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THE INFLUENCE OF INITIAL SHRINKAGE ON THE RESPONSE OF TENSIONED ELEMENTS

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Summary: *The concrete shrinkage strain leads to an initial shortening of the tensioned element and this factor must be taken into account in order to evaluate tension stiffening effects properly. The influence of initially developed concrete shrinkage strain on cracking and post-cracking axial force-strain response of tensioned reinforced concrete element is considered. The paper presents developed proposal for including concrete shrinkage strains in an analysis of tension stiffening effects in cracked tensioned reinforced concrete members.*

Keywords: *Tension stiffening, shrinkage strain, reinforced concrete, tensioned members*

1. INTRODUCTION

After cracking concrete in tensioned reinforced concrete element still continues to carry tensile stresses between cracks as a result of bond action, which effectively stiffens the element response and reduces elongation. This phenomenon is called tension stiffening and it plays an important role in assessing serviceability requirements after element cracking. The estimation of tension stiffening, which represents the contribution from concrete to element stiffness after cracking, is influenced by shrinkage and will lead to an underestimation of this value if the initial element shortening caused by shrinkage is not included in the element response calculations.

The shrinkage effect on tension stiffening in tensioned reinforced concrete elements is analyzed in theoretical and experimental study [1], where the simplified procedure for shrinkage account, based on linear elastic analysis, is proposed. The more accurate proposal for shrinkage account, which is based on application of algebraic constitutive relations for concrete, is proposed in reference [2]. That proposal is insignificantly complex related to proposal [1] and it is almoust equally appropriate for practical application. The influence of shrinkage in previous period of time on tensioned element cracking force at observed moment of time is detaily analized in [2] where these type of analysis is illustrated by numerical example. After application of corrections, whose are necessary to take into account shrinkage effects, tension stiffening effects in post

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cracking tensioned element response my be analyzed applying some of relevant code proposals [3] and [4], or applying some of actual researcher's proposals.

2. THE APPROACH FOR TAKING INTO ACCOUNT SHRINKAGE

Concrete can experience significant amounts of shrinkage before loading and even moist cured concrete will shrink to some extent. Shrinkage of the concrete leads to an initial element shortening and this factor must be taken into account in order to evaluate cracking force, tension stiffening effects and crack widths properly.

2.1 Uncracked tensioned element response before element loading

The uncracked element total strain after element shrinkage in period of time (t_0, t), which is equal to element strain change continuously developed during considered period of time, applying the algebraic constitutive relations for concrete is obtained as:

$$\varepsilon(t) = \Delta\varepsilon(t, t_0) = \frac{E_c^*(t, t_0) A_c}{E_c^*(t, t_0) A_c + E_s A_s} \varepsilon_{sh}(t, t_0) = \frac{1}{1 + n_s^* \mu_s} \varepsilon_{sh}(t, t_0) \tag{1}$$

where: $E_c^*(t, t_o)$ is age-adjusted effective modulus of concrete, $\varepsilon_{sh}(t, t_o)$ is unrestrained concrete shrinkage strain in period of time (t_0, t), n_s^* is corresponding modulus ratio $E_s / E_c^*(t, t_o)$ and μ_s is steel reinforcement ratio A_s / A_c.

The total element steel resultant compressive force $N_s(t)$ at the end of observed period of time (t_0, t), which is opposite to the total element concrete resultant tensile force $N_c(t)$, is finally obtained as:

$$N_s(t) = \Delta N_s(t, t_0) = -N_c(t) = -\Delta N_c(t, t_0) = E_s A_s \varepsilon_{sh}(t, t_0) \frac{1}{1 + n_s^* \mu_s} \tag{2}$$

The total steel and concrete forces, $N_s(t)$ and $N_c(t)$, are equal to steel and concrete force changes, $\Delta N_s(t, t_0)$ and $\Delta N_c(t, t_0)$, which are continuously developed during period of time (t_0, t).

2.2 Unstressed state of the concrete part of tensioned element cross-section

The fictitious element instantaneous elastic strain change at moment t which is necessary to bring previously tensioned concrete part of the cross-section in state of zero concrete stress is equal to:

$$\Delta\varepsilon_d(t) = -\frac{N_c(t)}{E_c(t) A_c} = \frac{E_c^*(t, t_0)}{E_c(t)} \varepsilon_{sh}(t, t_0) \frac{n_s^* \mu_s}{1 + n_s^* \mu_s} = \varepsilon_{sh}(t, t_0) \frac{n_s \mu_s}{1 + n_s^* \mu_s} \tag{3}$$

where: $E_c(t)$ is concrete modulus of elasticity at moment *t*, and n_s is steel to concrete modulus ratio $E_s / E_c(t)$. This fictitious state is usually called "fully decompression" state of concrete part of the element cross section, because previously developed stresses that need to be unstressed are usually compressive stresses. Here it is not so.
The fictitious instantaneus concrete and steel resultant force changes, $\Delta N_{c,d}(t)$ and $\Delta N_{s,d}(t)$, which corresponds to fictitious elastic strain change $\Delta\varepsilon_d(t)$, are equal to:

$$\Delta N_{c,d}(t) = -N_c(t) \tag{4}$$

$$\Delta N_{s,d}(t) = E_s A_s \Delta\varepsilon_d(t) \tag{5}$$

The fictitious concrete resultant force change $\Delta N_{c,d}(t)$ and the fictitious steel resultant force change $\Delta N_{s,d}(t)$ are in treated case compressive forces.
The fictitious force change $\Delta N_d(t)$ relating to fictitious elastic strain change $\Delta\varepsilon_d(t)$ is:

$$\Delta N_d(t) = \Delta N_{c,d}(t) + \Delta N_{s,d}(t) = E_s A_s \varepsilon_{sh}(t,t_0)\frac{1+n_s\mu_s}{1+n_s^*\mu_s} \tag{6}$$

The total strain in element cross-section at moment *t*, after fictive elastic unstressing of concrete part of section, is equal to sum of previously developed strain $\varepsilon(t)$ during time period of element shrinkage and fictitious elastic strain change $\Delta\varepsilon_d(t)$:

$$\varepsilon_d(t) = \varepsilon(t) + \Delta\varepsilon_d(t) = \varepsilon_{sh}(t,t_0)\frac{1+n_s\mu_s}{1+n_s^*\mu_s} \tag{7}$$

This total strain is ussually called "fully decompression" strain. At this fictitious section strain state the corresponding resultant force of total stresses in concrete part of section is equal to zero. The term "fully unstressed" strain is more precise.

$$N_{c,d}(t) = N_c(t) + \Delta N_{c,d}(t) = 0 \tag{8}$$

At fully unstressed state of concrete part of the tensioned element cross section the resultant force of total stresses in steel part of section is equal to sum of previously developed steel resultant force $\Delta N_s(t)$, during time period of element shrinkage (t_0, t), and fictitious elastic steel resultant force change $\Delta N_{s,d}(t)$:

$$N_{s,d}(t) = N_s(t) + \Delta N_{s,d}(t) = E_s A_s \varepsilon_{sh}(t,t_0)\frac{1+n_s\mu_s}{1+n_s^*\mu_s} \tag{9}$$

The total force applied on the cross section at fully unstressed state of the section concrete part is equal to total force carried by steel at this state:

$$N_d(t) = N_{c,d}(t) + N_{s,d}(t) = E_s A_s \varepsilon_{sh}(t,t_0)\frac{1+n_s\mu_s}{1+n_s^*\mu_s} \tag{10}$$

because element is not externally loaded during preceding period of time.

2.3 The cracking state of tensioned element cross section

The fictitious element instantaneous elastic strain change at moment t, which is necessary to bring previously unstressed concrete part of the cross-section in state immediately before concrete cracking, i.e. in state when concrete stress is equal his tensile strength $f_{ct}(t)$, is equal:

$$\Delta\varepsilon_{cr}^{d}(t)=\Delta\varepsilon_{c}^{d},_{cr}(t)=\frac{f_{ct}(t)}{E_c(t)}=\Delta\varepsilon_{s}^{d},_{cr}(t) \tag{11}$$

The fictitious force change $\Delta N_{cr}^{d}(t)$, which correspond to elastic element strain change $\Delta\varepsilon_{cr}^{d}(t)$ is equal:

$$\Delta N_{cr}^{d}(t)=[E_c(t)A_c+E_sA_s]\Delta\varepsilon_{cr}^{d}(t)=f_{ct}(t)A_c(1+n_s\mu_s) \tag{12}$$

The fictitious concrete and steel resultant force changes, which are related to considered elastic strain change $\Delta\varepsilon_{cr}^{d}(t)$, are equal to:

$$\Delta N_{c,cr}^{d}(t)=E_c(t)A_c\Delta\varepsilon_{cr}^{d}(t)=f_{ct}(t)A_c \tag{13}$$

$$\Delta N_{s,cr}^{d}(t)=E_sA_s\Delta\varepsilon_{cr}^{d}(t)=n_s\mu_sf_{ct}(t)A_c \tag{14}$$

The total strain in element cross-section at moment t, immediately before concrete cracking is equal to sum of previously developed unstressing strain and fictitious elastic strain change:

$$\varepsilon_{cr}(t)=\Delta\varepsilon_{cr}^{d}(t)+\varepsilon_d(t)=\Delta\varepsilon_{cr}^{d}(t)\left[1+\frac{1+n_s\mu_s}{1+n_s^*\mu_s}\frac{\varepsilon_{sh}(t,t_0)}{\Delta\varepsilon_{cr}^{d}(t)}\right] \tag{15}$$

The total force applied on the section immediately before cracking is equal:

$$N_{cr}(t)=\Delta N_{cr}(t)=\Delta N_{cr}^{d}(t)+\Delta N_d(t)=f_{ct}(t)A_c(1+n_s\mu_s)\left[1+\frac{n_s\mu_s}{1+n_s^*\mu_s}\frac{\varepsilon_{sh}(t,t_0)}{\Delta\varepsilon_{cr}^{d}(t)}\right] \tag{16}$$

The concrete and steel resultant force changes are equal to:

$$\Delta N_{c,cr}(t)=\Delta N_{c,cr}^{d}(t)+\Delta N_{c,d}(t)=f_{ct}(t)A_c\left[1+\frac{n_s\mu_s}{1+n_s^*\mu_s}\frac{\varepsilon_{sh}(t,t_0)}{\Delta\varepsilon_{cr}^{d}(t)}\right] \tag{17}$$

$$\Delta N_{s,cr}(t)=\Delta N_{s,cr}^{d}(t)+\Delta N_{s,d}(t)=f_{ct}(t)A_cn_s\mu_s\left[1+\frac{n_s\mu_s}{1+n_s^*\mu_s}\frac{\varepsilon_{sh}(t,t_0)}{\Delta\varepsilon_{cr}^{d}(t)}\right] \tag{18}$$

The total concrete and steel resultant forces related to cracking at moment t are:

$$N_{c,cr}(t)=\Delta N_{c,cr}(t)+N_c(t)=f_{ct}(t)A_c \tag{19}$$

$$N_{s,cr}(t)=\Delta N_{s,cr}(t)+N_s(t)=f_{ct}(t)A_cn_s\mu_s\left[1+\frac{1+n_s\mu_s}{1+n_s^*\mu_s}\frac{\varepsilon_{sh}(t,t_0)}{\Delta\varepsilon_{cr}^{d}(t)}\right] \tag{20}$$

The effects of an initial element shortening are shown on Fig. 1, indicating that element cracks at a lower external load, since the concrete is under an initial tensile stress:

$$\sigma_c(t)=\frac{\Delta N_c(t,t_0)}{A_c}=-E_c^*(t,t_0)\varepsilon_{sh}(t,t_0)\frac{n_s^*\mu_s}{1+n_s^*\mu_s} \tag{21}$$

The reduced external cracking force at moment *t*, can be expressed as part of element cracking force for the case when concrete shrinkage is ignored:

$$\Delta N_{cr}(t) = \Delta N_{cr}^{d}(t)\left[1 + \frac{n_s \mu_s}{1 + n_s^* \mu_s} \frac{\varepsilon_{sh}(t,t_0)}{\Delta \varepsilon_{cr}^{d}(t)}\right] = \Delta N_{cr}^{d}(t)\left[1 - \frac{\sigma_c(t)}{f_{ct}(t)}\right] \tag{22}$$

2.4 Post cracking response and tension stiffening effects of tensioned element

The point that has coordinates $(\varepsilon_d(t), N_d(t))$ is idealized origin of the new coordinate system with abscissa $\Delta\varepsilon^d(t)$ and ordinate $\Delta N^d(t)$ (Fig. 1). The coordinates of the idealized origin are obtained after application of the equations (7) and (10) and depends on the concrete creep and shrinkage characteristics in observed period of time (t_0, t) and on steel reinforcement coefficient μ_s. The initial concrete modulus of elasticity $E_c(t_o)$, the concrete creep coefficient $\varphi(t,t_o)$, the concrete unrestrained (free) shrinkage strain $\varepsilon_{sh}(t,t_o)$ and the concrete aging coefficient $\chi(t,t_o)$ have substantial influence.

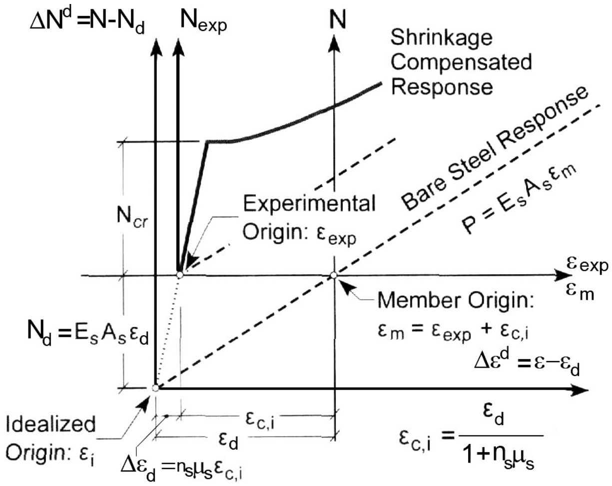

Figure 1. The effect of initial shrinkage on tensioned member response (ref. [2])

The new coordinates are defined as:

$$\Delta\varepsilon^d(t) = \varepsilon(t) - \varepsilon_d(t) = \Delta\varepsilon(t) - \Delta\varepsilon_d(t) \tag{23}$$

$$\Delta N^d(t) = N(t) - N_d(t) = \Delta N(t) - \Delta N_d(t) \tag{24}$$

Now, tensioned reinforced concrete member response under subsequent loading can be idealized as general instantaneous member response at moment *t* (see Fig. 2).

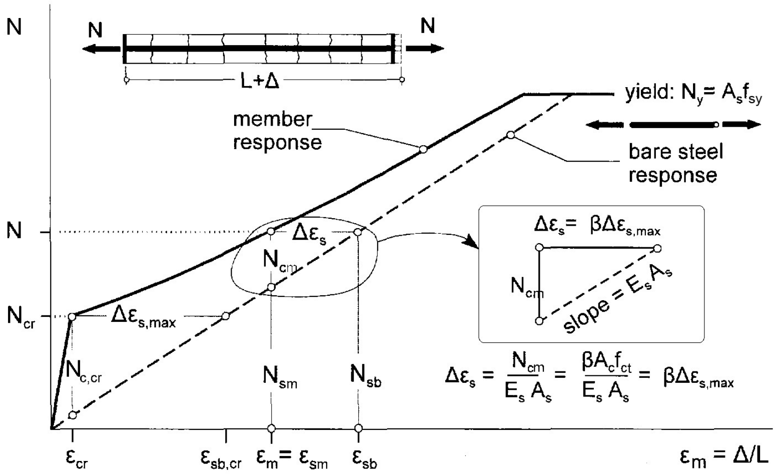

Figure 2. General instantaneus response of a reinforced concrete tension element before and after cracking without initial shrinkage (ref. [2])

3. CONCLUSIONS

The level of load that causes cracking of tensioned reinforced concrete element and post cracking tension stiffening response are affected significantly by shrinkage in previous period of time. The element behavior depends on the amount of time dependent developed concrete shrinkage and creep strain, and on reinforcement percentage.
Determining of element cross section dilatation, for which virtually complete unstressing of concrete part is realised, and her corresponding external axial force the process of inclusion of the initial shrinkage effects, in the period of time preceding load application, is performed. After moving to a new coordinate system the analysis of tension stiffening effects in post-cracking member response is implemented in a standard way.

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УТИЦАЈ ИНИЦИЈАЛНОГ СКУПЉАЊА НА ОДГОВОР ЗАТЕГНУТИХ ЕЛЕМЕНАТА

Резиме: *Дилатација скупљања бетона изазива иницијално скраћење затегнутог елемента и ова појава се мора узети у обзир приликом коректне процене повећања крутости елемента услед доприноса затегнутог бетона на деловима елемента између прслина. Разматран је утицај иницијалне дилатације скупљања бетона на појаву прслина и на зависност нормална сила-дилатација након појаве прслина код затегнутог армиранобетонског елемента. У раду је приказан унапређен предлог за обухватање иницијалних дилатација скупљања бетона у анализи повећања крутости испрскалих затегнутих армиранобетонских елемената.*

Кључне речи: *Повећање крутости при затезању, дилатација скупљања, армирани бетон, затегнути елемент*