tag{5}$$

results in system of equations which can be rewritten in matrix form:

$$\begin{bmatrix} R_x[0] & R_x[1] & \cdots & R_x[N] \\ R_x[1] & R_x[0] & \cdots & R_x[N-1] \\ \vdots & \vdots & \ddots & \vdots \\ R_x[N] & R_x[N-1] & \cdots & R_x[0] \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_N \end{bmatrix} = \begin{bmatrix} R_{sx}[0] \\ R_{sx}[1] \\ \vdots \\ R_{sx}[N] \end{bmatrix} \tag{6}$$

These equations are known as the Wiener–Hopf equations. Coefficients R represent autocorrelation of input signal and cross correlation of input and reference signal.

$$\begin{aligned} R_x[m] &= E\{x[n]x[n+m]\} \\ R_{xs}[m] &= E\{x[n]s[n+m]\} \end{aligned} \tag{7}$$

Notations: *x[n] - input signal*; *y[n] – output signal;* h_k *– Wiener filter coefficients; s[n] – reference signal;* respectively correspondent to the external temperatures - $T_e[n]$; estimated internal temperatures - $\tilde{T}_i[n]$; TTF coefficients - h_k; reference internal temperature - $T_i[n]$.

5. RESULTS AND DISCUSSIONS

The measurements have been taken from november 1st 2016 to febuary 8th 2017, what is more then 3 months ($98{,}78 day \cdot 24h \cdot 60min = 142250min$). The temperature measurements were recorded with a time step of 5 min. Thus, the length of all the measurements is 28450 samples, and the sampling rate is $f_s = \frac{1}{300}[Hz]$, or $f_s = 24 \cdot 60/5\ [1/day]$). The measurements are shown in Figure 3. Amplitudes of indoor temperature were much smaller than outdoor ones.

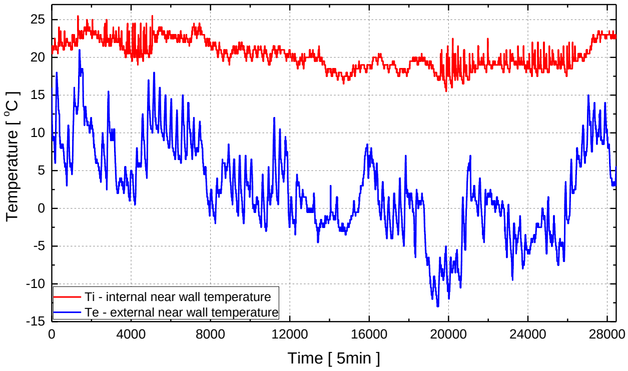
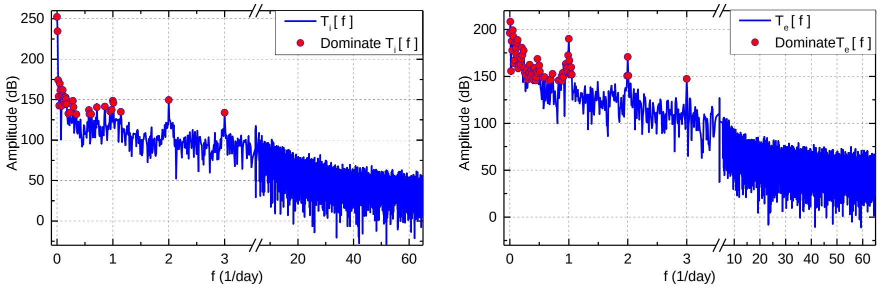


Figure 3. Measured air temperatures in a period of 3 months, with a time step of 5 min

A deeper analysis of the signals stems from the analysis of their DFT spectra, Figure 4. In the entire spectrums, for outside as well for inside temperature, dominate frequency components occur at the frequencies lower than $f = 3\ [1/\text{day}]$. Dominant frequencies (except zero frequency) are denoted with red circles in Figure 4.



a) b)

Figure 4. DFT amplitude spectrum of the measured signals

This analysis is important because TTF in complex domain could be analyzed only at the dominant frequencies of both signals. Otherwise, TTF will not be reliable.

Each measured data set is divided into two equal parts: 14225+14225 samples. First 14225 samples are used to find TTF as WF coefficients. The second 14255 samples are used to estimate $\tilde{T}_i$ from 14226 - 28450 samples using the known external air temperatures T_e, for the same time period and already identified TTF. This analysis showed how well the estimated TTF predict wall response to the new conditions. The number of the TTF coefficients – N (filter order), significantly affects the accuracy of the output signals. Generally, this number determines how many previous samples will be accounted for when performing the estimation. In our case, this means that we need to choose how many previous hours or days in module of 5 min, will have an impact on the current output.

In calculation of coefficients h_k it has been assumed that the maximum filter order should not be larger than 1500 samples what is about $1/20$ of the signal length, which correspondents to the maximum time of 5.2 days. This limit was taken to ensure that reliable auto- and cross-correlations in Eq.(6) can be computed from the measurements, but, based on the previouse reasrches carred out by the same autors, it was expected that the filter order would corespodent to the period of about 1 day (288 samples). The discrepancy between the reference and the estimate signals using already identified Wiener coefficients h_k, for all N, is calculated as Root Mean Square Error (RMSE) between these signals. A graphical representation of the errors is shown in Figure 5.

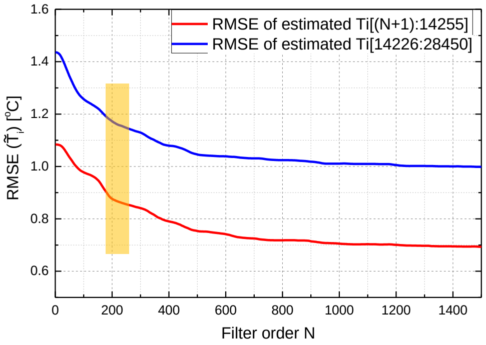


Figure 5. RMSEs for temperature estimations for the filter order ranging from 1 to 1500.

The region where the error becomes stady starts from the filter order $N = 1000$. In this region the error for the first set is about 0.7°C, what is below the level of accuracy of the data logger (see Figure 1.). Taking the filter order in the range from 100 to 200, the error decreses from 1.25°C (second dataset) to 0.9°C (first dataset), what is in the range of the logger accurancy. Thus, the range from 100 to 200 is a proposed range of the filter order and it is highlighted in Figure 5.

Graphical presentation of the obtained Wiener coefficients h_k but only for the filter order $N = 100$ and 150, which bilongs to the proposed range is presented in Figure 6.

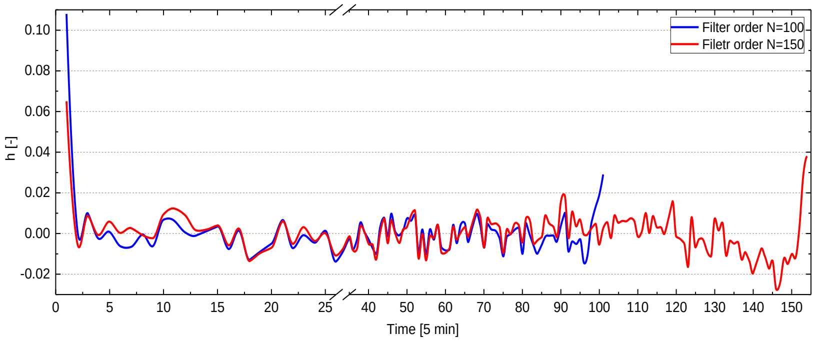


Figure 6. Wiener coefficients h_k. The first 25 coefficients are magnified.

The coeffocients h_k are the responses variation in internal air temperature, from a unit Dirac's temperature impulses imposed at the external side of the wall. Generally, when the amplitudes of the Dirac's impulses are not unit values, these coefficients are response factors with which input impulse must be multiplied. The rate of decline in the coefficients values determinates how the previous inputs impact the current output.

In Figure 7. estimation of the output $\tilde{T}_i$, for the first and the second dataset are presented.

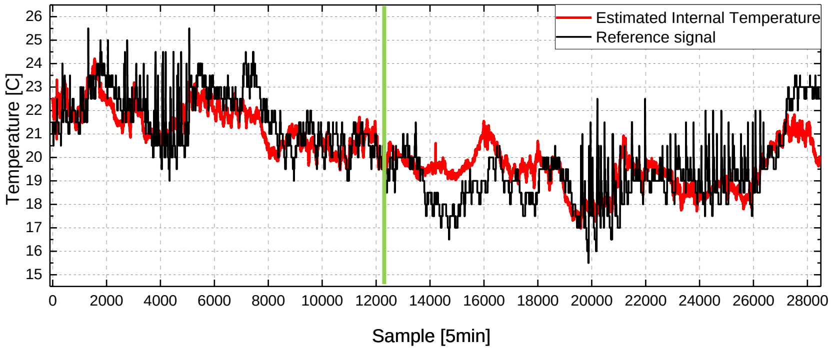


Figure 7. Estimated exterior near-wall air temperature $\tilde{T}_i$

The estimation is obtained by convoluting the coefficients h_k for the filter order $N = 150$. The reference signal at the diagram is presented by the thick black curve. A vertical green line visually separates estimation using the first and the second set of data.

Calculation of the dynamic factors, concerning the definitions of these factors, requires DTF in frequency domain. The DTF H is acquired by applying DFT to the Wiener coefficients, h_k. The ratio $|\hat{T}_{wi}(j\omega)|/|\hat{T}_{we}(j\omega)| = |H(j\omega)|$ from Eq.[1] represents DF between the internal and external air temperatures, while the TL is equal to phase shift between the complex amplitudes of the temperatures divided by ω, what is: $arg\left(\hat{T}_i(j\omega)\right) - arg\left(\hat{T}_e(j\omega)\right) = rg\big(H(j\omega)\big)$. The values of DTF, H, at some dominant angular frequency (Figure 4.), are presented in Table 2. by its module, $|H|$ and argument, $arg(H)$, what correspodent to DF an TL respecively.

Table 2 DF and TL values at the 5 dominante friquences

ω [rad/s]	T [day]	$DF = \lvert H(j\omega)\rvert$ [-]	$TL = arg\big(H(j\omega)\big)$ [h]
0.33	3	0.1702	-9.82
0.67	1.5	0.1292	-10.70
1	1	0.1076	-11.98
2	0.5(12h)	0.0821	-10.79
3	0.3 (8h)	0.3712	-6.89

It is necessary to highlight that derivative DF and TL correspond to the observed period and for the regime when the heating system has been turned on. The sign minus for the TL indicates that the internal air temperature is delayed.
Obtained values in $DF = 0.1076$ and $TL = -11.98h$ at the friquency $2\pi/day$ due to day-night variations, indicate very good dynamic thermal characteristics of the subjected wall in sense of low DF value and very high TL value. These values are also calculated by the procedure defined in the Standard ISO 13786 and for the temperature variations at the friquency $2\pi/day$ they are: $DF = 0.159, TL = -7.159$.
These values is based on the thermal characteristics of the wall layers and their thickness (Table 1). Comparing the values for DF and TL at the friquency $2\pi/day$, calculated by the Standard and as a module and argument of the H, a relatively hight disagreement can be observed. This could be explained by the fact that the Standard does not take into account the influence of the amplitudes and phases of the external and internal temperature harmonics.

6. CONCLUSIONS

In this paper, TTF of the building wall without a prior knowledge of thermal properties and dimensions of the façade wall has been determined using the Wiener coefficients. The coefficients have been calculated in time domain from the experimental data of the air temperatures on both sides of the facade wall.
Further, the coefficients were employed to predict inside air temperature for the next period, from the knowledge of the outside air temperature. RMSE between the predicted and reference signals is 1.22°C.
Obtained TTF was shifted to complex domain appling DFT, and complex coefficients were employed in estimation of the dynamic factors, TL and DF.

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ОДРЕЂИВАЊЕ ТРАНСФЕР ФУНКЦИЈЕ ТЕМПЕРАТУРЕ ПРИ ПРОЛАСКУ ТОПЛОТЕ КРОЗ ФАСАДНИ ЗИД

Резиме: *У раду је приказано одредјивање трансфер функције температуре из података добијених експерименталним мерењем. Подаци су прикупљани истовременим мерењем температуре ваздуха споља и унутар једне стамбене зграде у Београду у периоду од три месеца. Трансфер функција је базирана на моделу динамичког система са једним улазом и једним излазом и добијена је применом Wiener-овог филтера са конацним одзивом. Трансфер функција је одредјена у временском и комплексном домену, и даље тестирана на новом сету података. Из комплексне формефункције директно су одредјени декремент фактор и временско кашњење измедју температура.*

Кључне речи: *трансфер функција, пролаз топлоте, декремент фактор, фазно кашњење*